
Conventions and Notations

Categories

We follow the von Neumann–Bernays–Gödel set theory and distinguish between *sets* and *classes*. All categories are assumed to be *locally small* in the sense that the objects form a class and for each pair of objects the morphisms between them form a set. When a category is abelian or exact, we assume in addition that for each pair of objects the extensions (in the sense of Yoneda) form a set.

We denote by Set the category of *sets* and by Ab the category of *abelian groups*. The *cardinality* of a set X is denoted by $\text{card } X$.

Morphisms are composed from right to left. For the composite $X \xrightarrow{\alpha} Y \xrightarrow{\beta} Z$ we write $\beta\alpha$.

Functors $\mathcal{C} \rightarrow \mathcal{D}$ are by convention covariant. Replacing one of the categories by its opposite category identifies contravariant functors $\mathcal{C} \rightarrow \mathcal{D}$ with covariant functors $\mathcal{C}^{\text{op}} \rightarrow \mathcal{D}$ or $\mathcal{C} \rightarrow \mathcal{D}^{\text{op}}$.

Rings and Modules

All rings are associative and have a unit.

For a ring Λ we consider the category $\text{Mod } \Lambda$ of *right Λ -modules* but drop the adjective ‘right’. Left Λ -modules are identified with modules over the *opposite ring* Λ^{op} . The full subcategory of finitely presented Λ -modules is denoted by $\text{mod } \Lambda$, and $\text{proj } \Lambda$ denotes the full subcategory of finitely generated projective Λ -modules.

When Λ and Γ are k -algebras over a commutative ring k , then Γ - Λ -*bimodules* ${}_{\Gamma}M_{\Lambda}$ are identified with modules over the algebra $\Gamma^{\text{op}} \otimes_k \Lambda$.

Numbers

We denote by \mathbb{Z} the set of integers and write

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

for the set of non-negative integers.