BASIC PROPERTIES OF SWING-EXCITATION MECHANISM

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Abstract. We describe the basic process of coupling between dynamo and density waves in galaxies. The growth rate of magnetic field as a result of coupling is derived, applying the method of multiple time scales to the marginal state of disk dynamo. It is shown that the 1st-order resonance in a perturbation of density $\tilde{\sigma}/\sigma_0$, and thus the linear *swing excitation*, is possible. Moreover, the growth rate of magnetic field is always positive and does not depend on the initial phase difference between the magnetic and density waves. Both the numerical and analytical calculations show that $\omega = 2\omega_0$ (ω: *density-wave freq.*, ω_0: *dynamo freq.*) is still the best condition for resonance due to the linear effect of swing excitation.

Key words: Galactic Magnetic Fields – Spiral Arms – Resonance

1. Introduction

Galactic dynamos have been developed to explain the magnetic fields in nearby galaxies (Parker 1979, Ruzmaikin *et al.* 1988). However, the standard models have some serious problems, especially to explain the existence of bisymmetric magnetic field structure (BSS field). Chiba & Tosa (1990) (CT) proposed the role of velocity disturbance by spiral density waves; if the density wave disturbs the dynamo in such a way that the frequency of disturbance ω is twice of the dynamo frequency ω_0, the magnetic field grows via parametric resonance, or so called Swing Excitation mechanism (see also Hanasz *et al.* 1991). Recently, however, Schmitt & Rüdiger (1992) (SR) criticized CT suggesting that the case $\omega = \gamma\omega_0$ with $\gamma = 2$ does not play a special role in the field evolution.

In this contribution, we demonstrate that the argument in SR does not work to deny our former results (Hanasz & Chiba 1993); the fundamental state of magnetic fields adopted in SR (the marginal state of dynamo) is represented as only one dynamo wave, which is quite different from that in CT. This is the main reason they could not obtain the parametric resonance in their model. We show that considering the basic behaviors of disk dynamos, the magnetic field is powered by the density wave, especially by the wave with frequency $2\omega_0$.

2. Analytical approach for the coupling problem

2.1. Marginal state of disk dynamo

The dynamo equations disturbed by density waves are derived by CT. The phase of disturbance χ is given as $\chi = \omega t + \phi_0$. The basic equations are

$$\frac{\partial A}{\partial t} = \tilde{\alpha} B + \frac{\partial^2 A}{\partial z^2}, \tag{1}$$

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$$\frac{\partial B}{\partial t} = -D\frac{\partial A}{\partial z} + \frac{\partial^2 B}{\partial z^2} + \varepsilon\left(-R_\alpha f_1 \frac{\partial A}{\partial z}\cos\chi + f_2 B \sin\chi\right), \tag{2}$$

where A and B denote poloidal and toroidal magnetic fields, respectively. R_α and D are parameters for α-effect and $\alpha\omega$-dynamo, respectively, and $\tilde{\alpha}$ the vertical dependence of α-effect. We have extracted the smallness parameter $\varepsilon = \tilde{\sigma}/\sigma_0$, the perturbation of gas density, from f_1 and f_2 in CT, which are now defined as, $f_1 = \kappa^2/2\Omega$, and $f_2 = -\omega$. The analytical solutions of equs.(1) and (2) for $\varepsilon = 0$ are available when α is a step function, $\tilde{\alpha} = 1$ for $0 < z < 1$ and $\tilde{\alpha} = -1$ for $-1 < z < 0$ (Parker 1979). In this case, the internal solutions within a disk can be represented as the superposition of 4 modes;

$$\mathbf{B} = \mathrm{Re}\left(\sum_{n=1}^{4} \tilde{\mathbf{B}}_n \exp(\lambda t)\exp(-ik_n z)\right), \tag{3}$$

because the dispersion relation derived from equs.(1) and (2) is 4th order in the wavenumber k of magnetic fields, $(k^2+\lambda)^2 - i\tilde{\alpha}Dk = 0$. Parker (1979) obtained the phase velocities of 4 modes when the dynamo is marginal ($\mathrm{Re}\lambda = 0$ and $\omega_0 \equiv \mathrm{Im}\lambda \neq 0$). The 4 roots of dispersion relation satisfy $\sum k_n = 0$, therefore the components propagate with both the positive and negative phase speeds in z-direction. For example, in the limit $|i\omega_0/D^{2/3}| \ll 1$ (or $|A| \ll 1$ in Parker's notation), components of the oscillatory solution propagate in the z-direction with the phase velocities given by

$$V_{z1} \sim -\frac{2}{\sqrt{3}}\frac{\omega_0}{D^{1/3}}, \quad V_{z2} \sim \frac{2}{\sqrt{3}}\frac{\omega_0}{D^{1/3}}, \quad V_{z3} \sim \frac{3}{2}\omega_0 D^{1/3}, \quad V_{z4} \sim -\frac{1}{2}\frac{D^3}{\omega_0^4}. \tag{4}$$

Here it is important to notice that due to the boundary conditions this marginal state is composed of each of 4 waves, and that in the model of SR, there is only one component in marginal state (because another component with a different sign of phase velocity is largely damped). As we shall show later, this point is very crucial in the question of the coupling between the density waves and the marginal state of dynamo.

2.2. The first-order resonance

Now let us turn to the coupling problem between dynamo and density waves. Following the above discussion, suppose that the fundamental state of dynamo is simply given as $\exp i(\omega_0 t - kz) + \exp -i(\omega_0 t + kz) +$ c.c, where ω_0 and k are assumed to be real and positive and c.c denotes complex conjugation of all the preceding terms. The first wave propagates in the positive z-direction while the second toward $z = 0$. Then under the influence of density waves $\exp(2i\omega_0 t) + \exp(-2i\omega_0 t)$ (i.e. $\gamma = 2$ in SR), the new waves with a form of $\exp -i(\omega_0 t + kz) + \exp i(\omega_0 t - kz) +$ c.c are produced as well as other waves with frequencies $3\omega_0$ and $-3\omega_0$. It follows that the first component of the fundamental mode (with the positive phase velocity) coupled to the density wave powers the second component (with the negative phase velocity) and vice versa. In contrast, in the approach of SR, the term $\exp -i(\omega_0 t + kz)$ in the fundamental state and thus the term $\exp i(\omega_0 t - kz)$ in the new waves are absent, so that their fundamental wave $\exp i(\omega_0 t - kz)$ cannot be powered in this first coupling. This is the implication of the mentioned fact that their marginal state *contains only one wave* and in this respect *does not correspond to the properties of the disc dynamo.* If one properly considers the marginal state represented as the superposition of both the positive and negative phase velocity components, some of them are readily powered by density waves with frequency $2\omega_0$ in the first order in the density-wave amplitude. Therefore, it suggests the occurrence of the first-order resonance, or *linear swing excitation* when $\gamma = 2$.

2.3. Multiple time-scale method

We derive the actual solution for this coupling problem by applying the multiple time-scale method (see e.g. Nayfeh & Mook, 1981). In the following, we describe only the outline of the formulation. For details, see Hanasz & Chiba (1993).

We assume that $\tilde{\alpha}$ is a step function. Firstly, we expand equs. (1) and (2) in ε introducing $A = A_0 + \varepsilon A_1 + \ldots,\quad B = B_0 + \varepsilon B_1 + \ldots,\quad t_0 = \varepsilon^0 t,\quad t_1 = \varepsilon^1 t, \ldots,$ and assume the marginal state for $\varepsilon = 0$ as,

$$A_0 = \sum_{n=1}^{4} a_n(t_1) e^{-ik_n z} e^{i\omega_0 t_0} + c.c., \quad B_0 = \sum_{n=1}^{4} b_n(t_1) e^{-ik_n z} e^{i\omega_0 t_0} + c.c., \tag{5}$$

where ω_0 is real, and k_n a complex wavenumber. In order to describe the slow time evolution of the amplitudes of A_0 and B_0 we introduced the dependences on the slow time scale t_1 in $a_n(t_1)$ and $b_n(t_1)$.

Our present goal is not to derive the solution for A_1 and B_1, but rather to find the condition imposed on $a_n(t_1)$ and $b_n(t_1)$ in order to ensure the solutions for A_1 and B_1 the proper asymptotic behavior. We postulate that the solution is to be uniformly valid, what can be formally expressed by the requirement that in the limit $t_0 \to \infty$ the ratio $\varepsilon A_1/A_0 < \infty$. In order to obtain the uniformly valid solution, we have to postulate that the secular producing terms in equations for A_1 and B_1 (composed of $a_n(t_1)$ and $b_n(t_1)$ and their derivatives) vanish.

Let us concentrate on the case $\gamma = 2$ ($\chi = 2\omega_0 t_0$) and $|i\omega_0/D^{2/3}| \ll 1$. In this case we obtain $V_{z2} = -V_{z1}$ in the mentioned approximation, what means that the 'new wave' produced via coupling from the component associated to k_1 posses the same phase speed as the 'existing' fundamental mode associated to k_2, and vice versa. This fact makes the 1st-order resonance possible. Also, the other waves associated to k_3 and k_4 are powered together with these modes through the boundary condition at $z = 1$. Then the requirement of vanishing of the secular terms leads to the set of equations for $a_n(t_1)$ and $b_n(t_1)$, which posses the solutions proportional to $\exp(s_n t_1)$, where the growth rate s_n is given by

$$s_n = \frac{1}{4}\left(\left(f_2 + \frac{\omega_0}{D} R_\alpha f_1\right)^2 + \left(\frac{{k_n}^2}{D} R_\alpha f_1\right)^2\right)^{1/2}. \tag{6}$$

Notice that s_n is obtained from the 1st-order approximation in $\varepsilon = \tilde{\sigma}/\sigma_0$. Thus in the order of magnitude, s_n depends linearly on $(R_\alpha f_1, f_2)$, and the growth rate in the time scale t_0 is linear in the density-wave amplitude $(\varepsilon R_\alpha f_1, \varepsilon f_2)$. Therefore, when $\gamma = 2$, the rapid growth of the field is possible. In contrast, the results in SR are 2nd order in ε, because their fundamental state of dynamo does not allow the waves with the negative phase velocity so that the linear effect of coupling is null.

Let us turn to the cases $\gamma = 1$ and $\gamma = 3$. In such a case the density wave do not couple the components of our fundamental mode in the 1st order of ε, so that a_n does not depend on t_1 but the second time scale t_2. In other words, the oscillation $\exp(\pm i\gamma\omega_0 t)$ with $\gamma = 1$ or 3 produces the terms $\exp(\pm i\omega_0 t)$ after twice couplings with the fundamental mode $\exp(\pm i\omega_0 t)$. It means that the resonances for $\gamma = 1$ and $\gamma = 3$ are 2nd order in ε, and the resultant growth rates are smaller than that for $\gamma = 2$ by a factor ε.

3. Numerical results

We have also performed the numerical simulations of the disc dynamo represented by equs.(1) and (2) for a general function of $\tilde{\alpha}$. As in CT, we first calculate the unperturbed state ($\varepsilon = 0$), and obtain the dynamo number D and the frequency ω_0 in a marginal state. For even dynamo mode and $\tilde{\alpha} = \sin \pi z$, we obtain $D = 166.05$ and $\omega_0 = 13.31$. Next, we turn on the disturbance of density wave parametrized by $(\varepsilon R_\alpha f_1, \varepsilon f_2)$ with a phase $\chi = \gamma\omega_0 t + \phi_0$. On the base of each calculation done up to $t = 5000$ in a unit of diffusion time we compute the growth rate Re s of magnetic field resulting from the resonant coupling of the marginal mode of magnetic field and the density wave. The goal of these simulations is to check qualitatively our present and previous analytical results and make the comparison to the results of SR.

We check first, the suggestion of SR that there is no dependence of resonance on the initial phase relation between the dynamo and density waves. We investigate two cases with phases $\phi_0 = 0$ and $-\pi/2$. We find that even if the case $\phi_0 = -\pi/2$ starts to decay for a first moment, it turns to grow, and grows forever as in the case $\phi_0 = 0$. The final growth rates for both cases are exactly the same. Thus the present phenomenon of resonance does not depend on the initial phase relation as shown by SR. However, this property is *not* related to the beat phenomena as SR mentioned, otherwise the initial transient phase lasts and as a result the growth rate shall change with time.

Secondly, we attempt to clarify whether the resonance is of 1st-order or 2nd-order in the amplitude of density wave. For this purpose, we determine the dependence of the growth rate Re s on $(\varepsilon R_\alpha f_1, \varepsilon f_2)$. Here we redefine $(d, e) = (\varepsilon R_\alpha f_1, \varepsilon f_2)$ with the aim of simplifying notation and for making the differences between our results and that of SR more apparent. In Fig.1, we plot the numerically derived Re s when $\gamma = 2$. The distribution of each mark strongly suggests the linear dependency of Re s on the amplitude (d, e). In fact, we fit a function

$$\text{Re } s\ (d, e\ ; c_1, c_2) = |\ c_1(e + c_2 \frac{\omega_0}{D} d)\ | \quad (7)$$

to the discrete set of points, where c_1 and c_2 are factors. This functional form is the same as the analytically derived growth rate in eq.(6) when $k_n \to 0$, and is linear in d and e. In Fig.1, each line corresponds to the above function with (c_1, c_2)

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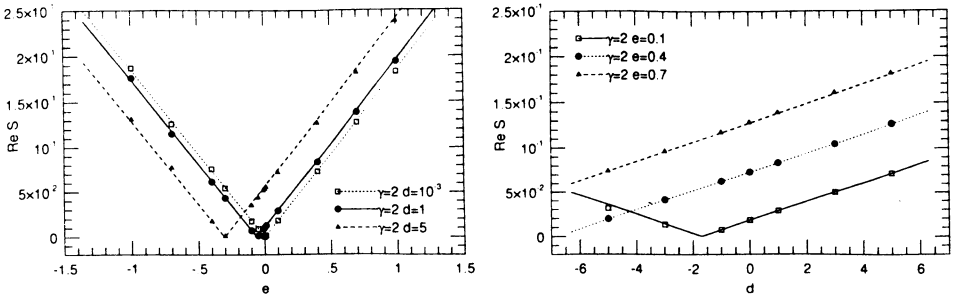

Fig. 1. Dependence of Re s on e (a) and d (b)

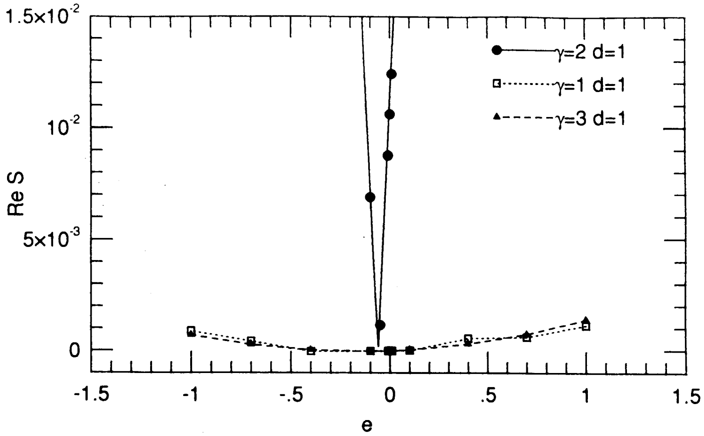

Fig. 2. Dependence of Re s on γ

obtained from the least square procedure. In both figures, the values of χ for fitting are of the order $O(10^{-3})$ to $O(10^{-5})$, which means the fitting to a straight line is good with high accuracy, In contrast, SR showed that Re s is quadratic in d and e (see their Fig.3), which disagrees with out present results.

Furthermore, Fig.2 shows how Re s depends on γ values. The solid line for $\gamma = 2$ is the above fitting function with $(c_1, c_2) = (0.185, 0.761)$, and dotted and dashed curves are ones by simply connecting among marks for the cases $\gamma = 1$ (circles) and 3 (boxes), respectively. This figure clearly indicates that the case $\gamma = 2$ leads to the largest growth rate, strongly inconsistent with SR (see their Fig.4). Moreover, we observe that the curves for $\gamma = 1$ and 3 are not linear in e but vary more or less in a quadratic manner, suggesting the higher-order resonance in d and e.

To conclude: both analytical and numerical calculations indicate that when $\gamma = 2$, the resonance in the 1st order of density-wave perturbation, or *linear swing excitation* is possible, and thus such a case gives the largest growth rate of magnetic

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fields in galaxies. This result confirms qualitatively our previous and present analytical results, however a small, but noticeable quantitative discrepancy concerning the value of c_1 is observed. For more details concerning this point see Hanasz & Chiba (1993).

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