8 Technicolor: a first attempt to explain hierarchies

In Chapter 5 we learned a great deal about quantum chromodynamics. In Section 4.5 we argued that the hierarchy problem is one of the puzzles of the Standard Model. The grand unified models of Chapter 6 provided a quite stark realization of the hierarchy problem. In an $SU(5)$ grand unified model we saw that it is necessary to adjust carefully the couplings in the Higgs potential in order to obtain light doublet and heavy color triplet Higgs. This is already true at tree level; loop effects will correct these relations, requiring further delicate adjustments.

Attempts to understand the hierarchy problem in a manner consistent with 't Hooft's naturalness principle fall into three broad categories: the dynamical breaking of electroweak symmetry, supersymmetry (in which it is still possible that the breaking of electroweak symmetry is dynamical), geometric approaches (large extra dimensions or warped space–times) and supersymmetry. The present chapter gives a brief introduction to dynamical models; Chapters 9–16 will deal with supersymmetry both as a possible new symmetry of nature and a possible solution to the hierarchy problem. We will discuss geometric solutions in Chapter 29 after we have learned about theories of space–time, i.e. general relativity and string theory.

The first proposal to resolve the hierarchy problem goes by the name *technicolor*. The technicolor hypothesis exploits our understanding of QCD dynamics. It elegantly explains the breaking of the electroweak symmetry. It has more difficulty accounting for the masses of the quarks and leptons, and simple versions seem incompatible with precision studies of the W and Z particles and now the discovery of a Standard-Model-like Higgs boson. In this chapter we will introduce the basic features of the technicolor hypothesis. We will not attempt to review the many models that have been developed to try to address the difficulties of flavor and precision electroweak experiments. It is probably safe to say that, as of this writing, none is totally successful nor particularly plausible. But it should be kept in mind that this may reflect the limitations of theorists; experiment may yet reveal that nature has chosen this path. In any case, the study of these theories will deepen our understanding of the Standard Model and of strongly coupled quantum field theories and will open our eyes to possibilities for new physics.

We will then turn briefly to dynamical alternatives to technicolor. One of the most interesting of these is the possibility that the Higgs particle is itself an approximate Goldstone particle, the result of the breaking of some accidental global symmetry. By itself this approach does not completely solve the hierarchy problem, but it suppresses the problem of quadratic divergences to higher orders and one might imagine that the phenomenon might arise in some more complete dynamical framework. It has the virtue that in it the Higgs is to a good approximation a fundamental field, as appears to be the case experimentally.

8.1 QCD in a world without Higgs fields

Consider a world with only a single generation of quarks and no Higgs fields. In such a world the quarks would be exactly massless. The $SU(2)_L \times SU(2)_R$ symmetry of QCD would be, in part, a gauge symmetry; $SU(2)_L$ would correspond to the $SU(2)$ symmetry of the weak interactions. The hypercharge Y would include a generator of $SU(2)_R$ and baryon number:

$$Y = 2T_{3R} + B. \tag{8.1}$$

The quark condensate,

$$\langle q_f \bar{q}_{f'} \rangle = \Lambda^3 \delta_{ff'}, \tag{8.2}$$

would break some of the gauge symmetry. Electric charge, however, would be conserved, so $SU(2) \times U(1) \to U(1)$.

In Appendix C it is shown that the quark condensate conserves a vector $SU(2)$ symmetry, ordinary isospin. This $SU(2)$ symmetry is generated by the linear sum

$$T_i = T_{iL} + T_{iR}. \tag{8.3}$$

So, the $SU(2)$ gauge bosons transform as a triplet of the conserved isospin. This guarantees that the successful tree level relation

$$M_W = M_Z \cos\theta \tag{8.4}$$

is satisfied. The $SU(2)$ which accounts for this relation is called a *custodial* symmetry (the Higgs potential of the Standard Model possesses, in fact, an approximate $O(4)$ symmetry which has a suitable $SU(2)$ subgroup).

To understand the masses of the gauge bosons remember that, for a broken symmetry with current j^μ, the coupling of the Goldstone boson to the current is

$$\langle 0 | j^\mu | \pi(p) \rangle = i f_\pi p^\mu. \tag{8.5}$$

This means that there is a non-zero amplitude for a gauge boson to turn into a Goldstone, and vice versa. The diagram of Fig. 8.1 is proportional to

$$g^2 f_\pi^2 p^\mu \frac{i}{p^2} p^\nu. \tag{8.6}$$

As the momentum tends to zero, this tends to a constant – the mass of the gauge boson. For the charged gauge bosons the mass is just

$$m_{W^\pm}^2 = g^2 f_\pi^2, \tag{8.7}$$

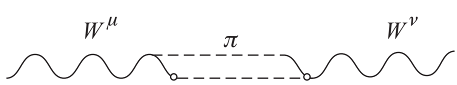

Fig. 8.1 Diagrammatic representation of technicolor.

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while for the neutral gauge bosons we have a mass matrix

$$f_{\pi^2}\begin{pmatrix} g^2 & gg' \\ gg' & g'^2 \end{pmatrix}, \tag{8.8}$$

giving one massless gauge boson and one with mass-squared $(g^2 + g'^2)f_\pi^2$.

All this can be nicely described in terms of the non-linear sigma model used to describe pion physics. Recall that the pions could be described in terms of a matrix field,

$$\Sigma = |\langle\overline{\psi}\psi\rangle| e^{i\vec{\pi}\cdot\vec{\tau}/2}, \tag{8.9}$$

which transforms under $SU(2)_\mathrm{L} \times SU(2)_\mathrm{R}$ as follows:

$$\Sigma \to U_\mathrm{L}\Sigma U_\mathrm{R}^\dagger. \tag{8.10}$$

Changes in the magnitude of the condensate are associated with excitations in QCD that are much more massive than the pion fields (the σ field of our linear sigma model of Section 2.2). So, it is natural to treat this as a constant. The field Σ is then constrained to take values on a manifold. As in our examples in two dimensions, a model based on such a field is called a non-linear sigma model. The Lagrangian is

$$\mathcal{L} = f_\pi^2 \, \mathrm{Tr}(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma). \tag{8.11}$$

In the context of the physics of light pseudo-Goldstone particles, the virtue of such a model is that it incorporates the effects of broken symmetry in a very simple way. For example, all the results of current algebra can be derived by studying the physics of such a theory and its associated Lagrangian.

In the case of the σ-model we have an identical structure except that we have gauged some of the symmetry, so we need to replace the derivatives by covariant derivatives:

$$\partial_\mu\Sigma \to D_\mu\Sigma = \partial_\mu\Sigma - i\frac{A_\mu^a\sigma_a}{2}\Sigma - i\Sigma\frac{\sigma_3}{2}B_\mu. \tag{8.12}$$

Again, we can choose a unitary gauge; we just set $\Sigma = 1$. The Lagrangian in this gauge is simply

$$\mathcal{L} = \mathrm{Tr}\left(\frac{A_\mu^a\sigma_a}{2}\Sigma + \Sigma\frac{\sigma_3}{2}B_\mu\right)^2. \tag{8.13}$$

This yields exactly the mass matrix as we wrote down before.

8.2 Fermion masses: extended technicolor

In technicolor models, the Higgs field is replaced by new strong interactions which break $SU(2) \times U(1)$ at a scale $F_\pi = 1$ TeV. However, the Higgs field of the Standard Model gives mass not only to the gauge bosons but to the quarks and leptons as well. In the absence of the Higgs scalar there are chiral symmetries which prohibit masses for any of the quarks and leptons. While our simple model can explain the masses of the Ws and Zs, it has no mechanism to generate mass for the ordinary quarks and leptons.

If we are to avoid introducing fundamental scalars, the only way to break these symmetries is to introduce further gauge interactions. Consider first a single generation of quarks and leptons. Enlarge the gauge group to $SU(3) \times SU(2) \times U(1) \times SU(N+1)$. The technicolor group will be an $SU(N)$ subgroup of the last factor. Take each quark and lepton to be part of an $N+1$ or $\overline{N+1}$ representation of this larger group. To avoid anomalies, we will also include a right-handed neutrino. In other words, our multiplet structure is:

$$\begin{pmatrix} Q \\ q \end{pmatrix}, \quad \begin{pmatrix} \bar{U} \\ \bar{u} \end{pmatrix}, \quad \begin{pmatrix} \bar{D} \\ \bar{d} \end{pmatrix}, \quad \begin{pmatrix} L \\ \ell \end{pmatrix}, \quad \begin{pmatrix} \bar{E} \\ \bar{e} \end{pmatrix} \quad \begin{pmatrix} \bar{N} \\ \bar{\nu} \end{pmatrix}. \tag{8.14}$$

Here $q, \bar{u}, \bar{d}, \ell$, etc., are the usual quarks and leptons; the fields denoted by capital letters are the techniquarks. Now suppose that the $SU(N+1)$ is broken to $SU(N)$ at a scale $\Lambda_{\text{etc}} \gg \Lambda_{\text{tc}}$ by some other gauge interactions, in a manner similar to that of technicolor. Then there is a set of massive gauge bosons with mass of order Λ_{etc}. Exchanges of these bosons give rise to operators such as

$$\mathcal{L}_{4f} = \frac{1}{\Lambda_{\text{etc}}^2} Q\sigma_\mu q^* \bar{U}\sigma^\mu \bar{u}^* + \text{h.c.} \tag{8.15}$$

Using the following identity for the Pauli matrices,

$$\sum_\mu (\sigma_\mu)_{\alpha\dot{\alpha}} (\sigma^\mu)^{\dot{\beta}\beta} = \delta_\alpha^\beta \delta_{\dot{\alpha}}^{\dot{\beta}}, \tag{8.16}$$

permits us to rewrite the four-fermion interaction as

$$\mathcal{L}_{4f} = \frac{1}{\Lambda_{\text{ETC}}} Q\bar{U}q^*\bar{u}^* + \text{h.c.} \tag{8.17}$$

We can replace $Q\bar{U}$ by its expectation value, which is of order Λ_{tc}^3. This gives rise to a mass for the u quark. The other quarks and leptons gain mass in a similar fashion.

This particular extended technicolor (ETC) model is clearly unrealistic on many counts: it has only one generation; there is a massive neutrino; there are relations among the masses which are unrealistic; there are approximate global symmetries which lead to unwanted pseudo-Goldstone bosons. Still, it illustrates the basic idea of extended technicolor models: additional gauge interactions break the unwanted chiral symmetries which protect the quark and lepton masses from radiative corrections.

One can try to build realistic models by considering more complicated groups and representations for the extended technicolor (ETC) interactions. Rather than attempt this here, we will consider some issues in a general way. We will imagine that we have a model with three generations. The extended technicolor interactions generate a set of four-fermion interactions which break the chiral symmetries acting on the separate quarks and leptons. In a model of three generations, there are a number of challenges which must be addressed.

1. Perhaps the most serious is the problem of flavor-changing neutral currents. In addition to four-fermion operators which generate mass, there will also be four-fermion operators involving just the ordinary quarks and leptons. These operators will not, in general,

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respect flavor symmetries. They are likely to include terms like

$$\mathcal{L}_{\Delta S=2} = \frac{1}{\Lambda_{\rm etc}} \bar{s}\bar{d}s^* d^*, \tag{8.18}$$

which violate strangeness by two units. Unless $\Lambda_{\rm etc}$ is extremely large (of order hundreds of TeV), this will lead to unacceptably large rates for $K^0 \leftrightarrow \bar{K}^0$.

2. Generating the top quark mass is potentially problematic; it is larger than the W and Z masses. If the ETC scale is large, it is hard to see how to achieve this.
3. The problem of pseudo-Goldstone bosons is generic to technicolor models, in just the fashion we saw for the simple model.

The challenge of technicolor model building is to construct models which solve these problems. We will not attempt to review the various approaches which have been put forward here. Models which solve these problems are typically extremely complicated. Instead, we briefly discuss another serious difficulty: the precision measurement of electroweak processes.

8.3 The Higgs discovery and precision electroweak measurements

In Section 4.5 we stressed that the parameters of the electroweak theory have been measured with high precision and compared with detailed theoretical calculations, including radiative corrections. One naturally might wonder whether a strongly interacting Higgs sector could reproduce these results. The answer is that it is difficult. There are two categories of corrections which one needs to consider. The first are, in essence, corrections to the relation

$$M_W = M_Z \cos\theta_W. \tag{8.19}$$

In a general technicolor model these will be large. But we have seen why this relation holds in the minimal Standard Model: there is an approximate global $SU(2)$ symmetry. This is in fact the case of the simplest technicolor model we encountered above. So this problem is likely to have solutions.

There are, however, other corrections as well, resulting from the fact that in these strongly coupled theories the gauge boson propagators are quite different from those in weakly coupled field theories. They have been estimated in many models and are found to be far too large to be consistent with the data. More details about this problem, and speculations on possible solutions, can be found in the suggested reading.

The discovery of a Higgs particle behaving very much as a simple fundamental doublet poses further challenges. In analogy with QCD, in general we would not expect to find scalars much lighter than the TeV scale, and would expect that any such scalars would be quite broad resonances. There is no reason to expect that they should be narrow, with couplings close to those of the Standard Model, never mind couplings as expected in the Minimal Supersymmetric Standard Model.

8.4 The Higgs as a Goldstone particle

An attractive possibility which has received much attention over the years is that the Higgs doublet is a pseudo-Goldstone particle of some approximate global symmetry. If the characteristic scale of the underlying theory is Λ, so that the next lightest excitations have masses of this order while the parameters of the Higgs potential are loop suppressed, we might hope that the doublet will behave like an elementary field up to terms suppressed by powers of Λ.

Necessarily this symmetry is broken by the gauge interactions. This is important, as such symmetry breaking is necessary to obtain a potential for the Higgs field. As an example, we might imagine that the underlying global symmetry is $SU(3)$, and the Goldstone bosons of this $SU(3)$ symmetry can be described by a non-linear sigma model with a field Σ living on the coset $SU(3)/SU(2)$. The components of Σ include the Higgs field. The difficulty with the simplest version is that the scales f (the Goldstone decay constant) and Λ are not appreciably separated. At one loop there are quadratically divergent corrections to the Higgs mass from gauge loops. These are cut off at some scale Λ. From considerations of unitarity – the scale Λ should be such that loop corrections are at most of order one – one expects that $\Lambda^2 < 4\pi f^2$. This is insufficient to explain precision electroweak breaking or the Higgs width.

To avoid this difficulty, models have been constructed with more intricate symmetries. Often, a phenomenon known as *collective symmetry breaking* is invoked. The basic idea is that there are several gauge interactions and only collectively do they break enough symmetry that one can generate a Higgs potential. In the resulting "little Higgs" theories the symmetries prevent a one-loop contribution to the Higgs mass at one-loop order, and the Higgs field appears to be elementary to the required precision.

It is important that the fermions also respect these larger symmetries. This requires, at a minimum, additional vector-like fields. At a more microscopic level one expects that these global symmetries are accidents of the underlying structure. Non-Abelian symmetries acting both on scalars and fermions in the required, rather intricate, ways may be challenging to discover. Some existing models invoke supersymmetry to achieve this.

Suggested reading

An up-to-date set of lectures on technicolor, including the problems of flavor and electroweak precision measurements, are given in the online article of Chivukula (2000). An introduction to the analysis of precision electroweak physics is provided by Peskin (1990); for an application to technicolor theories, see Peskin and Takeuchi (1990). The Particle Data group summary of technicolor theories surveys the status of dynamical models for electroweak symmetry breaking, in light of the Higgs discovery. Little Higgs theories are described in the reviews of Perelstein (2007) and Schmaltz and Tucker-Smith (2005).

Exercises

(1) Determine the relations between the quark and lepton masses in the extended technicolor model above.

(2) What are the symmetries of the extended technicolor model in the limit where we turn off the ordinary $SU(3) \times SU(2) \times U(1)$ gauge interactions? How many of these symmetries are broken by the condensate? Each broken symmetry gives rise to an appropriate Nambu–Goldstone boson. Some of these approximate symmetries are broken explicitly by the ordinary gauge interactions. The corresponding Goldstone bosons will then gain mass, typically of order $\alpha_i \Lambda_{\text{etc}}$. Some will not gain mass of this order, however. Which symmetry (or symmetries) will be respected by the ordinary gauge interactions?