OBSERVATIONAL FOUNDATIONS OF INHOMOGENEOUS UNIVERSE

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Abstract. Using new observational data at optical (Jagiellonian Field) and radio (GB region) frequencies, the distributions of galaxies and radio sources are analysed.

The results of the analysis show that there is a significant non-random surface distribution of galaxies irrespective of the scale of the field investigated. The observed clumpiness does not allow us to introduce discrete hierarchical structures. The distribution of galaxies shows rather a continuous hierarchical structure and for the description of the distribution of galaxies, some density indices should be used instead of imprecisely defined clusters of galaxies.

In the radio domain, the distribution of radio sources in the GB region and the number-flux density relations which come from the different sub-regions are investigated. The results of our investigation show that the surface density of radio sources at several flux density levels and the number-flux density relations vary from place to place in the GB region and the significance level of the observed variations is at least 1%.

The most important conclusion which can be drawn from the GB data is that results obtained from the observations of a single selected region of the sky cannot be generalized to obtain information about the whole population of radio sources in a given flux density range, and in addition the region itself – even chosen at random – ought not to be considered as a typical one.

1. Introduction

Numerous attempts have been made to investigate the distribution of luminous matter in the Universe but it still is one of the most important problems in extragalactic research. In the present report we would like to discuss this problem once more using new material in the optical and radio domains.

The basic optical observational material used for this study is a catalogue of 15650 galaxies in the Jagiellonian Field (Rudnicki *et al.*, 1972) which covers an area of sky, 6° × 6°, centred at $\mathrm{R.A.}_{2000} = 11^{\mathrm{h}}19^{\mathrm{m}}$, $\mathrm{Decl.}_{2000} = 35°53'$. The plates of the Jagiellonian Field were taken with the 48-in. Schmidt telescope of Palomar Observatory in three colours and at several exposure times. For the study of large scale effects, Zwicky's *Catalogue of Galaxies and Clusters of Galaxies* as well as other data were used.

The basic radio data came from a deep continuum survey in the northern part of the sky (GB region), defined by: $7^{\mathrm{h}}15^{\mathrm{m}} < \mathrm{R.A.} < 16^{\mathrm{h}}25^{\mathrm{m}}$, $45\overset{\circ}{.}9 < \mathrm{Decl.} < 51\overset{\circ}{.}8$, $b_{\mathrm{II}} > 25°$ (Maslowski, 1971, 1972b). The survey (GB survey), made at a frequency of 1400 MHz, contains 1086 radio sources down to a limiting flux density of 0.09 f.u. (1 f.u. $= 10^{-26}$ W m^{-2} Hz^{-1}) and overlaps the 5C1, 5C2 and the second part of the BP surveys (Kenderdine *et al.*, 1966; Pooley and Kenderdine, 1968; Bailey and Pooley, 1968). The total area of the GB region amounts to 0.1586 sr (about 521 sq deg). In addition to these some results of the GA survey (Davis, 1974) and the NRAO 5-GHz surveys (Davis, 1971; Pauliny-Toth and Kellermann, 1972; Pauliny-Toth *et al.*, 1972) were also taken into account. The GA and GB surveys have been made at the same fre-

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quency using the same 300-ft meridian radio telescope of the National Radio Astronomy Observatory*, Green Bank, U.S.A.

2. Optical Investigations

In order to determine the frequency distribution $p(m)$ of cells containing m galaxies each, the counts of galaxies were made in square grid cells. As the nature of the frequency distribution of galaxies may depend strongly upon the sizes of the cells in which galaxies are counted, the counts were repeated using various cell sizes (starting from cells 2.5 × 2.5 sq min of arc for the Jagiellonian Field and ending with cells of 144 sq deg for the North Galactic Polar Cap).

The observed distributions of galaxies were then analysed using three different tests:

(1) comparing the observed frequency distributions with a Poisson distribution;
(2) comparing the observed frequency distributions among themselves;
(3) using some index of surface density gradient.

Of course, the first method is the classical one and does not need explanation but the remaining two require some additional words. In order to compare two distributions with different numbers of objects we must first reduce them to the same size, throwing away the excess of elements in one sample randomly in all possible ways. The new distribution for the reduced sample is computed from the mean values of the distributions obtained by the throwing away procedure.

The frequency distributions of the reduced sample are given by

$$P_N(M)=\binom{N}{M}\sum_{m=M}^{n}\frac{\binom{n-N}{m-M}}{\binom{n}{m}}\,p_n(m); \qquad (M\leqslant N,\ m-M\leqslant n-N)$$

or

$$P_N(M)=\sum_{n=M}^{n}\binom{m}{M}\frac{\binom{n-m}{N-M}}{\binom{n}{N}}\,p_n(m),$$

where

n = the number of objects in the sample before the reduction, and
N = the number of objects in the sample after the reduction,
$p_n(m)$ = the frequency distribution of cells containing m objects each before the reduction, and
$P_N(M)$ = the frequency distribution of cells containing M objects each after the reduction.

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* Operated by Associated Universities, Inc. under contract with the National Science Foundation.

These formulae can be used interchangeably according to the number of objects in the sample and the mean surface density (mean number of objects in one cell). For a large number of objects or a large mean density the approximate formulae are quite satisfactory:

$$\binom{N}{M}\sum_{m=M}^{n}\frac{\binom{n-N}{m-M}}{\binom{n}{m}} \approx \sum_{m=M}^{n}\binom{N}{M}\left(\frac{m}{n}\right)^{M}\left(1-\frac{m}{n}\right)^{N-M}$$

$$\approx \sum_{n=M}^{n}\binom{m}{M}\left(\frac{N}{n}\right)^{M}\left(1-\frac{N}{n}\right)^{m-M}.$$

The factors occuring in these formulas can be easily calculated from binomial, Poisson or Gaussian distributions.

Now, we will define the index of density gradient and describe the way in which it is computed. First, for each two cells which are placed at the beginning and at the end of a chosen vector **r**, we must calculate the absolute differences between their densities. Then the sum s_1 of these absolute differences is computed from all possible pairs in the field considered. The index of density gradient is obtained by dividing the sum s_1 by the sum $\bar{s}$ which is the mean value of a random distribution of the variable s. This distribution is given mathematically by repeating the procedure of calculation for the s value for all possible situations obtained by moving the elementary cells in the investigated field without changing the value of $p_n(m)$.

All these investigations lead to the following general conclusion: there is a highly significant tendency to clumping of dense regions irrespective of the dimensions of the fields analysed as well as the cell sizes. For all cases considered (various fields and different cell sizes) the ratio of frequency of observed density to the frequency which results from theoretical random distributions has the same character which is schematically drawn in Figure 1.

As we can see, there are three qualitatively different classes of density. The empty and less populated cells as well as cells with large numbers of galaxies are in excess with respect to those expected from a random distribution. On the contrary, cells with average density are much less frequent. Such a situation is, of course, an obvious effect of the clumping of galaxies.

As was mentioned above, the overall shapes of the curves obtained for different fields at various cell sizes are the same, although there are differences in details. However, the observed differences are independent of both the dimensions of the field considered and the size of the cells in which galaxies are counted, as well as of the direction chosen in the field.

So, the general picture of the distribution of galaxies which can be drawn on the basis of our analysis is the following:

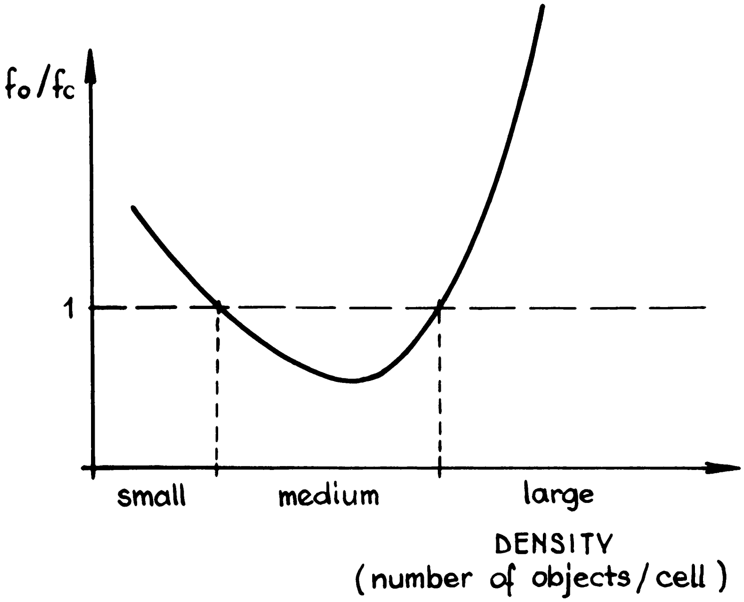

Fig. 1. The ratio of frequency of observed density (objects/cell), f_o, to the frequency f_c which results from a theoretical random distribution as a function of density.

(1) There is a highly significant non-random surface distribution of galaxies irrespective of the scale of the field investigated.

(2) The observed clumpiness does not allow one in general to introduce discrete hierarchical structures.*

(3) The distribution of galaxies shows rather a continuous hierarchical structure which means that between every two existing structures, a new one can always be located.

This picture suggests that for the description of the distribution of galaxies some density indices should be used rather than imprecisely defined clusters of galaxies and clusters of clusters of galaxies.

3. Radio Investigations

The second basic problem which is very important for extragalactic research is the problem of the distribution of weak radio sources. Although in the radio domain we do not have at our disposal such rich data as in the optical, some important conclusions can already be drawn. Since this problem may be less well-known we want to say a few words about the situation encountered in this field.

The investigations made at Cambridge on the basis of several low frequency source surveys (3C, 4C) showed – contrary to galaxies – lack of anisotropy and the absence of clustering of radio sources stronger than 0.2 f.u. at 178 MHz on a scale of angular

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* Similar effect was found by Kiang (1967).

size larger than half a degree (Holden, 1966; Hughes and Longair, 1967; Hinder and Branson, 1969). This result has been widely accepted by theoreticians as well as observers and has been interpreted as suggesting that all cosmological investigations based on homogeneity and isotropy of the distribution of matter are correct.

One of the most important conclusions drawn from the Cambridge analysis was the point of view which received general acceptance that any region of sky (excluding the galactic belt) can be considered as typical, i.e. a region in which the spatial distribution of radio sources (number-flux density relation) as well as their physical properties are representative of radio sources as a whole. Thus, it was considered that any combination of results obtained from a deep source survey of a small region of the sky with those from strong or intermediate source surveys can be used to investigate cosmological models and the evolution of radio sources, spectral effects, etc.

Recently, however, several papers have appeared in the literature (or are in press) in which the problem of the isotropy of the counts of weaker radio sources as well as spectral index distributions has been raised again (Kellermann, 1972; Maslowski, 1972a, 1973; Maslowski *et al.*, 1973; Pauliny-Toth and Kellermann, 1972; Yahil, 1972) on scales ranging from a few square degrees up to several steradians.

(1) Pauliny-Toth and Kellermann (1972) have found significant differences in both the source counts and the spectral index distributions observed between the northern and southern galactic hemispheres in analysing their NRAO 5-GHz source surveys.

(2) Maslowski (1972a) has reported for the first time apparent spectral differences between the 5C1 and 5C2 sources in the frequency range 1400 MHz–408 MHz. This effect has been related to the clustering of weak radio sources on a scale the size of the 5C areas. Besides, Maslowski (1973) found significant differences in the number-flux density relations derived for the GA and GB sources stronger than 0.5 f.u. Next, the above effect can be related to anisotropy in the distribution of radio sources on a scale of angular size of some tenths of a steradian.

The aim of this report is to give further observational evidence for anisotropy in the distribution of radio sources on a scale of some thousandths of a steradian on the basis of the GB data alone. The size of the GB region allowed us to select from it four rectangular sub-regions (labeled afterwards I, II, III and IV respectively) each two hours in right ascension, starting from 7^h30^m, and six degrees in declination (0.035 sr). These sub-regions were analysed separately and large, statistically significant differences in both the surface densities of radio sources at several flux density levels and the number-flux density relations were found between some of them.

The variations of the surface density of radio sources within the GB region at or above several flux density levels as a function of right ascension are shown in Figure 2. It was found that fluctuations in the surface density observed for the sources weaker than 0.13 f.u. gradually show significant non-randomness at or above 0.18 f.u. One can conclude that about half of the GB sources with $S(1400) \geqslant 0.18$ f.u. show a significant tendency toward clustering in some parts of the GB region, mainly in the sub-regions I and III. The significance level of the observed differences in the surface density between the four sub-regions is about 1%.

Since the observed variations in the surface density of sources are different at different flux density levels, they might come from different population laws existing in these sub-regions.

To check this, we have also examined the source counts for each sub-region separately and the results are shown in Figure 3 and in Table I. It was found that the similarity in the source distributions exists between sub-regions I and III (curve A) as well as between II and IV (curve B) but not between the adjacent ones. The number of sources involved in this analysis was sufficiently large (more than 220 in each sub-region) to make the differences in the counts statistically significant.

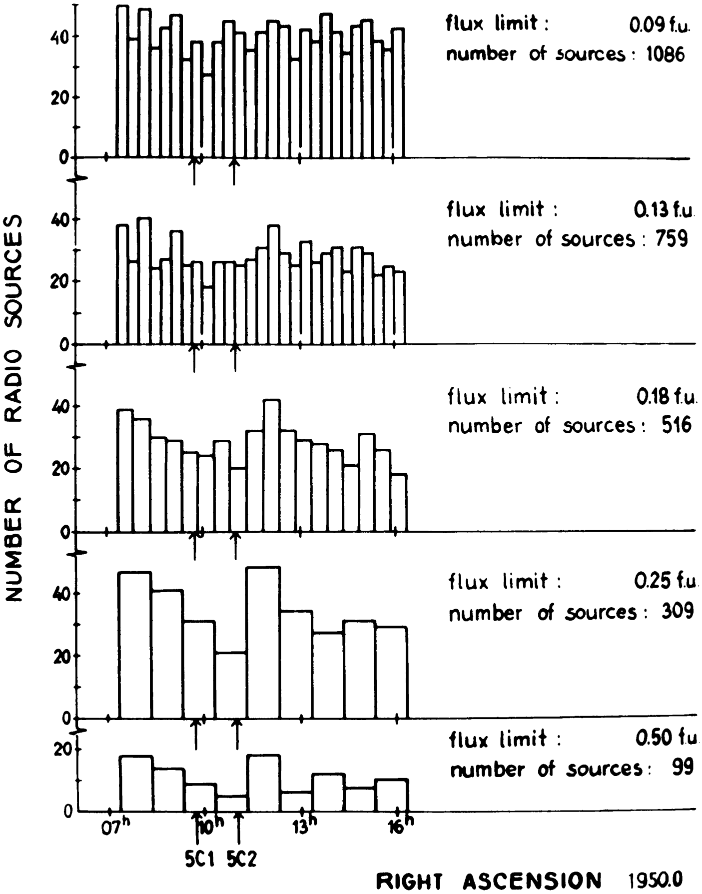

Fig. 2. Histograms of the number of sources observed in small sub-regions of the GB region at several flux density levels as a function of right ascension.

In order to investigate the differences observed in the slopes of the source counts for these sub-regions, we have used the flux density variable, $x = S_0/S$, and the results are given in Table I for the sources having $S \geqslant S_0 = 0.13$ f.u. One can see from this Table that the number of weaker sources having $x > 0.5$ is almost identical in all sub-

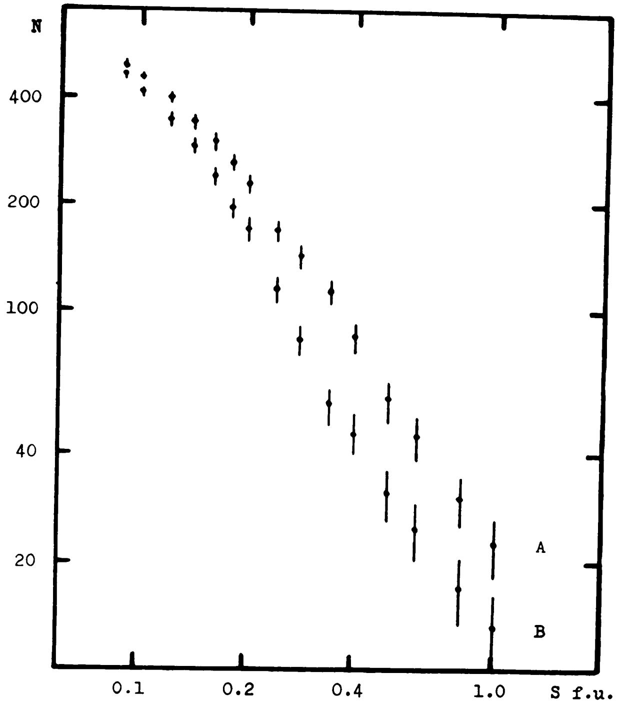

Fig. 3. Number – flux density relations for the GB subregions I + III (curve A) and II + IV (curve B). The numbers of sources involved in curves A and B amount to 491 and 473 respectively, and each of them came from an area of 0.0696 sr.

regions but the observed mean values of the flux density variable, x, are significantly different for adjacent sub-regions, but not for the pairs of sub-regions I and III and sub-regions II and IV. This may be explained assuming that the population law for the subregions I and III is not the same as for the sub-regions II and IV.

The above results have been further confirmed by way of numerical experiment. A large number (230) of random distributions of radio sources was generated by computer over the whole area of the GB survey according to the $N(S) \propto S^{-1.5}$ law. It was found that the probability of obtaining the observed density distributions between the sub-regions is very low (Machalski *et al.*, 1974).

TABLE I

Variation of the mean value of flux density variable $\bar{x} = n^{-1} \sum (S_0/S)^{1.5}$ for $S \geqslant S_0 = 0.13$ f.u.

Subregion	Number of objects	Number of objects with $x \leqslant 0.5/x > 0.5$	$\bar{x}$
I	181	104/77	0.454 ± 0.0215
II	148	76/72	0.517 ± 0.0237
III	185	108/77	0.455 ± 0.0212
IV	165	81/84	0.525 ± 0.0225

It is necessary to emphasize that the results presented here were obtained by comparing the relative differences observed between different sub-regions, covered by the same survey. Such a method guarantees the elimination of most of the errors resulting from noise, confusion and flux density scale.

One of the most important conclusions which can be drawn from the GB data and supported by other data mentioned earlier is that results obtained from the observations of a selected region of the sky cannot be generalized to obtain information about the whole population of radio sources in a given flux density range, and in addition the region itself – chosen even at random – ought not to be considered typical.

Our investigation and the results already described have provided further observational evidence against isotropy in the distribution of weaker radio sources ($S(1400) < 1$ f.u.).

Since in discussions of cosmology it is usually assumed that the Universe is highly isotropic, the present results cast doubt on the existing cosmological interpretations of extragalactic radio data and raise further questions as to the cosmological relevance of the source counts, at least of those based on the data collected from very restricted regions of sky.

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DISCUSSION

Kiang: Your conclusions regarding the spatial distribution of galaxies are almost word for word the same as in my 1967 paper (*Monthly Notices Roy. Astron. Soc.* **135**, 1–22, 1967). I would not have made this comment but for the fact that you seem to be unaware of this paper of mine.

Davis: I am quite surprised that you find a statistically significant effect in the GB survey, as a very similar analysis on the GA survey showed excellent agreement with the expected random distribution. We shall have to discuss this.