7
Strings on the Higgs branches

One common feature of supersymmetric gauge theories is the presence of moduli spaces – manifolds on which scalar fields can develop arbitrary VEVs without violating the zero energy condition. If on these vacuum manifolds the gauge group is broken, either completely or partially, down to a discrete subgroup, these manifolds are referred to as the Higgs branches.

One may pose a question: what happens with the flux tubes and confinement in theories with the Higgs branches? The Higgs branch represents an extreme case of type-I superconductivity, with vanishing Higgs mass. One may ask oneself whether or not the ANO strings still exist in this case, and if yes, whether they provide confinement for external heavy sources.

This question was posed and studied first in [102] where the authors concluded that the vortices do not exist on the Higgs branches due to infrared problems. In Refs. [211, 212] the $\mathcal{N} = 1$ SQED vortices were further analyzed. It was found that at a generic point on the Higgs branch strings are unstable. The only vacuum which supports string solutions is the base point of the Higgs branch where the strings become BPS-saturated. The so-called "vacuum selection rule" was put forward in [211, 212] to ensure this property.

On the other hand, in [103, 175] it was shown that infrared problems can be avoided provided certain infrared regularizations are applied. Say, in [103, 175] the infrared divergences were regularized through embedding of $\mathcal{N} = 1$ SQED in softly broken $\mathcal{N} = 2$ SQED. Another alternative is to consider a finite length-L string instead of an infinitely long string. In this case the impact of the Higgs branch was shown to "roughen" the string, making it logarithmically "thick." Still, the string solutions do exist and produce confinement for heavy trial sources. However, now the confining potential is not linear in separation; rather it behaves as

$$V(L) \sim \frac{L}{\ln L}$$

at large L. Below we will briefly review the string solutions on the Higgs branches, starting from the simplest case of the flat Higgs branch and then considering a more common scenario, when the Higgs branch is curved by the FI term.

7.1 Extreme type-I strings

In this section we will review the classical solutions for the ANO vortices (flux tubes) in the theories with the flat Higgs potential which arises in supersymmetric settings [103]. Let us start from the Abelian Higgs model,

$$S_{\rm AH} = \int d^4x \left\{ \frac{1}{4g^2} F_{\mu\nu}^2 + |\nabla_\mu q|^2 + \lambda \left(|q|^2 - v^2 \right)^2 \right\}, \tag{7.1.1}$$

for a single complex field q with the quartic coupling $\lambda \to 0$. Here

$$\nabla_\mu = \partial_\mu - i n_e A_\mu,$$

where n_e is the electric charge of the field q. Following [103], we will first consider this model with a small but nonvanishing λ and then take the limit $\lambda = 0$.

Obviously, the field q develops a VEV, $q = v$, spontaneously breaking the U(1) gauge group. The photon acquires the mass

$$m_\gamma^2 = 2 n_e^2 g^2 v^2, \tag{7.1.2}$$

while the Higgs particle mass is

$$m_q^2 = 4\lambda v^2. \tag{7.1.3}$$

The model (7.1.1) is the standard Abelian Higgs model which supports the ANO strings [36]. For generic values of λ the Higgs mass differs from that of the photon. The ratio of the photon mass to the Higgs mass is an important parameter – in the theory of superconductivity it characterizes the type of the superconductor in question. Namely, for $m_q < m_\gamma$ we have the type-I superconductor in which two well-separated ANO strings attract each other. On the other hand, for $m_q > m_\gamma$ we have the type-II superconductor in which two well-separated strings repel each other. This is due to the fact that the scalar field gives rise to attraction between two vortices, while the electromagnetic field gives rise to repulsion.

Now, let us consider the extreme type-I limit in which

$$m_q \ll m_\gamma. \tag{7.1.4}$$

We will assume the weak coupling regime in the model (7.1.1), $\lambda \ll g^2 \ll 1$.

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The general guiding idea which will lead us in the search for the string solution in the extreme type-I limit is a separation of different fields at distinct scales which are obviously present in the problem at hand due to the "extremality" condition (7.1.4). This method goes back to the original paper by Abrikosov [36] in which the tension of the type-II string had been calculated under the condition $m_q \gg m_\gamma$. A similar idea was used in [103] to calculate the tension of the type-I string under the condition $m_q \ll m_\gamma$.

To the leading order in $\ln m_\gamma/m_q$ the vortex solution has the following structure in the plane orthogonal to the string axis: The electromagnetic field is confined in a core with the radius

$$R_g \sim \frac{1}{m_\gamma} \ln \frac{m_\gamma}{m_q}. \tag{7.1.5}$$

At the same time, the scalar field is close to zero inside the core. Outside the core the electromagnetic field is vanishingly small, while the scalar field behaves as

$$q = v \left\{ 1 - \frac{K_0(m_q r)}{\ln(1/m_q R_g)} \right\} e^{i\alpha}, \tag{7.1.6}$$

where r and α are polar coordinates in the orthogonal plane (Fig. 3.6). Here K_0 is the (imaginary argument) Bessel function[1] with the exponential fall-off at infinity and logarithmic behavior at small arguments,

$$K_0(x) \sim \ln(1/x) \text{ at } x \to 0.$$

The reason for this behavior is that in the absence of the electromagnetic field outside the core the scalar field satisfies the free equation of motion, and (7.1.6) presents the appropriate solution to this equation. From (7.1.6) we see that the scalar field slowly (logarithmically) approaches its boundary value v.

The tension of this string is [103]

$$T = \frac{2\pi v^2}{\ln\left(m_\gamma/m_q\right)}. \tag{7.1.7}$$

The main contribution to the tension in (7.1.7) comes from the logarithmic "tail" of the scalar field q. It is given by the kinetic term for the scalar field in (7.1.1). This term contains a logarithmic integral over r. Other terms in the action are

[1] It is also known as the McDonald function.

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suppressed by inverse powers of $\ln(m_\gamma/m_q)$ as compared with the contribution quoted in (7.1.7).

The results in Eqs. (7.1.5) and (7.1.7) imply that if we naively take the limit $m_q \to 0$ the string becomes infinitely thick and its tension tends to zero [103]. This apparently means that there are no strings in the limit $m_q = 0$. As was mentioned above, the absence of the ANO strings in the theories with the flat Higgs potential was first noted in [102].

One might think that the absence of the ANO strings means that there is no confinement of monopoles in the theories with the Higgs branches.

We hasten to say that this is a wrong conclusion.

As we will see shortly confinement does not disappear [103]. It is the formulation of the problem that has to be changed a little bit in the case at hand.

So far we considered infinitely long ANO strings. However, an appropriate setup in the confinement problem is in fact slightly different [103]. We have to consider a monopole–antimonopole pair at a large but finite separation L. Our aim is to take the limit $m_q \to 0$. This limit will be perfectly smooth provided we consider the ANO string of a finite length L, such that

$$\frac{1}{m_\gamma} \ll L \ll \frac{1}{m_q}. \tag{7.1.8}$$

Then it turns out [103] that $1/L$ plays the role of the infrared (IR) cutoff in Eqs. (7.1.5) and (7.1.7), rather than m_q. The reason for this is that for $r \ll L$ the problem is two-dimensional and the solution of the two-dimensional free equation of motion for the scalar field given by (7.1.6) is logarithmic. If we naively put $m_q = 0$ in this solution the McDonald function reduces to the logarithmic function which cannot reach a finite boundary value at infinity. Thus, as we mentioned above, infinitely long flux tubes do not exist.

However, for $r \gg L$, the problem becomes three-dimensional. The solution to the three-dimensional free scalar equation of motion behaves as

$$(q - v) \sim 1/|\vec{x}|$$

where x_n $(n = 1, 2, 3)$ are the spatial coordinates in the three-dimensional space.

With this behavior the scalar field reaches its boundary value at infinity. Clearly, $1/L$ plays the role of an IR cutoff for the logarithmic behavior of the scalar field.

Now we can safely put $m_q = 0$. The formula for the radius of the electromagnetic core of the vortex takes the form

$$R_g \sim \frac{1}{m_\gamma} \ln\left(m_\gamma L\right), \tag{7.1.9}$$

while the string tension now becomes [103]

$$T = \frac{2\pi v^2}{\ln\left(m_\gamma L\right)}. \tag{7.1.10}$$

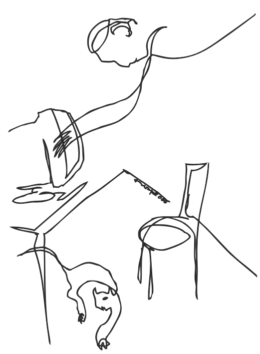
The ANO string becomes "thick." Nevertheless, its transverse size R_g is much smaller than its length L,

$$R_g \ll L,$$

so that the string-like structure is clearly identifiable. As a result, the potential acting between the probe well-separated monopole and antimonopole confines but is no longer linear in L. At large L [103]

$$V(L) = 2\pi v^2 \frac{L}{\ln\left(m_\gamma L\right)}. \tag{7.1.11}$$

The potential $V(L)$ is an order parameter which distinguishes different phases of a given gauge theory (see, for example, [71]). We conclude that on the Higgs branches one deals with a new confining phase, which had never been observed previously. It is clear that this phase can arise only in supersymmetric theories because we have no Higgs branches without supersymmetry.



7.2 Example: $\mathcal{N} = 1$ SQED with the FI term

Initial comments regarding this model are presented in Part I, see Section 3.2.2. The SQED Lagrangian in terms of superfields is presented in Eq. (3.2.1), while the component expression can be found in (3.2.5). For convenience we reiterate here crucial features of $\mathcal{N} = 1$ SQED, to be exploited below.

The field content of $\mathcal{N} = 1$ SQED is as follows. The vector multiplet contains the U(1) gauge field A_μ and the Weyl fermion λ^α, $\alpha = 1, 2$. The chiral matter

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multiplet contains two complex scalar fields q and $\tilde{q}$ as well as two complex Weyl fermions ψ^α and $\tilde{\psi}_\alpha$. The bosonic part of the action is

$$S_{\text{SQED}} = \int d^4x \left\{ \frac{1}{4g^2} F^2_{\mu\nu} + \bar{\nabla}_\mu \bar{q} \nabla_\mu q + \bar{\nabla}_\mu \tilde{q} \nabla_\mu \bar{\tilde{q}} + V(q, \tilde{q}) \right\}, \quad (7.2.1)$$

where

$$\nabla_\mu = \partial_\mu - \frac{i}{2} A_\mu, \quad \bar{\nabla}_\mu = \partial_\mu + \frac{i}{2} A_\mu.$$

Thus, we assume the matter fields to have electric charges $n_e = \pm 1/2$. The scalar potential of this theory comes from the D term and reduces to

$$V(q, \tilde{q}) = \frac{g^2}{8} (|q|^2 - |\tilde{q}|^2 - \xi)^2. \quad (7.2.2)$$

The parameter ξ is the Fayet–Iliopoulos parameter introduced through ξ_3.

The vacuum manifold of the theory (7.2.1) is the Higgs branch determined by the condition

$$|q|^2 - |\tilde{q}|^2 = \xi. \quad (7.2.3)$$

The dimension of this Higgs branch is two. To see this please observe that in the problem at hand we have two complex scalars (four real variables) subject to one constraint (7.2.3). In addition, we have to subtract one gauge phase; thus, we have $4 - 1 - 1 = 2$.

In general, the physics of the massless modes in theories with the Higgs branches can be described in terms of an effective low-energy sigma model

$$S_{\text{LE}} = \int d^4x g_{MN}(\varphi) \partial_\mu \varphi^N \partial_\mu \varphi^M, \quad (7.2.4)$$

where φ^M are massless scalar fields parametrizing the given Higgs branch and g_{MN} is the metric which depends on φ.

For example, the squark fields in $\mathcal{N} = 1$ SQED subject to the constraint (7.2.3) can be parametrized as follows:

$$q = \sqrt{\xi}\, e^{i\alpha + i\beta} \cosh \rho,$$
$$\bar{\tilde{q}} = \sqrt{\xi}\, e^{i\alpha - i\beta} \sinh \rho, \quad (7.2.5)$$

where α is an (irrelevant) gauge phase while $\rho(x)$ and $\beta(x)$ are two massless fields living on the Higgs branch. With this parametrization the sigma model (7.2.4) on the Higgs branch takes the form [175]

$$S_{\text{LE}} = \xi \int d^4x \left\{ \cosh 2\rho \left[(\partial_\mu \rho)^2 + (\partial_\mu \beta)^2 \tanh^2 2\rho \right] \right\}. \quad (7.2.6)$$

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From this expression one can immediately read off the two-by-two metric tensor.

The mass spectrum of $\mathcal{N} = 1$ SQED with the FI term, as it is defined in Eqs. (7.2.1) and (7.2.2), consists of one massive vector $\mathcal{N} = 1$ multiplet, with mass

$$m_\gamma^2 = \frac{1}{2} g^2 v^2, \tag{7.2.7}$$

(four bosonic + four fermionic states) and one chiral massless field associated with fluctuations along the Higgs branch. The VEV of the scalar field above is given by

$$v^2 = |\langle q \rangle|^2 + |\langle \tilde{q} \rangle|^2. \tag{7.2.8}$$

Next, following [175], let us consider strings supported by this theory. First we will choose the scalar field VEV to lie on the base point of the Higgs branch,

$$q = \sqrt{\xi}, \quad \tilde{q} = 0. \tag{7.2.9}$$

Then the massless field $\tilde{q}$ plays no role in the string solution and can be set to zero. This case is similar to the case of non-Abelian strings in $\mathcal{N} = 1$ SQCD described in detail in Section 5.1. On the base of the Higgs branch we do have (classically) the BPS ANO strings with the tension given by (4.2.12). In particular, their profile functions are determined by (3.2.18) and satisfy the first-order equations (3.2.19).

Now consider a generic vacuum on the Higgs branch. The string solution has the following structure [175]. The electromagnetic field, together with the massive scalar, form a string core of size $\sim 1/(g\sqrt{\xi})$. The solution for this core is essentially given by the BPS profile functions for the gauge field and massive scalar q. Outside the core the massive fields almost vanish, while the light (massless) fields living on the Higgs branch produce a logarithmic "tail." Inside this "tail" the light scalar fields interpolate between the base point (7.2.9) and the VEVs of scalars q and $\tilde{q}$ on the Higgs branch (7.2.3). The tension of the string is given by the sum of tensions coming from the core and "tail" regions,

$$T = 2\pi\xi + \frac{2\pi\xi}{\ln\left(g\sqrt{\xi}\, L\right)} l^2, \tag{7.2.10}$$

where l is the length of the geodesic line on the Higgs branch between the base point and the VEV,

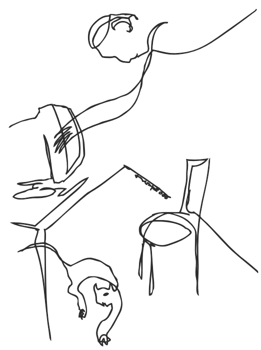
$$l = \int_0^1 dt \sqrt{g_{MN}\left(\partial_t \varphi^N\right)\left(\partial_t \varphi^N\right)}, \tag{7.2.11}$$

where g_{MN} is the metric on the Higgs branch, while φ^N stand for massless scalars living on the Higgs branch (see e.g. (7.2.6)). For example, for $v^2 \gg \xi$

$$l^2 = v^2/\xi,$$

and the "tail" contribution in (7.2.10) matches the result (7.1.10) for the string tension on the flat Higgs branch.

In (7.2.10) we consider the string of a finite length L to ensure infrared regularization. It is also possible [175] to embed $\mathcal{N} = 1$ SQED (7.2.1) in softly broken $\mathcal{N} = 2$ SQED much in the same way as it was done in Section 5.1 for non-Abelian strings. This procedure slightly lifts the Higgs branch making even infinitely long strings well defined. Note, however, that within this procedure the string is *not* BPS-saturated at a generic point on the Higgs branch.



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