THE MOTIONS OF URANUS' SATELLITES: THEORY AND APPLICATION

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As spacecraft and sophisticated ground-based observations measure physical properties of many planets and satellites, dynamical theory and astrometry remain a principal source of such knowledge of the Uranian system. Study of the motions of Uranus' satellites thus has broad application to planetary studies as well as to celestial mechanics. Moreover, the structure and dynamics of the system provide important cosmogonical constraints; any theory of solar system origin and evolution must account for the formation within it of analogous systems of regular satellites.

The five known satellites of the Uranian system have nearly circular, regularly spaced, coplanar orbits. To remain coplanar, they must lie near Uranus' equatorial plane, consistent with spectroscopic measurements of Uranus' rotation, which show rotation in the same direction as the satellites' motion. Small, irregular satellites, like Jupiter and Saturn's, have yet to be discovered. Despite the internal regularity of the known system, it has a highly irregular, 98° obliquity, the origin of which is an important question for dynamical astronomy.

The rings of Uranus, recently discovered interior to the other satellites' orbits (Elliot et al. 1977), have properties which raise interesting dynamical problems; they are apparently narrow, in contrast to the broad rings of Saturn, and in conflict with theories of collisional evolution (Goldreich and Nicholson 1977); they are not all circularly symmetric, again in conflict with theory; and, since they are not visible in reflected solar light, the ring material must be very dark (albedo < 5%)(Smith 1977), surprising for outer solar system material. Moreover, dynamical constraints on masses strongly suggest that Miranda and Ariel are icy in contrast to the nearby ring material.

Masses are determined from satellites' mutual perturbations. One diagnostic effect is precession. Dunham (1971) found Titania's precession rate to be $2^\circ\!.9 \pm 1^\circ\!.5$/yr. Assuming this behavior to be dominated by Oberon (UIV), he found the latter's mass to be $(0.8 \pm 0.6) \times 10^{-4}$ (herein Uranus' mass $\equiv 1$). Dunham's precession rate for Oberon is too

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imprecise for any useful mass constraint.

Other mutual perturbations are the enhanced variations in longitude due to the near-commensurability amongst the inner three satellites, Miranda (UV), Ariel (UI) and Umbriel (UII). Their mean motions nearly obey the Laplace relation, $n_V - 3n_I + 2n_{II} = 0$. The relation implies that the combination of orbital longitudes $\Theta \equiv \lambda_V - 3\lambda_I + 2\lambda_{II}$ varies slowly. Θ can be interpreted geometrically as the angle between the longitudes of UV and UI when UI and UII are in conjunction (i.e. when $\lambda_I = \lambda_{II}$). Θ circulates through 360° in 12.5 yr, slow compared with orbital periods, so geometrical configurations of the three satellites repeat periodically, enhancing perturbations. Earlier workers (Harris 1949, Dunham 1971) assumed such effects were negligible, because the relation amongst mean motions is only approximate, not exact as for three Galilean satellites. As I shall show, their assumption was not justified a priori for plausible satellite masses.

Analysis of the Laplace relation is more complicated than common two-satellite commensurabilities; the critical argument, Θ, does not appear in the Fourier expanded disburbing function. Only when the perturbation theory is extended to second order in satellite masses do combinations of terms appear which have Θ, or multiples of Θ, as arguments of sines and cosines. Then, but not before, other terms with short-periods can be neglected to study the effects of the commensurability. In the Galilean case, first order terms with arguments $2\lambda_2 - \lambda_1$ and $2\lambda_3 - \lambda_2$ have long periods. They dominate and thus simplify the theory [cf. Professor Hagihara's (1972) lucid account of Souillart's theory].

In the case of the Uranian satellites, many other first-order terms have significant contributions to the second-order long-period terms. Development of the theory is a much more tedious procedure. The first partial treatment was made by Sinclair (1975). I have described in previous publications (Greenberg 1975a, 1976) an approach to the theory that uses some numerical shortcuts. Applying my methods, I find

$$dn_V/dt = \mu_I \mu_{II} n_V^2 \left[-11.5 \sin\Theta + 5.8 \sin 2\Theta + 1.1 \sin 3\Theta + \ldots\right]$$

$$dn_I/dt = \mu_V \mu_{II} n_I^2 \left[83.9 \sin\Theta - 42.5 \sin 2\Theta - 8.3 \sin 3\Theta + \ldots\right]$$

$$dn_{II}/dt = \mu_V \mu_I n_{II}^2 \left[-274 \sin\Theta + 36.2 \sin 2\Theta + 9.9 \sin 3\Theta + \ldots\right]$$

where μ's are satellites' masses. Based on known visual magnitudes, and the assumption of albedos and densities identical to Oberon's, Dunham estimated $\mu_V = 3 \times 10^{-6}$, $\mu_I = 6 \times 10^{-5}$, and $\mu_{II} = 2 \times 10^{-5}$. For these values, and the known behavior of Θ, integration gives the following amplitudes of longitude variations: $\Delta\lambda_V \sim 15^\circ$, $\Delta\lambda_I \sim 0\overset{\circ}{.}9$, $\Delta\lambda_{II} \sim 3\overset{\circ}{.}5$. In fact, the observed amplitudes are $< 5^\circ$ for UV (Greenberg 1976) and $< 0.1^\circ$ for UI and UII (Dunham 1971). From these we derive the following limits on mass products: $\mu_I \mu_{II} \lesssim 10^{-9}$, $\mu_V \mu_I \lesssim 5 \times 10^{-12}$ and $\mu_V \mu_{II} \lesssim 6 \times 10^{-12}$.

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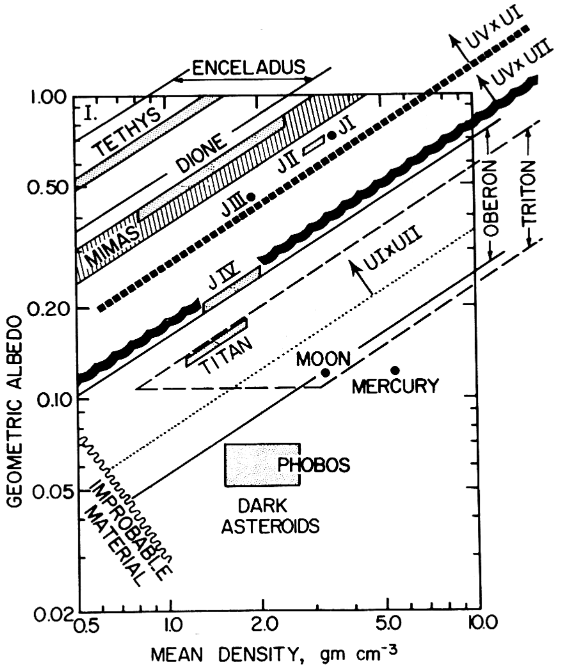

Figure 1. Density vs. Albedo plot showing limits for Uranian satellites and other planetary bodies.

These limits combined with visual magnitudes, place constraints on bulk physical properties of the satellites. Consider a plot of density versus albedo (Figure 1) on which a body's known mass and magnitude define a line of slope 2/3. For comparison limits for other planets and satellites are shown. For UV and UI the limit is shown assuming each has the same density and albedo. One might lie below this limit, but the other must then lie at least as far above it to satisfy the mass product constraint. Similar limits are shown for the pairs UI-UII and UV-UII. These limits support Dunham's assumption that Titania's precession is dominated by Oberon, whose boundaries are also shown. UI and UII are not both dark or carbonaceous material, although one of them might be. Also UV or UI must lie in the region of the plot suggestive of icy material. Although the region admits bright-rock-like albedos, this would require a low bulk density such as that of ice. Conversely, rock-like densities would imply high albedos such as a surface of ice or of evaporite salts, as has been suggested for Io (Fanale et al. 1977). The inner satellites are much brighter material than the rings and significantly brighter and/or less dense than Oberon.

Another application of Uranian Satellite dynamics is in the determination of Uranus' oblateness, a basic constraint on interior structure models. Measurements of the optical oblateness, ε, have been made by Dollfus (1970) and by Danielson et al. (1972), who found .03 ± .008 and

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.01 ± .01, respectively. From the apsidal precession rate of Ariel's nearly circular orbit, based on the apparent orientation of the apsides at two epochs. Dunham found J_2 = 0.012. From the apsidal and nodal precession of Miranda, Whitaker and Greenberg (1973) obtained a lower value, J_2 = 0.005. For a uniform fluid model and the long-accepted rotation period 10.8 hours (Moore and Menzel 1930), both values of J_2 correspond to ε values larger than observed optically. Greenberg (1975b) suggested that the rotation should be remeasured using modern equipment. Recent measurements show the period to be 24 ± 3 hr (Hayes and Belton 1977, Trafton 1977). Thus Dunham's J_2 corresponds to $\varepsilon \approx 0.025$ and Whitaker and Greenberg's to $\varepsilon \approx 0.013$, both in agreement with optical values. However, Podolak (1976) suggested that Dunham's J_2 is not consistent with realistic interior models, while J_2 = .005 is quite plausible.

Whitaker and Greenberg did note explicitly that their determination depended on the assumption that the Laplace relation had negligible effects on Miranda's longitude. Yet we have seen that the Laplace relation may be much stronger than previously thought. The next step will be to fit the complete theory of motion to the full set of observations, including the significant data base acquired in the last few years. It should be possible to separate the effects of planetary oblateness and satellite masses and to solve for these parameters. A great deal remains to be learned about the physical properties of the Uranian system by the methods of celestial mechanics.

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