BOOK REVIEWS

BONSALL, F. F. AND DUNCAN, J., *Numerical Ranges II* (Cambridge University Press, 1973), vii + 179 pp., £2·40.

The notion of numerical range for operators on a Banach space or elements of a Banach algebra has been intensely studied during the last decade and much additional work was stimulated by the authors' first book on the subject. In the present volume the authors report on some of this work and begin with the improvement of the Bishop-Phelps theorem on support functionals due to Bollobás and on its application to the spatial numerical range. A result of Zenger that the spatial numerical range contains the convex hull of the point spectrum is also proved along with a related result of Crabb and Sinclair.

Algebra numerical ranges are considered in the next chapter with mapping theorems, inequalities, and extremal algebras for these inequalities being touched upon first. Results on hermitian elements of Banach algebras and spectral operators are also obtained along with a presentation of Harris' elementary proof that the closed convex hull of the unitary elements in a unital B^*-algebra is the set of contractive elements.

Lastly, the authors describe some work on essential numerical ranges, joint numerical ranges, and matrix-valued numerical ranges. The main thrust of most of this work concerns operators on Hilbert space. The authors conclude with some proposed axioms for the notion of numerical range for operators. The book is quite readable, despite the technical nature of some of the material, covers many interesting topics including many omitted in the above description, and is highly recommended to all mathematicians interested in operator theory or the theory of Banach algebras.

R. G. DOUGLAS

GREENSPAN, D., *Discrete Models* (Addison-Wesley, 1973), 181 pp., £8·80.

The subject matter of this book is perched, somewhat precariously perhaps, between the mathematical analysis demanded of the physical applied mathematician and the finite difference methods of the numerical analyst. It is unquestionably of interest to workers in both fields. As indicated by its title it is concerned with discrete mathematics. However, in spite of the physical fields from which it draws its examples, it most emphatically does not accept the conventional viewpoint that discretisation is basically an approximation, more or less accurate according to the care taken and efforts made, to some " exact " expression of physical laws in continuum language. By contrast, it argues right from the outset that, although the usual approach is to postulate a continuous model from discrete data and then (frequently) to approximate to the differential equations inherent in this model by first differences, the intermediate continuous model can in many cases be replaced by a discrete model and that this intermediate intrusion of continuum language is logically inconsistent. Accordingly, laws governing problems in particle, continuum and even relativistic mechanics are formulated in terms of their applicability at a finite number of space and time points.

Such an approach has both advantages and disadvantages. The advantages include the shrugging off of the concept of approximations and errors—what is laid down are discrete *laws* and within the axioms the mathematical development is self-contained and exact. Even the concept of stability is rigorously defined in Definition 1·7—rigorously within the axioms, that is—but the purist may wince at a formal definition of stability in which it is demanded that the dependent variable is nowhere " in absolute