PENETRATIVE CONVECTION IN STARS

Jean-Paul ZAHN
Observatoire de Nice - FRANCE

I. Introduction

Penetrative convection occurs in a fluid whenever a convectively unstable region is bounded by a stable domain. This situation is encountered in many stars, and it is also a very common circumstance on Earth: in the oceans and in the atmosphere. One would therefore expect that the astrophysicists may largely benefit from the experience accumulated on this subject by the geophysicists.

However, this is only partly the case. In the ocean, salinity plays a very important role and especially so at the interface between a stable and an unstable (mixed) region. In the atmosphere, the behavior of the convective planetary boundary layer is dominated by the 24 hour thermal cycle, so that a steady state is never achieved , as it is in a star (at least in one that is not pulsating). Furthermore, the ratio between viscosity and conductivity, as measured by the Prandtl number, is of order unity for water and air, but it drops to 10^{-6} and less in a star. Finally, the effects of stratification are much stronger in stars where convective regions often span several density scale heights.

For all these reasons, the astrophysicists have developed methods of their own to describe stellar convection, even though some are widely inspired by those used by the geophysicists. The same is true for convective penetration, whose study cannot be separated from that of convection itself. The purpose of this review will be to recall those methods, and to verify if they are suited to describe the penetration of convective motions into stable surroundings.

II. Phenomenological approaches

In those approaches, one hypothesizes a flow which is plausible in that it does not seemingly contradict the laws of fluid dynamics and that it conserves heat and kinetic energy. One then calculates the gross parameters that characterize this flow: convective flux, mean temperature gradient. The most commonly used of such procedures are based on the concept of mixing length, and have already been discussed in this colloquium by D.O. Gough.

https://doi.org/10.1017/S0252921100112448 Published online by Cambridge University Press

1. Non-local mixing-length treatments

All mixing-length procedures applied to stellar convection are in fact based on the two differential equations describing:

i) the variation with height z of the density excess $\delta\rho$ between a convective element and the surrounding medium, in which the densities are respectively ρ^* and ρ

$$\frac{d}{dz}(\delta\rho) = \frac{d\rho^*}{dz} - \frac{d\rho}{dz} \quad , \tag{1}$$

ii) the variation of the kinetic energy of that convective element

$$\frac{d}{dz}\left(\frac{1}{2}\rho v^2\right) = -\delta\rho\, g \quad , \tag{2}$$

where g is the gravity.

The standard prescription (Vitense 1953) is to replace these equations by

$$\delta\rho = \left[\frac{d\rho^*}{dz} - \frac{d\rho}{dz}\right]\frac{\ell}{2} \quad , \tag{3}$$

$$\frac{1}{2} v^2 = -C\, g\, \frac{\delta\rho}{\rho}\, \frac{\ell}{2} \quad , \tag{4}$$

ℓ being the mixing length and C an efficiency factor which allows for the production of turbulent energy. In this treatment, both the density excess and the convective velocity are functions of local quantities only (the mixing length and the density gradients); by construction the convective motions cannot penetrate into the stable adjacent regions.

That constraint may however be relaxed by treating the original differential equations in a less crude way. This was done by Shaviv and Salpeter (1973), Maeder (1975a) and Cogan (1975), to be specifically applied to the overshooting from a convective stellar core. The differential equations are integrated over one mixing length (or up to the point where the velocity vanishes, whichever happens first):

$$\delta\rho = \int_{z_i}^{z}\left[\frac{d\rho^*}{dz} - \frac{d\rho}{dz}\right] dz \quad , \tag{5}$$

$$\frac{1}{2} v^2 = -C\int_{z_i}^{z} g\,\frac{\delta\rho}{\rho}\, dz \quad , \qquad z - z_i \leqslant \ell \; . \tag{6}$$

https://doi.org/10.1017/S0252921100112448 Published online by Cambridge University Press

(To formally recover certain results of the standard scheme, Maeder identifies the integration distance with half the mixing length). The density stratification $d\rho/dz$ of the ambient medium is adjusted until the constancy of the total energy flux (convective plus radiative) is realized.

This non-local mixing-length treatment permits the description of many features of penetrative convection in the laboratory or in the Earth's atmosphere. A convective element ceases to be buoyant at some distance from the unstable region, where also the convective flux vanishes; from there on its momentum carries it still further into the stable region, and since it is cooler than the surrounding medium, the convective flux is of opposite sign. In a stellar core, the Péclet number is very high and thus the convection is extremely efficient; it follows that the whole domain where the motions occur is kept nearly adiabatic.

The main weakness of this approach, as one may expect, is that all quantitative predictions depend on the assumption made for the mixing length. Another parameter plays here also some role, and it too can only be guessed: it serves to measure the fraction of space filled by the convective elements. In the bulk of the unstable domain this parameter is probably close to unity, but in the overshooting region, it drops to one half and possibly much less, because it is unlikely that many downwards moving elements are present there.

In a generalization of the mixing-length procedure proposed by Spiegel (1963), the number of convective elements is not fixed a priori, but is governed by an equation of conservation similar to the radiative transfer equation. Travis and Matsushima (1973) have applied this non-local theory to the solar atmosphere, and they obtain an appreciable overshooting into the photosphere. In order to match the solar limb-darkening observations, they must choose a ratio of mixing length to pressure scale height of 0.35 or less. Unfortunately, this value is too small to yield the correct solar radius, within the assumptions that can be made for the chemical composition. Travis and Matsushima suggest that this discrepancy be removed by allowing the above mentioned ratio, between mixing length and scale height, to vary with depth.

https://doi.org/10.1017/S0252921100112448 Published online by Cambridge University Press

2. Other procedures

A different approach has been used by the meteorologists to model cloud dynamics (Stommel 1947). It is based on the concept of thermals, and has since been applied to a variety of other problems; it was Moore (1967) who brought it to the attention of the astronomical community. A thermal is an organized cell which, like the eddy of the mixing-length treatment, exchanges heat and momentum with the surrounding medium, but has also the property of gaining or loosing matter through entrainment or turbulent surface erosion.

The only serious attempt to apply this concept to an astrophysical case was made by Ulrich (1970 a, b), who used it to build a model of the solar atmosphere. He had to overcome such difficulties as the absence of any ground level (from where the thermals start on Earth), fragmentation (since the thermals are bound to move over several scale heights) and radiative exchanges (the Péclet number becomes rather small above a certain level). His model displays substantial overshooting well into the photosphere, but one may wonder whether this is not due mainly to a simplifying assumption he made for the correlation between the velocity of a thermal and its temperature excess. Another consequence of this is that there is no sign change of the convective flux in the stable region.

A similar treatment has been proposed recently by Nordlund (1976), in which the medium is organized in two streams of rising and falling fluid. Those behave like the thermals in the sense that they too exchange matter, heat and momentum, but here there is no ambient medium. Dimensional arguments are invoked to write down the equations governing the exchanges between the two streams. Solar models constructed with this procedure are characterized by an appreciable penetration up to an optical depth of $\tau = 0.1$; the quantitative predictions of course depend on the choice of the dimensionless parameters that occur in the equations.

https://doi.org/10.1017/S0252921100112448 Published online by Cambridge University Press

III. Direct approaches

In the past ten years a new approach has been explored thanks to the fast computers with large memory storage that are now available: one can start directly from the fluid dynamics equations, instead of replacing them by simpler ones that are more tractable. Of course, it is not feasible yet to treat the most general problem: as we will see, the solutions obtained to date all suffer from some kind of restriction. But at least they help to build up an intuition which has been lacking so far. We shall consider here only the nonlinear investigations; the main interest of the linear studies has been to determine the critical conditions (Gribov and Gurevich 1957, Stix 1970, Whitehead 1971), but they cannot be used to predict the extent of penetration, which is strongly related to the amplitude of the solution.

1. Boussinesq convection

The prototype of penetrative convection in the laboratory is the ice-water experiment suggested by Malkus (1960) and performed among others by Townsend (1964) and Myrup *et al.* (1970). Water has the peculiar property of presenting a density maximum at 4°C, so that a tank of water whose bottom is kept at 0°C will be convectively unstable up to the level of maximum density, and stable above. Veronis (1963) gave the criterion for the onset of the instability, which is of the finite amplitude type. Thereafter Musman (1968) made the first quantitative predictions for the extent of penetration, using the so-called mean-field approximation (Herring 1963). The next improvement came from Moore and Weiss (1973), who solved the two-dimensional problem without further simplification.

A slightly different experiment is that of a fluid heated in its bulk by Joule effect, in which the parabolic temperature profile creates two superposed domains of respectively unstable and stable stratifications (Tritton and Zarraga 1967). This experiment has been modelled by Strauss (1976), again with a two-dimmensional code; his results are similar to those of Moore and Weiss (1973).

These two-dimensional studies are fairly successful in predicting, at moderate Rayleigh numbers, the mean temperature profile and thus the extent of penetration. But it is doubtful that they can be extrapolated to the parameter range which is of astrophysical interest (high Rayleigh numbers and low Prandtl numbers). Moreover, these two-dimensional studies are unable to describe the time-dependent temperature fluctuations which are observed at the boundary of the well-mixed region. These seem to be excited randomly, and are essentially three-dimensional in their nature. The astrophysical importance of these oscillations must not be underestimated: in the Sun, they would occur just at the base of the photosphere and would generate gravity waves.

Another suggestion that the two-dimensional studies may be somewhat misleading comes from the results obtained by Latour *et al.* (1977). They analyze the penetration of convective motions from an unstable slab into the stable adjacent regions. The solutions are expanded into orthogonal modes in the horizontal, and a finite differences scheme is used in the vertical. In the special case of a single mode with a two-dimensional planform, this procedure reduces to the mean-field approximation of Herring used by Musman (1968). But one can also choose a three-dimensional planform representing, for instance, prismatic cells of hexagonal base. The comparison of solutions derived with the two types of planforms reveals that penetration is much stronger when the convective motions are allowed to be three-dimensional (Figure 1). In the simplest three-dimensional case, where only a single planform is retained, the solutions are asymmetrical: the overshooting occurs mainly on the side to which the centerline flow is directed in the hexagonal cells. The mean temperature profile becomes symmetrical again when one superposes two patterns of hexagonal cells with opposite centerline velocities; remarkably enough, the total kinetic energy of the flow does not vary as one switches from the one-mode solution to this two-mode solution. And the total extent of penetration too remains unchanged, if it is defined as the sum of the penetration depths at either side of the unstable layer.

2. Convection in a stratified medium

In the laboratory (or Boussinesq) case, the extent of penetration is related to the only natural length that characterizes the problem, namely the thickness of the unstable layer. But what should one expect in a stratified medium, such as the solar convection zone, where the unstable domain spans several density or pressure scale-heights?

This question has not been answered yet. Toomre *et al.* (1976) have studied the penetration from the deeper convection zone of an A-type star; this zone is due to the second ionization of helium, and it measures about one pressure scale height. Using the technique mentioned above of truncated modal expansion, and retaining only one single three-dimensional mode, they find that the motions penetrate up to one scale height into the stable region below. More recently, they have established that the convective motions penetrate also above, as far as to build a link between the deeper convection zone and the upper one, which is caused by the ionization of hydrogen. But the situation considered is admittedly not one of severe stratification, and these results cannot be extrapolated to the Sun, for instance. Moreover, the solutions obtained so far are all stationary, missing thereby the time-dependent character of penetrative convection which may be of primordial importance.

Another difficulty with these drastically truncated modal calculations is that they depend on the choice made for the horizontal wavelength of their single planform. Fortunately, the results are not too sensitive to this parameter, which is felt mainly in the horizontal heat exchanges; it does not play the dominant role of the mixing length in the phenomenological approaches.

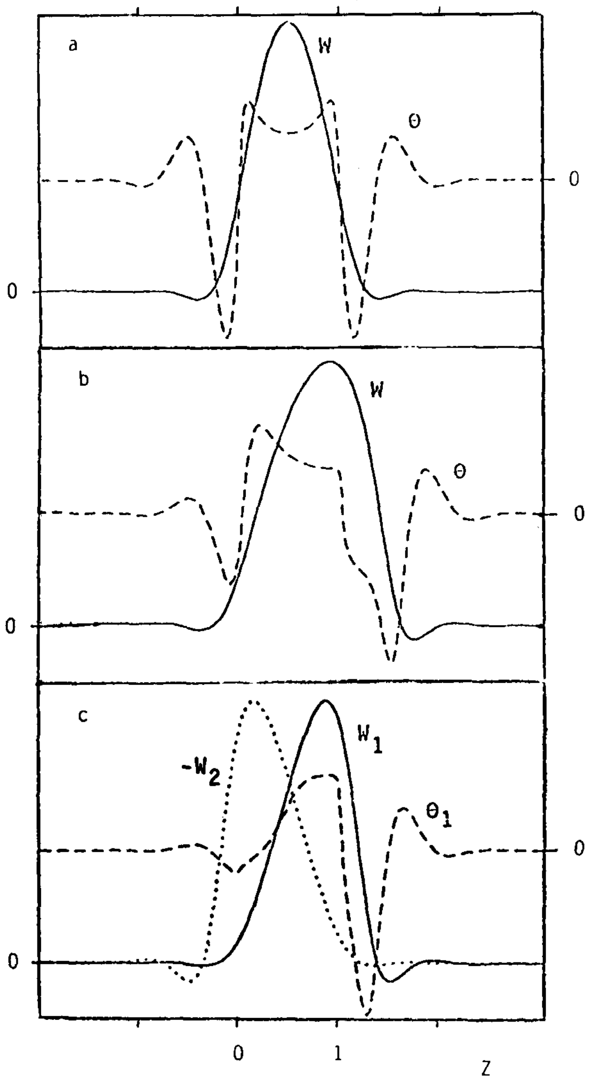

Figure 1. Modal solutions for penetrative Boussinesq convection.

The unstable layer, which extends in depth from z = 0 to z = 1, is imbedded in an infinite stable domain from which only a fraction of thickness $\Delta z = 2$ on each side is shown here. The same Rayleigh number $R = 10^5$ characterizes the stability and the instability of the three superposed layers (it corresponds to about thousand times critical). The amplitudes of the vertical velocity, W, and of the temperature fluctuations, Θ, are displayed as functions of z. Figure 1a shows a single two-dimensional mode (which may be visualized as a horizontal roll), figure 1b a single three-dimensional mode of hexagonal horizontal planform, and figure 1c two non-interacting three-dimensional modes of that same geometry. In all cases, the value of the horizontal wavenumber is 2, and the Prandtl number is 1. Notice that the overshooting into the stable surroundings is much more pronounced with the three-dimensional motions.

The only way to avoid any extra assumption would of course be to directly integrate the basic equations in three-dimensional space. This has been done by Graham (1975), whose latest results are presented in this colloquium. But even the most powerful computers which are presently available set a rather low limit on the number of gridpoints that can be used. This in turn fixes the highest Rayleigh or Reynolds numbers that can be reached: typically one hundred times critical. There is thus still a very long road to go before meeting the numbers characterizing a stellar convection zone, but in the meanwhile these numerical experiments are very useful as a workbench to test the various approximations that have been proposed.

IV. Observational tests

It is relatively easy to confront theoretical predictions of Boussinesq penetrative convection with laboratory experiments. But, as we were already reminded by K.H. Böhm, the comparison of astrophysical models with stellar or solar observations is more delicate, for the physical parameters that can be determined often depend on other factors than just the properties of convection.

For the stars, one is forced to rely on the few gross parameters which can be observed. In principle the classical tests for probing the internal structure of a star may be used to determine the extent of the regions which are in nearly adiabatic stratification, at least once their location is roughly known. These tests can complement each other: the apsidal motion test (see Schwarzschild 1958) is more sensitive to the overall mass concentration in a star, whereas the pulsational period of a variable star (see Ledoux and Walraven 1958) depends more on the stratification of its envelope. There is even a slight hope to interpret the properties of the dynamical tide in a close binary system, which are closely related to the size of the quasi-adiabatic core of the two components (Zahn 1977).

But the most promising tests are probably those which sense the inhomogeneities in chemical composition. Prather and Demarque (1974) and Maeder (1975b, 1976) have included some amount of overshooting in their calculations of evolutionary stellar models. They find that the evolutionary tracks, lifetimes and cluster isochrones all are appreciably modified by an increase of the convective core. Prather and Demarque obtain the best fit between their theoretical isochrones and the cluster diagram of M 67 for a penetration depth of about 10% of the pressure scale height; Maeder's value is slightly less and he uses it to calibrate his non-local mixing-length procedure.

The thickness of a convective envelope (together with its penetrative extension) may be inferred from the abundance of elements which undergo nuclear destruction at moderate remperatures, such as lithium, beryllium and boron. In the case of the Sun, additional information can be gathered from the composition of the solar wind (Bochsler and Geiss 1973). But when interpreting such observations, one must keep in mind that other instabilities than convection may also lead to a thorough mixing of the stellar material.

It looks at first sight as if the Sun should be the ideal object on which to check the theories of penetrative convection. The solar atmosphere becomes convectively unstable below optical depth $\tau = 1$, which means that the overshooting motions should occur in the photosphere and thus be visible. The difficulty however is to distinguish in the observations of Doppler-shifted lines what is due to waves or oscillations, and what is due to genuine penetrative convection. The accuracy of correlation measurements between velocities and temperature fluctuations is still not sufficient to permit the separation of both types of motions (for a recent and complete review on such measurements, see Beckers and Canfield 1976). And one encounters the same problem when it comes to the interpretation of the non-thermal energy flux: the convective (enthalpy) flux is blended with the flux of kinetic energy, which is carried by both convection and waves. But the solar observations are rapidly progressing toward better precision and spatial resolution, and one may hope that these questions will be settled in the not too distant future.

Bibliography

Beckers,J.M., Canfield,R.C. 1976, Physique des Mouvements dans les Atmosphères Stellaires, R.Cayrel and M.Steinberg eds., CNRS, p.207

Bochsler,P., Geiss,J. 1973, Solar Phys. 32, 3

Cogan,B.C. 1975, Astrophys. J. 201, 637

Graham,E. 1975, J. Fluid Mech. 70, 689

Gribov,V.N., Gurevich,L.E. 1957, Soviet Phys. JETP 4, 720

Herring,J.R. 1963, J. Atmos. Sci. 20, 325

Latour,J., Toomre,J., Zahn,J.P. 1977 (in preparation)

Ledoux,P., Walraven,Th. 1958, Handbuch der Physik, t.51, p.353 (Springer)

Maeder, A. 1975a, Astron. & Astrophys. 40, 303

Maeder, A. 1975b, Astron. & Astrophys. 43, 61

Maeder, A. 1976, Astron. & Astrophys. 47, 389

Malkus,W.V.R. 1960, Aerodyn. phenomena in stellar atmosph., p.346 (Thomas edit)

Moore,D.W. 1967, Aerodyn. phenomena in stellar atmosph., p.405 (Thomas edit.)

Moore,D.R., Weiss,N.O. 1973, J. Fluid Mech. 61, 553

Musman,S. 1968, J. Fluid Mech. 31, 343

Nordlund,Å. 1976, Astron. & Astrophys. 50, 23

Prather,M.J. Demarque,P. 1974, Astrophys. J. 193, 109

Schwarzschild,M. 1958, Structure and evolution of stars, p.146 (Dover)

Shaviv,G., Salpeter,E.E. 1973, Astrophys. J. 184, 191

Spiegel,E.A. 1963, Astrophys. J. 138, 216

Stix,M. 1970, Tellus 22, 517

Stommel,H. 1947, J. Meteorol. 4., 91

Strauss,J.M. 1976, Astrophys. J. 209, 179

Toomre,J., Zahn,J.P., Latour,J., Spiegel,E.A. 1976, Astrophys. J. 207, 545

Townsend,A.A. 1964, Quart. J. Roy. Meteorol. Soc. 90, 248

Travis,L.D., Matsushima,S. 1973, Astrophys. J. 138, 216

Tritton,D.J., Zarraga,M.N. 1967, J. Fluid Mech. 30, 21

Ulrich,R.K. 1970a, Astrophys. & Space Sci. 7, 71

Ulrich,R.K. 1970b, Astrophys. & Space Sci. 7, 183

Veronis,G. 1963, Astrophys. J. 137, 641

Vitense,E. 1953, Z. für Astroph. 32, 135

Whitehead,J.A., Chen,M. 1970, J. Fluid Mech. 40, 549

Whitehead,J.A. 1971, Geophys. Fluid Dynamics 2, 289

Zahn,J.P. 1977, Astron. & Astrophys. 57, 383

https://doi.org/10.1017/S0252921100112448 Published online by Cambridge University Press