CORRESPONDENCE.

ON THE APPLICATION OF THE DIFFERENTIAL AND INTEGRAL CALCULUS TO "INTEREST" QUESTIONS.

To the Editor of the Assurance Magazine.

SIR,—Having been kindly favoured by Professor De Morgan with some remarks on my Paper, lately read before the Institute, and to be inserted, I believe, in your present Number, your readers may be glad to have the following additional illustration of the application of the Calculus to Interest questions, as afforded by so eminent a mathematician. The illustration may be used to indicate, that if but one mean annual rate of interest be assumed for all periods alike, whether long or short, as is commonly done, that such rate can only be properly taken as typical of average accumulation, by supposing all investments broken up into yearly periods, with annual repayment of principal and annual chance of unproductiveness, or of reinvestment at such rates as may yearly happen to obtain.

Without enlarging on the manner in which the differentials of the second and higher orders disappear, the Professor says:

"Let $\phi x dx$ be the chance that the year's rate of interest per £1 lies between x and $x+dx$; let $x_1, x_2, \ldots x_n$ be the years' rates actually occurring in the n separate years; then the £1 will amount to $(1+x_1)(1+x_2)\ldots.(1+x_n)$, and the chance of such event is $\phi x_1 dx_1 . \phi x_2 dx_2 \ldots\ldots \phi x_n dx_n$: whence the equivalent certainty for such a case is $(1+x_1)(1+x_2)\ldots.(1+x_n).\phi x_1 \phi x_2 \ldots. \phi x_n . dx_1 dx_2 \ldots. dx_n$; and for all possible cases, α and β being the extreme possible rates,

$$\int_\alpha^\beta \int_\alpha^\beta \ldots \int_\alpha^\beta (1+x_1)(1+x_2)\ldots(1+x_n).\phi x_1 \phi x_2 \ldots \phi x_n . dx_1 dx_2 \ldots dx_n,$$

which, by reason of the independence of the limits, and the factorial separability of the functions, is

$$\int_\alpha^\beta (1+x_1)\phi x_1 dx_1 \times \int_\alpha^\beta (1+x_2)\phi x_2 dx_2 \ldots. \times \int_\alpha^\beta (1+x_n)\phi x_n dx_n,$$

reducible to

$$\left\{\int_\alpha^\beta (1+x)\phi x\, dx\right\}^n.$$

If all cases be equally likely, $\phi x dx = \dfrac{dx}{\beta-\alpha}$, and

$$\frac{1}{\beta-\alpha}\int_\alpha^\beta (1+x)dx = \frac{(1+\beta)^2-(1+\alpha)^2}{2(\beta-\alpha)} = 1+\frac{\beta+\alpha}{2},$$

so that the mean value for the end of the nth year is $\left(1+\dfrac{\beta+\alpha}{2}\right)^n$, or the common result when the mean rate of interest is assumed for all periods."

A more striking instance, perhaps, could not have been given, of how completely the Calculus adapts itself to the investigation of even the commonest assumptions in actuarial subjects; and thus enables us to ascertain the exact conditions with which such assumptions are really connected.

Thus, while the above illustration shows that the common assumption of the mean rate of interest between the limits is a fair one when loans and repayments are dealt with year by year, the table in the Paper continues the series and indicates how such an assumption should be modified to represent the average accumulations of longer transactions. A real and not a fictitious tabular simplicity of results may thus, it is hoped, be gradually brought about in actuarial calculations; and with the greater and greater effect, as the supposed difficulties of variation, instead of being evaded, become more and more thoroughly studied.

To prevent undue inferences, it is right to state that the eminent mathematician alluded to, is not to be considered as in any way answerable for the contents of the paper in question, nor indeed as an implied authority either for or against the principles therein enunciated.

Your obedient Servant,
EDWIN JAS. FARREN.

Hanover Chambers, Buckingham Street, Strand,
London, February 2nd, 1855.

ON THE FACILITIES AFFORDED BY MR. THOMSON'S ACTUARIAL TABLES IN MAKING CERTAIN CALCULATIONS.

To the Editor of the Assurance Magazine.

SIR,—It would, I think, be useful, if you were to invite communications from your readers of such questions as they may meet with in practice, and which are not to be found in the text books. If you approve of this suggestion, and think the accompanying case worthy of a place in your *Magazine*, perhaps you will kindly insert it. The facility with which the formula is worked out affords another instance of the usefulness of Mr. Thomson's Actuarial Tables, and of the consequent benefit they confer on the profession.

I am, Sir, yours truly,
ROBERT TUCKER.

Lombard Street, 8th February, 1855.

What single and annual premium should be charged to secure £100 per annum to A, aged 32, after the death of B, aged 40, provided B die within 5 years (Carlisle 3 per cent.)?

The value of that portion of the annuity which may be enjoyed by A during the first five years is evidently $_{\overline{5|}}A - {_{\overline{5|}}AB}$; and it is equally clear, that the value of the remaining portion is $^{5}A \times {_{\overline{5|}}\mathfrak{B}}$ ∴ the total value of the annuity is $_{\overline{5|}}A - {_{\overline{5|}}AB} + {^{5}A} \times {_{\overline{5|}}\mathfrak{B}}$.

Thomson, Table 1, Single Lives.		Table 2, Single Deaths.
$A = 19{\cdot}13521$	$^{*}AB = 14{\cdot}30229$	$\mathfrak{B} = {\cdot}47158$
$A_{-\vert 5} = 14{\cdot}69049$	$_{-\vert 5}AB = 9{\cdot}91515$	$_{-\vert 5}\mathfrak{B} = {\cdot}40884$
$_{\overline{5\vert}}A = 4{\cdot}44472$	$_{\overline{5\vert}}AB = 4{\cdot}38714$	$_{\overline{5\vert}}\mathfrak{B} = {\cdot}06274$
$4{\cdot}38714$		
$_{\overline{5\vert}}A - {_{\overline{5\vert}}AB} = 0{\cdot}05758$		

* AB being taken equal to a single life of 50.

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