NONADIABATIC, NONLINEAR PULSATIONS OF BUMP CEPHEIDS - A NEW APPROACH

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Abstract

Finite amplitude pulsations of realistic stellar models are analyzed within the framework of a promising asymptotic perturbation approach and the results are compared with those of numerical hydrodynamic studies.

The pulsations of Cepheid variables are characterized by the fact that the growthrates of the excited modes are fairly small compared to their oscillation frequencies. While this is a distinct drawback for the numerical hydrodynamic modelling of the nonlinear behavior of such stars, it can be used to one's advantage in a nonadiabatic nonlinear perturbation formalism (**NNPF**) (Buchler & Goupil 1984; **BG84**). Previous perturbation approaches (e.g. Buchler 1978, Regev & Buchler 1981, Dziembowski & Kovacz 1984, Takeuti & Aikawa 1982) have assumed the pulsations to be quasiadiabatic throughout the whole star, that is also in the obviously nonadiabatic outer layers, and thus conveniently used the real adiabatic eigenvectors as a perturbation basis. While quite useful for analytical considerations, the quasiadiabatic approximation unfortunately does not lead to an unambiguous practical computational scheme (Pesnell & Buchler 1984). In contrast, the NNPF includes the nonadiabatic effects already in lowest order through the use of the linear nonadiabatic (**LNA**) eigenvectors. The price one has to pay is that these eigenvectors are not only complex, but, in addition, necessitate the introduction of the left (adjoint) LNA eigenvectors, which form a dual set orthogonal to the right eigenvectors. One of the small parameters of problem is then the ratio r of the growthrate, κ, to the oscillation frequency, Ω. Since r is a global parameter, nonadiabatic effects can be arbitrarily large as long as r remains small. The basic assumption of the NNPF is that the LNA modes can be split into two groups, one containing the strongly damped (slave) modes and one containing the marginally unstable, excited modes together with those marginally stable modes which may get entrained either because of a low order internal resonance or because of a nonlinear loss of stability. When these conditions are satisfied the center manifold theorem and the theory of normal forms guarantee us that amplitude equations (**AE**) can be found, which involve only the amplitudes of the second group of modes.

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So far, we have applied (Klapp, Goupil and Buchler 1984; **KGB84**) the NNPF to the case of Cepheid models, of population I (Bump Cepheids) and of population II (paradigmatized by BL Her), in the regime where they have an internal resonance of the type $2\Omega_0 \simeq \Omega_2$. The subscripts 0 and 2 refer to the fundamental and second overtone, respectively. The apposite AE are given by (BG84)

$$\frac{da_0}{dt} = \sigma_0 a_0 + N_0 a_0^* a_2, \quad \text{and} \quad \frac{da_2}{dt} = \sigma_2 a_2 + N_2 a_0^2 \quad , \qquad (1)$$

where $\sigma_\nu = i\Omega_\nu + \kappa_\nu$ are the LNA eigenvalues, a_0 and a_2 are the complex amplitudes for the two resonant modes and the coefficients N_0 and N_2 involve the quadratic nonlinearities of the momentum and heat flow equations, expanded around hydrostatic and thermal equilibrium. These quadratic operators are then sandwiched between appropriate right and left LNA eigenvectors (BG84). In lowest order the displacement can be written as

$$\delta R(m) = 0.5 \left[a_0(t)\ \xi_0(m) + a_2(t)\ \xi_2(m) + \text{c.c.} \right] + \ldots, \qquad (2)$$

where the $\xi_\nu(m)$ are the radial parts of the right LNA vectors.

The solution of eq. 1 provides the time dependence of the amplitudes in eq. 2. By introducing moduli and phases for the amplitudes, $a_\nu(t) = A_\nu(t)\exp(i\Omega_\nu t)\exp(i\theta_\nu(t))$, it is possible to reduce the two complex AE to a set of three real AE involving A_0, A_2 and $\Gamma = \theta_2 - 2\theta_1$ only. Of particular interest are the solutions with constant A_0, A_2 and Γ (stable fixed points of the AE), corresponding to oscillations in which the amplitudes of the two modes are constant. Interesting, more complicated behavior is also possible (KGB84), namely periodic energy transfer between the two modes or irregular (chaotic) transfer.

The physical content of our AE (1) is very simple. The fundamental mode is unstable, saps thermal energy from the star and converts it initially exponentially fast into mechanical (pulsational) energy. As the oscillation amplitude gets sufficiently large energy is shared with the resonant overtone through the nonlinear terms. The latter mode, bring vibrationally stable, restores energy back to the thermal reservoir. Stable fixed points (and more general bounded solutions as well) can result from a balance between these effects.

Figure 1 shows an H-R diagram for a specific set of models of 4 solar masses. Our AE assume a near resonance to hold and we have indicated lines of constant period ratio, P_2/P_0 marked 0.48, 0.50 and 0.52. Also exhibited is the line (F) below which the fundamental mode is unstable. Finally to the right of the line marked (FPB) our AE have no fixed points.

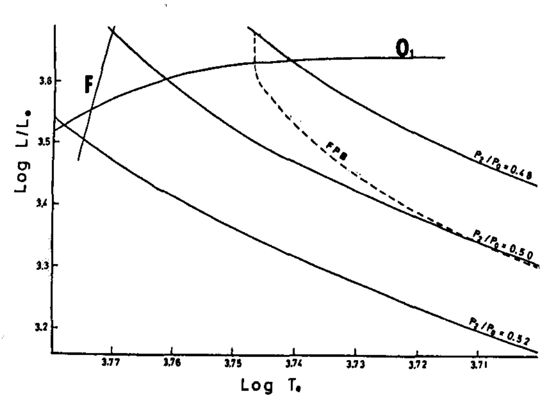

Fig. 1 H-R Diagram

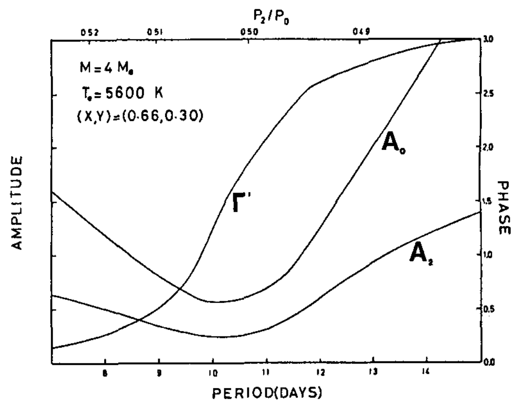

Fig. 2 Sequence of models

Fig. 2 shows the behavior of the amplitudes and phases for a sequence of models of constant effective temperature. This behavior, namely the variation of $\Gamma' \equiv \Gamma + \pi$ through $\pi/2$ and the dip of the amplitudes near the resonance is typical and can readily be derived from a mere inspection of the AE (KGB84). Similarly it can be shown that these features do not occur exactly at the resonance, the deviation depending on the magnitude of the ratios (Re N_ν/Im N_ν). This, while having been held against a resonance origin of the bumps (Vemury & Stothers 1978), in fact corroborates it. If the quadratic terms were evaluated with adiabatic eigenvectors, the N_ν would be purely imaginary. The size of these ratios and the shift of the above features from exact resonance are then an overall measure of the nonadiabaticity of the pulsations.

In fig. 3 we show the (scaled) surface velocities for the sequence of models of fig. 2, clearly exhibiting the so-called Herzsprung progression of the bump. The calculated surface velocities agree to within a factor of two with those obtained with comparable numerical hydrodynamic models.

We have studied a variety of resonant Cepheid models (KGB84). In all the models in which our AE predict a steady bump, hydrodynamic models indeed also find one.

To be fair we should also mention some of the limitations of the AE. The AE cannot claim to reproduce all the physics of the envelope, for example, shock waves, which involve high nonlinearities. Similarly they cannot pretend to reproduce the skewed observed velocity profiles in the atmosphere which occur in the dilute atmosphere. However, as long as the shock waves or the

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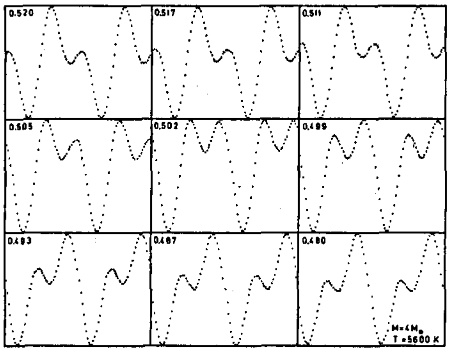
Fig. 3 Surface velocity for the sequence of models of fig. 2.



outermost dilute layers are not instrumental in the saturation mechanism, our AE are a useful and fast computational tool for determining the gross features of the nonlinear behavior of a given stellar model, i.e., the type of finite amplitude behavior (stable fixed point, limit cycle, irregular attractor), the magnitude of the velocities and the position (phase) of the bump. The NNPF is thus seen to be complementary to the much more (hundredfold) expensive, but more detailed numerical hydrodynamic approach.

We conclude that the good agreement with the hydrocode suggests that our formalism captures the basic saturation mechanism and can be used with some confidence to study the nonadiabatic nonlinear behavior of large classes of stellar models. To finish we want to emphasize that the NNPF is not limited to the resonant models considered in this contribution, but applies to other models as well.

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