62. LARGE SCALE OSCILLATIONS OF GALAXIES

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Abstract. The observational evidence that may indicate the presence of large scale modes of oscillation in galaxies is reviewed, and some results of theoretical modal calculations are described.

1. Observational Evidence

Although to a first approximation the matter in galaxies rotates in circular orbits about a center, there are also indications of large scale systematic and non-random departures from purely circular motion. A natural interpretation of these motions is that they indicate the presence of various free modes of oscillation of the galaxy that have been excited at some time in the past, either during the formation of the galaxy or as a result of a tidal interaction with another galaxy. Three important pieces of evidence of large scale non-circular motions are listed below.

(i) Kerr (1962) found that the Northern and Southern hemisphere 21 cm maps of our Galaxy fitted together better if there is a general radial outflow of 7 km s^{-1} in the solar neighborhood.

(ii) It has been noticed in many instances that the derived rotation curves of external galaxies are not symmetrical about a center. One example is M31 for which Burke *et al.* (1964) found different rotation curves for the opposite NF and SP sections. Such differences can be explained by the presence of modes of odd angular wave number m. Presumably $m = 1$ modes are the most fundamental of these and are the most likely to be prominent.

(iii) Roberts' (1966) detailed map of the observed radial velocity field for M31 (his Figure 2) is markedly different from that which would be observed for pure circular motion. In particular the minor axis is not a line of constant radial velocity (the systemic radial velocity) but shows an inflow of matter towards the center at distances less than 9 kpc, and an outflow at greater distances.

This interpretation assumes that M31 is flat. An alternative explanation is possible in terms of a bending of the plane of M31. Reasons for rejecting this explanation are that substantial bending is required on account of the high inclination of M31, whereas the theoretical analysis of Hunter and Toomre (1969) showed that bending of the central regions of a galaxy is hard to maintain. Our Galaxy, for example, is bent substantially only in the outer parts (Kerr *et al.*, 1957).

Roberts remarked on the similarity of opposite quadrants of his radial velocity map, and this indicates the predominance of motions of even angular wave number. Figures 1 and 2 show the isovel maps of the upper half of M31 that result from adding a prescribed radial velocity in the plane of the galactic disk of M31 to the circular velocity derived by Roberts. The form of the radial velocity used was chosen to agree

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roughly with that shown in Roberts' map along the minor axis, and was then extended over the whole plane of the disk. Figure 1 results from the addition of an axisymmetric non-circular motion. Isovel lines in the left hand quadrant have the more involved forms away from the central regions, and in some areas slope counter to the direction they would for pure circular motion (Roberts' Figure 3). This is in qualitative agreement with Roberts' Figure 2. The features noted above are more marked in my

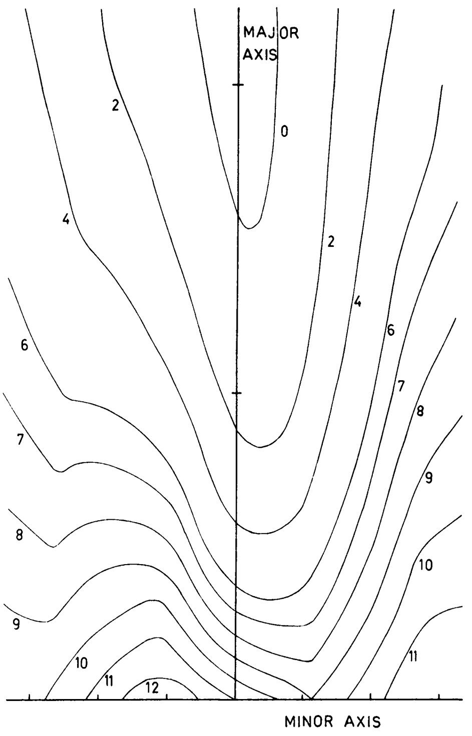

Fig. 1. Curves of constant radial velocity for an axisymmetric radial motion added to the circular velocity field of M31. A unit change between isovels corresponds to a jump of -21 km s^{-1}, and 11 represents the systemic velocity of -310 km s^{-1}. The marks along the axes are at 4 kpc intervals.

Figure 2 in which the added radial motion has an $m=2$ angular variation. Greater radial motions are present in this case, and the figure changes considerably with the orientation of the motion. Since the beam width used for the observations was relatively large, the true isovel map is still somewhat uncertain, but the indications are that it is consistent with the presence of organized large scale non-circular motions in the plane of M31.

2. Theoretical Calculations of Modes

The most straightforward calculations of free modes of oscillation are those obtained using the 'cold disk' model. A particular equilibrium model is selected, typically by

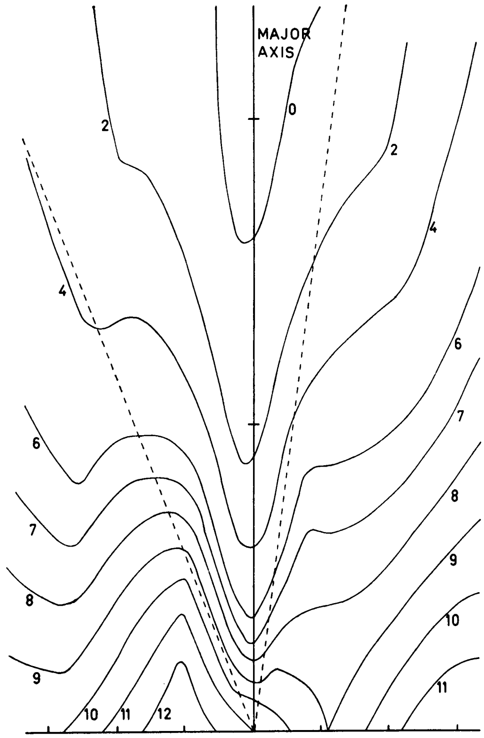

Fig. 2. Curves of constant radial velocity for an $m = 2$ radial motion added to the circular velocity field of M31. The dashed lines show the directions of maximum amplitude of the radial motion.

taking a given rotation curve and then finding a finite mass distribution whose gravitational attraction provides the necessary centrifugal force. Possible free oscillations are determined by solving in a fully self-consistent manner the zero pressure hydrodynamic equations that govern departures from the unperturbed state of circular motion (Hunter, 1965). Although such an analysis produces unstable modes of short wavelength where there is nothing to prevent the growth of Jeans instability, a few stable large scale modes are also found which remain discrete when the shorter wavelength modes tend to form continua (Hunter, 1969). Figure 3 shows the radial and circular velocity components of the first two axisymmetric modes of a model of M31 derived by fitting Roberts' combined $n=\frac{3}{2}$ rotation curve within 25 kpc by a suitable mass distribution. The first mode is highly concentrated towards the center and has a higher frequency of 37.5 km s^{-1} kpc^{-1} than that of 10.8 km s^{-1} kpc^{-1} of the second mode. The latter also has an associated radial velocity field more like that observed for M31. Non-radial modes may also by computed though, in this theory, steady modes show no spiral structure. Figure 4 of Hunter (1969) shows the circular velocity field associated with a large scale and centrally concentrated $m = 1$

mode for another galactic model which would cause an asymmetrical rotation curve to be observed for it.

Recently I have been calculating modes of oscillation by using a more realistic set of hydrodynamic equations for a galaxy of stars that takes account of stellar random velocities. The latter are supposed small compared with circular velocities, and the collisionless Boltzmann equation is expanded in terms of an appropriate small parameter. This expansion leads to a set of equations for the moments of the perturbed distribution function. An early truncation of the expansion leads to the equations of zero pressure hydrodynamics, but the continuation of the expansion one stage further gives a more complicated set of hydrodynamic equations with general stress terms. These stresses can be related to the systematic velocities so that a closed set of equations is obtained.

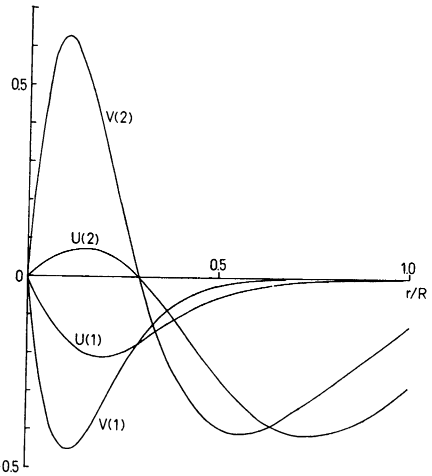

Fig. 3. The spatial variation of the radial velocity U and circular velocity V for the first two axisymmetric modes of M31. Here r measures radial distance from the galactic center, and the outer radius R is at 25 kpc.

The distribution of stellar random radial velocities in the unperturbed galaxy must be specified in this theory. Toomre (1964) has shown via a local analysis of the full collisionless Boltzmann equation that all short wavelength axisymmetric oscillations are stable for a Schwarzschild distribution of random velocities provided the mean radial random velocity $c > 3.36\, G\sigma/\kappa$, where G is the constant of gravitation, σ is the surface density and κ is the epicyclic frequency. A similar stability condition can be established for the stellar hydrodynamic equations; the only difference being that the coefficient 3.36 is replaced by 3.0.

Modal calculations have been performed with the distribution of mean radial random velocity given by $c^2 = 4\pi^4\beta(G\sigma/\kappa)^2$. Here β is a parameter that is varied between calculations, $\beta = 0$ corresponding to a cold disk, and $\beta = 0.023$ corresponding to the theoretical stability limit described above. The results of axisymmetric calculations confirm that the theoretical stability limit, derived strictly only for short wavelength disturbances, is more generally valid, though the critical value of β may be more like 0.025 than 0.023. The precise point of stabilization is hard to determine because of the sensitivity of the calculations to numerical errors. Non-axisymmetric calculations, principally for $m=2$, show that these modes are not so readily stabilized even at considerably larger values of β where instabilities still persist. The instabilities of large scale do not have any clear spiral form.

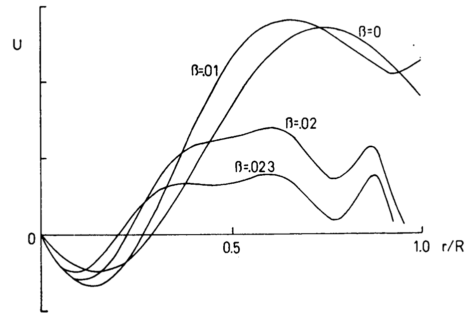

Fig. 4. The radial velocity U for the second axisymmetric mode of M31 for increasing random velocities as specified in the text.

The increase of random velocities causes changes in the shape of modes. Figure 4 shows how the radial velocity of the second axisymmetric mode of the present M31 model is affected. Generally there is a tendency for the smooth shapes of cold disk modes to break down. This is much more marked with the first axisymmetric mode which disintegrates for values of β much less than the critical one. Thus the slower second axisymmetric mode seems from the calculations to be the more fundamental mode of a galaxy and the rough agreement between the non-circular velocities observed on the minor axis of M31 and the calculated shapes of Figure 4 support the idea that this mode is present in M31.

Acknowledgements

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