IMPROVING THE DYNAMICAL REFERENCE FRAME THROUGH MINOR PLANET ORBIT CORRECTION USING CROSSING POINT OBSERVATIONS

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ABSTRACT. We have begun a program to establish a dynamical reference frame based on the motions of minor planets. The program will utilize observations from the Hubble Space Telescope, and will ultimately tie the HIPPARCOS reference system to a dynamical base. Thirty-four minor planets, 20 of which are suitable for observation with the Hubble Space Telescope, have been selected. Ground based observations, particularly crossing-point observations with long focus reflectors, have been initiated.

A computer program to simultaneously solve for the corrections of the orbits of the 34 minor planets including the crossing-point observations, was successfully run. The observations are treated by the method of W. H. Jeffreys. Using simulated data, solutions with and without crossing point observations demonstrate the value of those observations to produce a homogeneous and coherent set of results.

1. INTRODUCTION

In the past, transit circle observations of the principal planets have been utilized to establish the equinox and equator of fundamental reference frames, e.g. (Newcomb 1895), (Morgan 1951). For practical applications, the reference frame is realized by an associated catalog of star positions and motions. Night observations of the outer planets are more reliable than day observations of Mercury, Venus and the Sun, which may be subject to sizable random and systematic errors. It is a complex and frustrating task to relate such day observations to a reference catalog based on night observations. Minor planets, observed at night, provide images which are more star-like, and in the case of smaller asteroids, avoid the necessity for phase or limb-to-center corrections. The use of such objects to establish a dynamical frame to be used as a standard against which to compare fundamental star catalogs has been detailed by several investigators, (Dyson 1928), (Numerov 1933), (Brouwer 1935, 1941). The advent of photographic astrometry has provided observations of fainter minor planets, over a greater range of their apparent longitude than can be obtained with the transit circle. The observations of sixteen minor planets selected by Brouwer have been analyzed by Pierce (1971) and the derived equinox and equator compared with the FK4 by Fricke (1982).

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The development of the ESA HIPPARCOS telescope and the Hubble Space Telescope (HST) in conjunction with more precise ground-based astrometric techniques makes feasible a new effort to establish a dynamical reference frame. To this end, a fifteen year program of observation of 34 selected minor planets with ground-based and space telescopes has begun. Basic to this effort is the use of "crossing-point" observations in addition to classical astrometric observations to produce a homogeneous, interlocking set of minor planet orbits which can be used to investigate the accuracy of the fundamental reference frame.

2. DIFFERENTIAL ORBIT CORRECTION WITH CROSSING-POINT OBSERVATIONS

The method used here for determining differential corrections to minor planet orbital elements employs crossing-point observations. A crossing-point observation is defined to be the observed angular separation between two minor planets as measured against the same stellar background. These observations are currently made by photographing the minor planets as close as possible to the predicted times (generally different) at which the apparent paths of the minor planets intersect. The resulting photographic plates contain a number of background stars in common. The positions of the minor planets are measured relative to these common background stars. The final observed quantities are then obtained by differencing the measured positions of the minor planets to arrive at the angular separation in right ascension and declination. Differencing the measured positions of the minor planets eliminates errors which may be present in the positions of the reference stars. The elimination of the systematic errors in the star positions is shown formally in Eq. 1, where carets denote the measured quantities, $\varepsilon_{s\alpha}$ and $\varepsilon_{s\delta}$ represent the errors in the star positions, and $\varepsilon_{1\alpha}$, $\varepsilon_{1\delta}$, $\varepsilon_{2\alpha}$, and $\varepsilon_{2\delta}$ contain all other errors in the measurements.

$$\begin{aligned}
\hat{\alpha}_1 &= \alpha_1+\varepsilon_{s\alpha}+\varepsilon_{1\alpha} & \hat{\delta}_1 &= \delta_1+\varepsilon_{s\delta}+\varepsilon_{1\delta} \\
\hat{\alpha}_2 &= \alpha_2+\varepsilon_{s\alpha}+\varepsilon_{2\alpha} & \hat{\delta}_2 &= \delta_2+\varepsilon_{s\delta}+\varepsilon_{2\delta} \\
\Delta\hat{\alpha} &= (\hat{\alpha}_1-\hat{\alpha}_2)\cos\hat{\delta}_1 = (\alpha_1-\alpha_2+\varepsilon_{1\alpha}-\varepsilon_{2\alpha})\cos(\delta_1+\varepsilon_{s\delta}+\varepsilon_{1\delta}) \\
\Delta\hat{\delta} &= \hat{\delta}_1-\hat{\delta}_2 = \delta_1-\delta_2+\varepsilon_{1\delta}-\varepsilon_{2\delta}
\end{aligned} \tag{1}$$

Typical errors for classical observations (α and δ) are a few tenths of a second of arc whereas the errors for crossing-point observations are only 0.02 seconds of arc. This improvement in observational accuracy is not without its price. Since the crossing-point observations concern only the relative positions of the minor planets, information about the absolute orientations of the orbits in space is lost. Classical observations are therefore required to orient the minor planet orbits. This situation is analogous to the HIPPARCOS reference system which will be an extremely accurate, rigid frame of reference but which will require external observations to determine the rigid body orientation of the system.

A system of computer programs has been written to perform differential orbit correction using combinations of classical and crossing-point observations. The system uses Jefferys' (1980, 1981) iterative, nonlinear method of least squares. This method is well suited to the problems posed by the use of two types of data with the attendant two forms of the conditional equations. The iterative nature of the solution requires the recomputation of the minor planet positions at the times of observation following each correction of the orbital elements. The minor planet positions are computed by numerical integration of the equations of motion as perturbed by the nine major planets which are modeled using the JPL DE200 ephemeris (Standish, et. al., 1976). The crossing-point observations introduce correlations between the elements of the minor planets involved in the observations, and by inference, with the elements of many or all minor planets in the program. For example, if minor planet 148 Gallia is involved in a crossing-point observation with 51 Nemausa, and Nemausa is involved in crossing-point observations with 475 Ocllo, 502 Sigune, and 1108 Demeter, then correlations will exist between 148 Gallia and 475 Ocllo, 502 Sigune, and 1108 Demeter, as well as with 51 Nemausa. These correlations necessitate the simultaneous solution for the corrections to the orbital elements of all 34 minor planets in the program. The present system of programs performs this simultaneous solution.

3. TESTS WITH SIMULATED DATA

To test the effectiveness of crossing-point observations, a set of simulated "plates" was generated and used to create two sets of "observations" which were run through the software system. The "plates" involved 47 crossing-point observations and 314 classical observations spanning the interval from 23 October 1982 to 29 November 1988. "Plates" of all 34 minor planets were included in the data set but the number and distribution of the observations varied among the minor planets. The "plates" were generated by numerically integrating the minor planet orbits from starting positions derived from the Ephemerides of Minor Planets for 1984 to obtain "exact" positions. The measurement of the "plates" was simulated by adding normally distributed random errors to each computed right ascension and declination. The errors had two components as shown in Eq. 1. The first component of the error was the star system error. These errors had a mean of zero and a variance 0.25 seconds of arc. The same random error was added to both components of a crossing-point observation to simulate the fact that the error is introduced by the positional errors in the same background stars. The second component of the introduced errors had a mean of zero and a variance of 0.01 seconds of arc. Finally, normally distributed random errors with a mean of zero and a variance of 0.5 seconds of time were added to the times of observation.

These "plate measurements" were then used to generate two data sets. In the first data set, all of the positions were used as classical observations. In the second data set, the pairs of "plates" which could be combined into crossing-point observations were used to generate one classical and one crossing point observation. Thus, the total number of "observations" was the same in both data sets. Since the random star system errors which were added to the "exact" positions were the same for both components of the crossing-point observations, the actual errors introduced into the crossing-point observations were on the order of 0.01 seconds of arc. To obtain initial values of the orbital elements of the minor planets, the elements from the Ephemerides of Minor Planets for 1984 were rounded to one less significant digit than the published values. Thus, the initial errors were on the order of 10^{-7} AU for the semimajor axis, 10^{-7} for the eccentricity, and 10^{-5} degrees for the angular elements. The program required four iterations to converge when the crossing-point observation data set was used and only one iteration when the classical observation data set was used. This is not surprising since the lack of crossing-points in the classical observations data set meant that the program was performing 34 simultaneous, but independent, classical differential orbit corrections. Convergence in only one iteration is to be expected given the small initial errors in the elements. On the other hand, the correlations introduced by the crossing-point observations resulted in the simultaneous correction to all 204 parameters (34 minor planets times 6 orbital elements per minor planet). The adjustment of this many parameters naturally required more iterations than was required for the classical solution.

To compare the solution obtained from the crossing-point observations data set with that obtained from the classical observations data set, the two sets of final orbital elements were used to compute positions at 100 day intervals from 24 March 1983 to 24 November 1988. The equations of motion were again integrated using full planetary perturbations. In addition, the true elements from the Ephemerides of Minor Planets for 1984 were used to compute true positions at these dates. Figure 1 shows the differences between the true and classical, and the true and crossing-point solutions in right ascension and declination respectively.

The points for minor planets 61, 965, 1474, 1584, and 1626 have been removed from Figure 1 because there were no crossing-point observations of these minor planets. The solutions for these minor planets were identical for both data sets. It is significant that while all 34 sets of elements were corrected at once this did not contaminate the solution of a minor planet for which there were only classical observations.

A notable feature of Figure 1 are the points with high scatter (> 1 arcsec.) for both the classical and crossing-point data sets. Upon examination, these points are all found to belong to minor planets 599, 652, 846, 1222, and 1252. This high divergence from the true solution indicates that both solutions are poor. The "observations" of these minor planets are the source of the poor solutions. In the cases of minor planets 599, 846, 1222, and 1252 the "observations" cover less than one-half of the period of the minor planets.

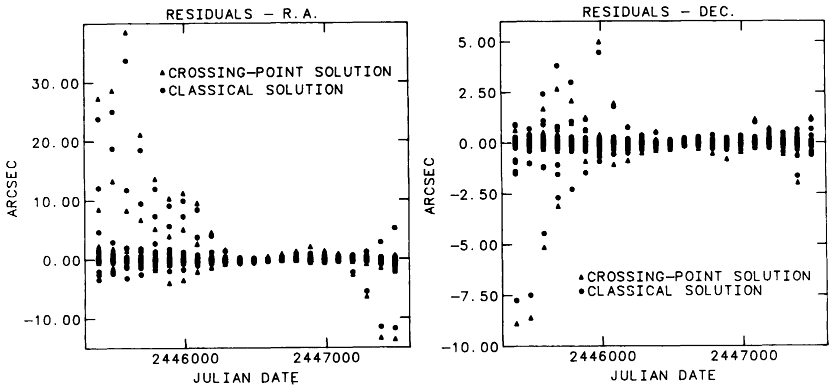

Figure 1. Classical and crossing-point solution test-point residuals.

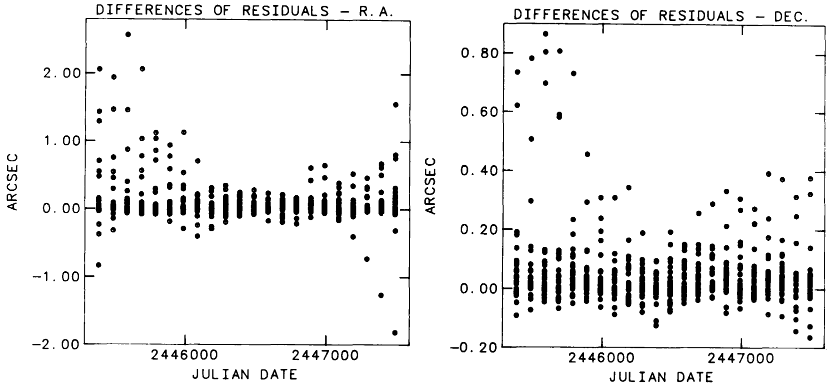

Figure 2. Differences between the classical and crossing-point test-point residuals.

In the case of minor planet 652, the "observations" cover one and one-third orbits but are clustered around only two points on the orbit.

The elimination of these minor planets leaves 24 for which there were sufficient "observations" for meaningful solutions and for which there were crossing-point observations. To compare the quality of the crossing point and classical solutions, the difference of the absolute values of the residuals between the true solution and the classical solution, and the residuals between the true solution and the crossing-point solution, in right ascension and declination, are plotted in Figure 2. The mean of the difference between the solutions in right ascension is 0.091 seconds of arc and the variance is 0.319 seconds of arc. The mean of the difference between the solutions in declination is 0.054 seconds of arc and the variance is 0.180 seconds of arc. Over the 2200 days of the integrated arc, the solution that was derived from the crossing-point observations averaged a tenth of a second of arc better than the solution derived from only classical observations. The outlying negative points in Figure 2 all belong to minor planet 849 Ara. The "observations" of this minor planet span only the period from JD 2445691 to JD 2447015. Over this period the accuracy of the crossing-point solution and the classical solution are comparable. The large difference between the solutions occurs outside of the period of observation.

4. CONCLUSION

It has been shown that a method of differential orbit correction which uses crossing-point observations, developed as a part of this project, can realize a positional improvement over orbits derived from only classical observations. Since this positional improvement was realized in a six year span with only 47 crossing-point observations, the much larger number of crossing-point observations which will be made over the fifteen years of the project is therefore anticipated to yield a dynamical reference system fully capable of detecting and mapping systematic star system errors.

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