CHAPTER 5

SEISMOLOGICAL INVESTIGATIONS OF COMPACT STARS

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ABSTRACT. Since Landolt's discovery of the first ZZ Ceti star, the ranks of the compact pulsators have swollen to include planetary nebula nuclei, hot pre-white dwarf stars, hot and cool white dwarf stars, and possibly even neutron stars. The discovery of multi-periodic variations in these objects has helped to usher in a new era of rapid progress in the field of stellar seismology. For the compact stars this rapid progress is the result of a combination of four fortunate circumstances: their physical structure is relatively simple; typical amplitudes are small enough to be both readily observable and amenable to linear analysis; the wealth of observed periods in each object provides many independent clues to the underlying structure of the compact stars; and the periods are short enough that we can observe many cycles in a night, thereby completely resolving the period structure in many of these stars.

In this review I will discuss the current state of our understanding of these objects, with special emphasis on their distribution in the H-R diagram, range of pulsation properties, possible mode selection mechanisms, excitation of the pulsations, problems of convection, and particularly the measurement and interpretation of period changes. In addition to the results of recent research, I will review the techniques used to obtain them, pointing out both their strengths and weaknesses. I will close with a discussion of some of the important problems facing us in the study of these objects, and attempt to identify potentially fruitful directions for future research.

1. INTRODUCTION

The goal of this paper is to provide an introduction and overview of seismological investigations of compact stars. Although I will take great care to get the main features of the historical record straight, I will make no attempt to provide an encyclopedic review of the literature. Instead, I will attempt to identify the major "turning points" in the field, and explore the strengths and weaknesses of the observational and theoretical techniques that have brought us to our current understanding of the compact pulsators. Also, in keeping with the spirit of this symposium, I will signpost the many pitfalls into which we have stumbled along the way, with the thought that others may learn as much from our mistakes as from our successes.

1.1. An Overview of the Compact Pulsators

Potentially the most interesting of the compact pulsators are the neutron stars. The status of this field, as well as the study of the other main class of compact pulsators, the white dwarf

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and related objects, have been summarized recently in the excellent and comprehensive review by Van Horn (1984). Seismological studies of these objects may well prove to be the most exciting area of astrophysics in the coming decade—providing unprecedented direct tests of elementary particle physics in conditions likely to remain beyond the reach of laboratories for the foreseeable future. However, the status of this work is very similar to that of the pulsating white dwarf stars twenty years ago: we have a basic notion of the oscillation properties, thanks largely to the work of McDermott and Thorne and their collaborators (see Van Horn 1984 for references), but we are basically ignorant of the possible excitation mechanisms, and lack any definitive observations of pulsating neutron stars. Partly for these reasons, and mostly because of my boundless ignorance of neutron stars, I will defer further discussion of these objects in the remainder of this paper. The lesson of the white dwarfs, on the other hand, suggests we would be well advised to keep a close eye on this promising field.

With apologies to the neutron stars, I will use the term compact pulsators to refer to the white dwarf stars and their progenitors. All of these objects are multi-periodic with periods in the range from about one hundred seconds up to several thousand seconds. Unlike the solar type pulsators, these objects display only certain of the possible spectrum of nonradial oscillation modes, so that some selection mechanism is clearly at work in even the most complex pulsators. We can readily divide them into four classes (see Figure 3).

In increasing order of age, and taking the zero time to be the onset of nebula formation, we first find the pulsating planetary nebula nuclei (or PNNV stars). The presence of an observable nebula argues that these objects are probably less than 10^4 yr old. Next, we find the hot DO variables (DOV stars), these objects are thought to be in the transition stage between white dwarfs and planetary nebula nuclei, with their cooling dominated by plasmon neutrino emission from the deep interior. Although there is some evidence for a wind in these objects, no observable nebulosity remains indicating ages of order 10^5 yr.

The last two known classes of compact pulsators are true white dwarf stars, in the sense that they evolve at essentially constant radius: virtually all of their support against gravity is due to the pressure of their degenerate electrons, a mechanism independent of temperature. The youngest are the pulsating helium white dwarfs (DBV) stars, with ages of about 10^7 yr. The DBV stars are substantially cooler than the DOV stars; their evolution is dominated by normal photon radiation, although the plasmon neutrinos still contribute a significant fraction of the total energy loss. The oldest class are the pulsating hydrogen white dwarfs (DAV stars, known also by the less descriptive appellation of the ZZ Ceti stars). These stars are of order 10^9 yr old, and are the coolest known compact pulsators.

1.2. Why are these objects interesting?

A glance at these proceedings is sufficient to reveal that there are an impressive number of stars which are known, or are suspected, to be nonradial pulsators. Of these stars the compact pulsators are arguably the simplest in their equilibrium structure and, in a related way, in their pulsation properties. The simple equilibrium structure of these objects is the result of their highly evolved state. Nuclear energy sources, possibly present in the hottest classes, are no longer important in the evolution of these stars. Their thermal structure is the result of a slow cooling process. Their physical structure also is extremely simple: a highly degenerate C/O core with extremely thin, mostly nondegenerate layers of nearly pure He and possibly H on top. This simple structure is the direct result of gravitational settling.

In many ways the pulsation properties are ideal for ground based photometric observations. The amplitudes of the pulsations are large enough to be readily detectable yet in many cases small enough for the pulsations to be linear. The periods are typically short enough for many cycles to be observed in one night; as a result, several of the light curves have been completely resolved. They therefore provide a nearly ideal laboratory for studying nonradial stellar pulsations. In addition, there are a wealth of observed periods providing many independent clues to the underlying structure of these stars, a subject interesting in its own right.

The seismology of the compact pulsators has already provided, and will continue to provide, unique insights into the physics of matter under extreme conditions of temperature and density. It also gives us an opportunity to study observationally the chemical diffusion of matter in the presence of strong gravitational fields and to confront current diffusion theory with these observations.

Most importantly, these objects are the oldest stars in our galaxy and contain in their interiors an archeological record of primordial star formation. As we will see below, the compact pulsators provide us with a means of measuring the ages of these objects—yielding an accurate and independent determination of the age of the disk and thereby the age of the universe.

2. OBSERVATIONS OF COMPACT PULSATORS

2.1. Basic Properties of the White Dwarf Stars

Before we proceed to the pulsators, it will be useful for us to review the basic properties of the white dwarf and pre-white dwarf stars. Currently, the best evidence suggests that these stars are formed from objects with masses less than $\sim 5 - 8M_\odot$; thus nearly all stars end up producing planetary nebulae, and the nuclei become white dwarfs. These compact objects span the entire range in the H-R diagram from $3 \gtrsim \log(L/L_\odot) \gtrsim -4.5$.

In spite of their remarkably diverse origins and luminosities, they are a remarkably homogeneous class of objects. After the initial collapse from the hot PNN stage, the stars not in binaries have a $\log g \simeq 8$, indicating a narrow mass distribution tightly clustered around $0.6M_\odot$. The white dwarf stars are observed to come in two principle "flavors": those with essentially pure Helium atmospheres (spectroscopic type DB), and those with essentially pure hydrogen atmospheres (spectroscopic type DA). The DA and DB stars comprise about 80% and 20%, respectively, of the total population of all white dwarfs, with a smattering of mixed compositions and compositions containing elements heavier than helium. This ratio, however, is a sensitive function of temperature; the evidence at this time strongly suggests that at least some of these stars can change their spectroscopic type—possibly more than once during their evolution.

Models describing pre-white dwarf evolutionary history insist that these objects have C/O cores, with the very thin surface layers of hydrogen or helium rendered chemically pure by the effects of gravitational settling. The thicknesses of these layers is only weakly constrained by evolution theory to lie in the range $-16 < \log\,(M_H/M_\odot) < -4$ and $-16 < \log\,(M_{He}/M_\odot) < -2$. The lower mass limits arise from the consideration that the surface layer must be optically thick throughout the temperature range in which they are observed. The upper limit arises from the consideration of previous nuclear burning stages. Clearly there is great uncertainty in the values of these basic parameters although here, as in other areas, seismological investigations have something to say.

2.2. What, If Anything, Is a Pulsating White Dwarf? or Discovery History of the Compact Pulsators, or "Kicking the Ball Around the Infield"

In the 40's, and 50's there had been suggestions that the compact objects may pulsate on time scales of 10 s or less (radial pulsation timescales for these objects)(Sauvenier-Goffin 1949, Schatzman 1958). Ultimately, this work prompted a surveys of white dwarfs aimed at discovering high frequency variability. However, as is so often the case in science, the original discovery of pulsations in compact objects was an accident. Landolt (1968) found variations of about 0.3 mag with a quasi-period of about 750 s, in HL Tau-76. These variations clearly did not meet with the theoretical expectations; the timescales were too long by two orders of magnitude and it was not clear that they represented pulsations at all. Indeed, Warner and Nather (1970) initially suggested that the variations might represent some sort of flare activity.

In the years following the initial discovery by Landolt "high-frequency" variability was reported in a number of other compact objects. These variations were reported in white dwarfs of all spectral types (*i.e.*, all surface compositions) and colors (see discussion in McGraw 1977). Some theorists greeted these new discoveries with a great deal of agony and gnashing of teeth, others with militant indifference. The inhomogeneous character of the variables was particularly disturbing since it implied that the observations held no clues at all to the cause of the variability. Thus, it seemed at the time, the variations were not the result of a temperature effect such as a pulsational instability strip, nor did they have anything to do with the surface composition. All indications suggested that the cores of these objects were pretty much the same (degenerate carbon/oxygen of various temperatures); the evidence that the only properties which differed from star to star (temperature and surface composition) didn't matter in determining whether or not the star was a variable was both confusing and depressing.

Fortunately, in 1972 R. E. Nather introduced a major innovation to the observation of high-frequency variability in astronomical objects: the two-star high-speed photometer. This tool, described in a modest publication in *Vistas of Astronomy* (Nather 1973), completely revolutionized the observation of compact variables; with it J. T. McGraw, E. L. Robinson, and Nather were able to create order out of the chaos in the newly developing field.

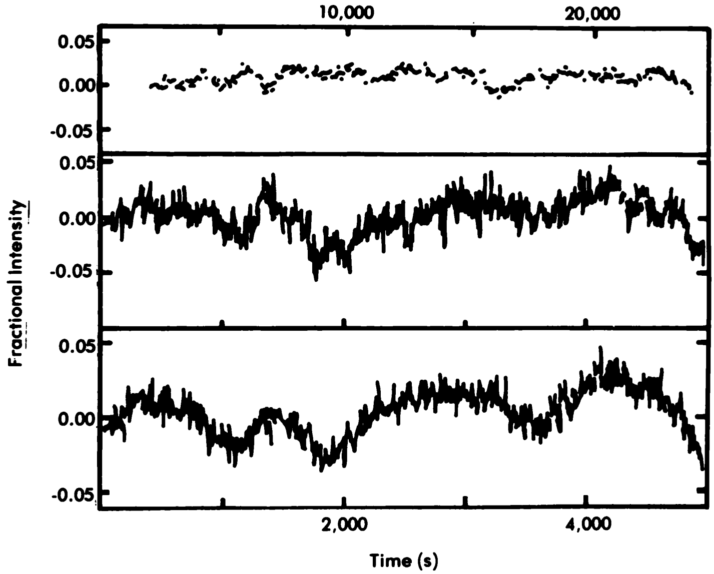

Figure 1. Light curves of constant stars in middle and bottom panels obtained under conditions visually photometric. For comparison, top panel is light curve of the DOV K1-16 under actual photometric conditions.

To appreciate the impact and utility of this tool a few light curves are worth the proverbial thousand words. Figure 1 illustrates the light curves of three stars. These light curves display quite similar amplitudes and periods—within the range typical of the compact pulsators. The top light curve is that of the compact pulsator K1-16 obtained by A. Grauer using a two-star photometer under completely photometric conditions, as determined by the simultaneous observation of a comparison star in the same field. The middle light curve is another compact object, which is *not* a variable star at all (within observational limits). In fact, the lower light curve is that of a comparison star in the same field obtained simultaneously using a two-star photometer, and it appears to show the same sort of variations as the compact object. This illustrates the profound importance of two-star photometry. The conditions at the time the middle and lower light curves were obtained appeared *to the eye* to be completely photometric. The eye is only sensitive to variations greater than about 20%, thus, as is dramatically illustrated in the bottom two panels of Figure 1, variations in transparency or sky brightness can mimic the low amplitude variations commonly observed in the real variables; an observer with only a single photometer channel would be completely unaware of these conditions and would reasonably conclude that the object in the middle panel was a compact pulsator.

In addition to the problem of producing spurious variability in nonvariables, the all-too-common non-photometric conditions discussed above can also effect the results for known variables. Such conditions can produce inaccurate measurements of periods and amplitudes. This makes two-star photometry essential for the study of the long term behavior of the periods and amplitudes of known variables. Clearly data obtained under conditions when the comparison star in the second channel shows variations are unusable.

The use of two-star photometers enabled McGraw and Robinson (1976) to weed out the "ringers," or non-variables, and demonstrate that the compact variables which remained formed a highly homogeneous class of pulsating variable stars: the DAV stars. They demonstrated that the variables were all of spectral type DA, all had very similar photometric colors, and all were otherwise normal DA white dwarf stars. This demonstration led to rather rapid theoretical progress on the pulsating white dwarf stars. The improved understanding then led to the theoretical prediction of a new class of compact pulsators, the DBV stars. This prediction was soon confirmed observationally. The independent discovery of two additional kinds of pulsating compact objects, the DOV (McGraw *et al.* 1979, Bond *et al.* 1984) and PNNV stars (Grauer and Bond 1984), brings us to our current total of four classes of compact pulsators.

The lesson of this brief discovery history is that if you use data to determine periods or amplitudes, or even just to demonstrate the variability of an object, you must be *certain* conditions don't just appear photometric but *are* photometric. An example of the level of certainty required is given in Figure 2, where the top curve is a high-speed two-star photometry run on a DBV star , and the bottom curve is that of the simultaneously observed comparison star. The moral of this brief discovery history is very simple: *You can't do photometry when it isn't photometric, and you can't always tell by looking.*

2.3. Basic Physical Properties of the Compact Pulsators

This material has been reviewed recently (in much more detail than is presented here) by Winget (1986) for the DAV and DBV stars, and by Cox (1986) for the DOV and PNNV stars. The locations in the HR diagram of the four classes of compact pulsators are shown in Figure 3. As previously noted, these objects span an impressive range in both luminosity and temperature; in spite of this, the pulsation properties of all of these objects are remarkably similar.

The observed amplitudes of the variations in the light curves are often several tenths of a magnitude, yet the amplitudes of the individual frequencies in the power spectra are typically only of the order of several percent. The discrepancy between these two amplitudes is the result of the multi-periodic character of the pulsators. All of these objects have at least four frequencies simultaneously excited, many have more than ten, and at least one

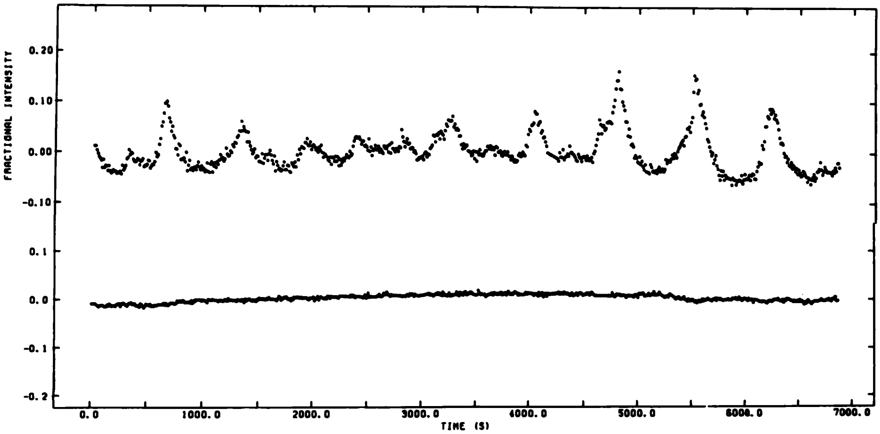

Figure 2. Light curve of the DBV GD 358 and comparison star, obtained with a two-star photometer under photometric conditions (compare with bottom panels in Fig. 1).

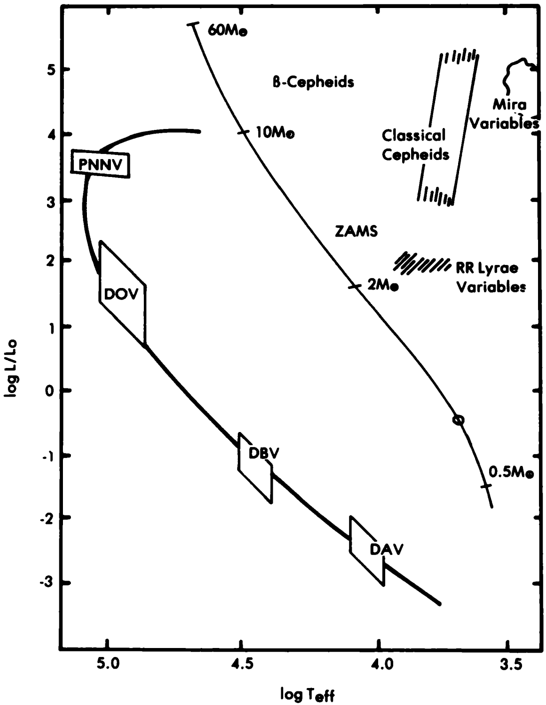

Figure 3. The compact pulsators in the H-R diagram.

has more than 25 (GD 358, Winget *et al.* 1982b). The beating together of these oscillations results in occasionally very high and very low amplitudes in the light curves.

The spectroscopic information is particularly revealing for the two cooler classes of compact pulsators. The DBV stars have only surface helium present in their spectra, and the only thing which sets them apart from the nonvariable DB stars is that they are found only near the He I opacity maximum. By the same token the DAV stars have only surface hydrogen present, and are found only near the very well defined opacity maximum for hydrogen. For these objects the observational message is quite clear: the pulsations are associated with the development of the partial ionization zones in the surface layers. Thus the variability is just a result of the normal evolution of a DA or DB white dwarf into the temperature domain where significant partial ionization begins. In addition, these stars have surface gravities in the same range as the nonvariables, no detectable magnetic fields, and no anomalously rapid rotation rates; the only difference between the variables and nonvariables is their *effective temperature*. This has the important implication that the variables are in no way "pathological" and that anything we learn about the DAV and DBV stars through seismological studies applies also to the other, nonpulsating DA and DB stars.

The meaning of the spectroscopic evidence is much less clear for the hotter pulsators, the PNNV and DOV stars.† Although no hydrogen is observed in these objects, with their high temperatures it is entirely possible that some significant amount of hydrogen may still be present; the current limit is only $He/H > 1$ (cf. Wesemael *et al.* 1985). In addition to this uncertainty, it is virtually impossible to tell the relative abundance of the elements—in part due to the difficulty of constructing meaningful model atmospheres of objects with these luminosities and temperatures. At this time it is only possible to say that significant amounts of He II, C IV, and O VI, and perhaps a few other species, are present. What makes the situation even more difficult is that several objects which are spectroscopically identical have *no* detectable pulsations. For these reasons, the observations to date have very little to say about the physical cause of the pulsations in these objects.

Because the spectroscopy of the DBV and DAV stars is a little more manageable than that of the DOV and PNNV stars, comparatively more is known about the temperature bounds for these variables. Best understood are the coolest of the compact pulsators, the DAV stars. The most recent work by Weidemann and Koester (1984) shows that the bulk of the DAV stars are confined to a very narrow interval in effective temperature: $11,200 < T_e < 12,500$ K, with the absolute value of the temperatures possibly 500 K cooler, depending on the adopted calibration of the multichannel photometry. Weidemann and Koester also show that the hottest and coolest DAV stars have temperatures of 13,000 K and 10,000 K, respectively. The statistics of the population of the instability strip have been investigated by Fontaine *et al.* (1982) using Greenstien's(1982) multichannel observations of DA white dwarfs, and more recently by Fontaine *et al.* (1985) who used a much larger sample of DA stars with a homogeneous set of Strömgren photometric colors. These investigators conclude that the data are consistent with the proposition that most, and quite possibly all, DA white dwarfs within the boundaries of the observed instability strip are variable.

The DBV stars are considerably hotter than the DAV stars and correspondingly less is known about the temperature boundaries of the DBV instability strip. This is due in part to the poor statistics (only about 12 hot DB stars near the instability strip, and only 4 known pulsators to date), in part to problems with comparison of the *IUE* and optical temperature scales, and in part to problems in the theoretical atmosphere calculations. After allowing for the various uncertainties the most conservative position is probably that the the blue edge of the DBV instability strip is around 27,000 ± 3,000 K with a width of about 3,000 K (for a detailed discussion of these problems see Koester *et al.* 1985 and Liebert *et al.* 1986 and references therein).

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† Because these stars show somewhat similar ionization species in their spectra (Wesemael *et al.* 1985), and because there is only one confirmed PNNV, they are sometimes lumped together. The presence of nebulosity around the PNNV coupled with the fact that its characteristic pulsation timescales are about a factor of 3 longer, however, suggests that while it may have the same basic type of excitation mechanism as the DOV stars, it is physically a different kind of object.

The temperature boundaries of the DAV and DBV instability strips have interesting implications for understanding the cataclysmic variables (interacting binaries containing a white dwarf as the mass acceptor). The question has often been raised as to why do we not find a DAV star in a cataclysmic variable system since the accreted material is usually very rich in hydrogen. The answer, based on the location of the DAV instability strip, is that the accreting star will always be much hotter than 12,000 K, due to the process of accretion and hence will be hotter than the DAV blue edge and so it will not pulsate.

On the other hand, there are at least two cataclysmics which are interacting binary white dwarfs (G61-29, and PG 1346+082), and which have (spectroscopically) a pure helium composition. The accreting object may well have a temperature near 27,000 K and hence could pulsate in the same manner as the DBV stars. The system PG 1346+082 may be doing just that. This system displays large amplitude (~3.6 mag) outbursts on timescales of days. On its way to maximum it has been observed to display a light curve very reminiscent of the DBV stars with many periods present from 200 – 400 s which are coherent at the limit of the run length (Wood *et al.* 1987). The spectroscopic data indicate that this object may indeed pass through the DBV temperature instability strip during the transition to outburst (Wood *et al.* 1987). If this is the case, we have the exciting prospect that the location and properties of the DBV instability strip can be mapped out to a reasonable extent from the observations of a single object.

2.4. The Pulsation Properties

All of the compact pulsators are intrinsically faint and the amplitudes of the pulsations are quite low. Also, the absorption lines are pressure broadened due to the high gravities. These factors combine to make time resolved spectroscopy virtually impossible with current telescopes and instrumentation. Because of this the basic tool for studying the compact pulsators is the high-speed photometer which is typically used in unfiltered light in order to improve the photon statistics.

Pulsation timescales in these objects range from hundreds to thousands of seconds. These timescales are sufficiently short that it is usually possible to obtain many cycles of the pulsation modes in one night. However, because these objects are all multi-periodic, it is usually necessary to run all night on a single object in order to resolve the intrinsic modes from those which arise from beating processes. Because of the short timescales, and the need for large blocks of continuous data, only relative photometry has so far been possible. Sky measurements are short—usually about a minute—and only possible every hour or so, and standard star observations are possible only at the beginning and end of each run. For these reasons, the amplitude is a very poorly determined quantity with scatter of 10-15% from run to run.

Particularly because of the long periods of the variations, the question of their physical cause naturally arises: are they the result of of radius changes, or temperature changes, or some combination of both? Multi-color *UBV* high-speed photometry by Nather and Warner (1972) of HL Tau 76, and multicolor Strömgren high-speed photometry by McGraw (1979) of HL Tau 76 and G29-38 suggested that the luminosity variations were due entirely temperature variations. Later Robinson *et al.* (1982), and Kepler (1984b) used photometric observations of the wings of the H_γ line to demonstrate conclusively that the luminosity variations in R548 and G117-B15A were due entirely to temperature variations. This was the first observational evidence, independent of the lengths of the periods, that the pulsation modes are nonradial g-modes, and it also ruled out the possibility of r-mode pulsations for these DAV stars.

Recently similar conclusions were reached by Barstow *et al.* (1986) for the DOV star PG 1159-035 on the basis of EXOSAT soft x-ray observations combined with optical observations. They showed that, within the errors of the observations (about 0.1 cycles), the x-ray and optical light curves were in phase. This was the first time pulsations had been detected from observations of photospheric x-rays.

The light curves of the compact pulsators are qualitatively similar: they all have roughly similar amplitudes and frequencies, and they are all multi-periodic so they can be expected to show the same type of beating in their light curves. For the simplest of the pulsators this similarity is more than qualitative. For example, the DBV star PG 1351+489 is the simplest of all the compact pulsators. It's light curve is very nearly mono-periodic, dominated by a single frequency, ν_o, and its harmonics at 2 ν_o and 3 ν_o, with amplitudes down each by a factor of three from the preceding peak. It also has a significant pulsation frequency near 3/2 ν_o, at an amplitude about 1/10 that of the main peak. The most interesting thing is that this same basic structure, a main frequency ν_o with relatively large amplitude, its harmonics with decreasing amplitudes, and a low-amplitude peak near 3/2 ν_o is found in at least two other compact pulsators: GD 154 and G191-16, both DAV stars. The reason for this "signature" frequency structure in the simple pulsators is at present unknown, but the fact that we see it in DAV stars and a DBV star may be telling us about a fundamental similarity between the two kinds of objects.

The more complex pulsators have presented a different problem. The power spectra do not repeat from night to night causing many investigators to conclude that these objects are not stable pulsators. This led to the habit of reviewers placing the pulsating white dwarfs into separate categories according to the observed stability of their pulsation periods and amplitudes —as measured by the power spectra of their individual light curves (cf. Robinson 1979, Dziembowski 1979, Hansen 1980, or Winget and Fontaine 1982). This instability was interpreted as arising as the result of "mode switching" due to the nonlinear effects which theorists believed must be important in these objects. For one of the DAV stars, GD 385, the actual measurement of a growth rate was reported, based on the changing amplitude of an individual frequency. The possibility of observational measurements of growth rates and the opportunity to study non-linear mode interactions were of such great theoretical interest that S. O. Kepler, D. O'Donoghue, and their collaborators have begun the enormous task of systematically observing the "unstable" pulsators for the hundreds of hours necessary to demonstrate any instabilities, and possibly resolve the period structure of the objects.

The first results of these extensive observational undertakings are quite sobering. Three promising candidates for unstable pulsations have had their light curves completely resolved into their component pulsation modes: BPM 31594 (O'Donoghue, 1986), G 226-29 (Kepler *et al.* 1983), and GD 385 (Kepler 1984a). Far from demonstrating the eagerly anticipated nonlinear instabilities and "mode switching" type behavior expected, all of these objects have demonstrated long term coherence when they were studied with adequate data. *All* the apparent instabilities were the result of overinterpreting undersampled data and resulted not from the nonlinear effects anticipated but from the *beating of closely spaced frequencies.* With 20/20 hindsight, we can easily see that we should have *expected* this behavior from multi-periodic pulsators. Occam would have been embarrassed by our assumption of instabilities.

The moral of this can be stated, with tongue only slightly in cheek, in the following way: observation of one cycle does not constitute a period measurement; the detection of two frequencies does not prove "mode switching;" and different amplitudes from different measurements do not constitute the measurement of a growth rate.

3. THEORY OF THE COMPACT PULSATORS

As we noted above, the early calculations of white dwarf oscillation properties, starting with Sauvenier-Goffin (1949), focused on radial modes. This presumably is because the known pulsators of the day were all presumed to be radial mode pulsators. It was then natural to follow up these calculations with calculations aimed at determining if the oscillations were self-exciting. G. Vauclair (1971a,b) did exactly that using a quasi-adiabatic pulsation code and white dwarf envelope models with realistic surface compositions. He found that his DA white dwarf models were pulsationally unstable around 10,000 K due to the action of the κ- and γ-mechanisms in the hydrogen partial ionization zone. This was right in the neighborhood of the several of the compact pulsators including HL Tau 76.

This probably would have aroused a great stir of interest but for one fact: the observed pulsations are one to two orders of magnitude too long to be due to radial pulsations. This puzzle was soon resolved, independently by G. Chanmugam (1972) and by Warner and Robinson (1972) who suggested that the oscillations were nonradial g-modes. This suggestion was put on firmer footing by the work of Osaki and Hansen (1973) who carried out numerical calculations of the g-mode periods of crude white dwarf evolutionary models with homogeneous compositions. They found that that the periods of g-modes with low ℓ and low radial "quantum number," k, corresponded to the shortest observed periods (~100 s), and the longer periods (~1,000 s) could be accounted for by resorting to higher values of k. The mystery then to be solved was how the nonradial g-modes were excited.

The stage was set, and it remained only for someone to apply the quasi-adiabatic (if not nonadiabatic) computational tools of Osaki and Hansen to the calculation of the stability of realistically compositionally stratified DA white dwarf models with temperatures near 10,000 K. The observations, erroneous as we now know they were, indicated that the spectral type (surface composition) and temperature of the white dwarf were not relevant to the variability. Thus no effort was made at this time to use realistic compositions, and the theorists happily computed away using models with solar, pure C, and even pure Fe compositions.

It was, surprisingly, some time after the observational picture was clarified by McGraw and collaborators before the theorists could be dragged (largely through the efforts of McGraw and Robinson) kicking and screaming to consider the pulsation properties of models with realistically stratified surface compositions. The initial attempt was by Dziembowski and Koester (1981) who found instabilities in g-modes with the correct periods for the DAV stars. These instabilities, however, were the result of driving from the underlying He partial ionization, and so were not confined to the observed temperature domain near the hydrogen opacity maximum. In addition, in order to get the driving the surface hydrogen layers were made sufficiently thin that they were likely to be dynamically unstable due to the underlying He convection zone (cf. Winget and Fontaine 1982 for a discussion of this). In spite of these objections, this work was extremely important because it was the first to demonstrate that modes of the observed periods could be self-exciting.

Shortly after Dziembowski and Koester's (1981) work, and independent of it, another dramatic breakthrough occurred. Dolez and Vauclair (1981), and others nearly simultaneously (Winget 1981, Winget *et al.* 1982a, and Starrfield *et al.* 1982), showed that g-modes with periods long enough to match those observed were excited by hydrogen partial ionization zones in models with relatively thicker, dynamically stable, surface hydrogen layers. Further, because of the crucial nature of the hydrogen partial ionization, the instabilities were confined to a narrow temperature domain roughly consistent with the observations. The puzzle of why the (then known) variable white dwarfs pulsate had been solved.

This solution led Winget (1981) and Winget *et al.* (1982a) to examine the possibility that DB white dwarfs may pulsate in a similar way due to driving by the surface partial ionization of He. This work showed that DB white dwarfs should also pulsate, but in a temperature range appropriate to surface He partial ionization: the region where the He I opacity is near a maximum. A systematic survey of the helium white dwarfs showed that those with the broadest He I lines did indeed pulsate (cf. the discussion in Van Horn 1984). This result was important in that it confirmed both the pulsation description of the variations itself and the role of surface partial ionization zones in driving the pulsations.

A partial ionization zone driving mechanism was soon found for the DOV and PNNV stars by S. Starrfield and collaborators (Starrfield *et al.* 1983, 1984, and 1985). This work showed that the partial ionization of C and O could drive the pulsations in these stars. This theoretical prediction provided an opportunity for a direct observational test: significant amounts of oxygen, not previously detected in these objects, should be present at the surface, perhaps even the photosphere. Subsequently the oxygen lines were indeed found in each of the DOV and PNNV pulsators (Sion *et al.* 1985), thus lending strong support to the partial ionization zone explanation for the driving in these stars.

We now have a possible partial ionization zone driving mechanism explanation for the pulsations for each of the classes of compact pulsators. Lest we get too carried away with the success of the partial ionization, particularly for the DOV and PNNV stars, Cox (1986) reminds us that the inclusion of even 25% by mass of He in the theoretical models is sufficient to quench the driving of the pulsations by the κ- and γ-mechanisms. Given the spectroscopic presence of the strong He II absorption features in all of the DOV and PNNV stars, the presence of He in these quantities is entirely plausible. Thus we should explore other possible driving mechanisms.

A natural alternative to consider is the ϵ-mechanism, arising from a nuclear shell burning source, presumably He-burning, in the DOV and PNNV stars; indeed, standard models of the planetary nebula nuclei (hereafter PNN) do contain active shell burning sources (*e.g.*, Schönberner 1979, Iben (1984), Iben and Tutukov 1984). Recent work by Kawaler (1986), and Kawaler *et al.* (1986a), Kawaler *et al.* (1986b) has addressed this possibility using a wide range of evolutionary PNN models. For *all* models with with an active shell burning source and $\log(L/L_{\odot}) < 3.3$, Kawaler and collaborators do indeed find driving no matter how small the shell's contribution to the total luminosity. This driving mechanism also provides a sharp filter mechanism picking out modes with large amplitudes in the nuclear burning region. Unfortunately, these modes always have periods in the range of 50–200 s, a factor of at least 3 to 4 too short to agree with the observations. Kawaler points out that this result seems to be essentially independent of the free parameters associated with uncertainties in the input physics: as long as there is a He-burning shell the object pulsates. Therefore, since the pulsations in this period range are not observed, we are forced to conclude that most PNN, including the PNNV, do not have active He-shell burning sources.

4. COMPARISON OF THEORY AND OBSERVATION OR SEISMOLOGY IN ACTION

The purpose of this conference is to discuss the use of pulsations to probe the hidden interiors—for this purpose anything below the photospheres—of the sun and the stars. There has been a great deal of talk about the promise of the future for this sort of technique, with views expressed ranging from "it'll never work," to "it'll give us the age of the universe." Before getting to the obligatory wild claims for future progress in the study of the compact pulsators (so necessary to ensure future employment), it is rewarding to review what we've already learned. Perhaps much to the surprise of the strong "it'll never work" contingent, the seismology of the compact stars has already revealed something of their internal structure and evolution.

A map of the theoretical pulsational instabilities as a function of surface hydrogen layer mass for $0.6M_{\odot}$ DA models is sketched in Figure 4 (taken from Winget and Fontaine 1982). The "DA forbidden zone" indicates that DA models with $\log(M_H/M_{\star}) < -14$ are not stable against mixing from the underlying He convection zones and hence are not representative of DA white dwarfs. The arrow marked "ML2" shows the effect of assuming more efficient convection in the equilibrium models by taking the ratio of the mixing length to the pressure scale height to be 2; this has the effect of moving the entire diagram about 1,600 K to the left. Comparison of the results summarized in this figure with the observations immediately teaches us a number of things about the DAV stars.

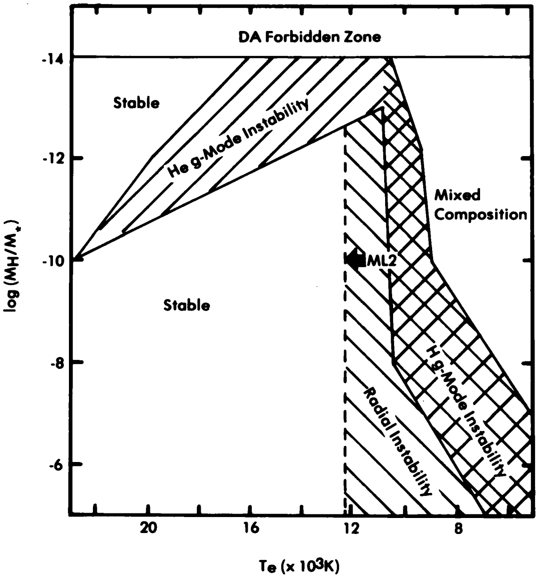

Figure 4. Locations of the theoretical DA instability strips as a function of surface H-layer mass.

First, since the blue edge of the DAV instability strip is sharp, then the appropriate instability strip is the H g-mode instability strip driven by hydrogen partial ionization. Second, the absence of any extremely hot variables implies there are few, if any, DA stars with $M_H/M_\star \ll 10^{-10}$. Third, the paucity of nonvariables within the instability strip implies that $M_H \ll 10^{-7}$ for most DA stars. This in turn, implies that most DA stars convectively mix when they cool off sufficiently for the hydrogen convection zone to penetrate to the underlying He layer ($T_e \ll 10,000$ K). This is consistent with the observed large increase in the ratio of non-DA to DA stars (Sion 1984). In fact, the recent work by Greenstein (1986) indicates that there may not be any true DA stars below 10,000 K; all seem to contain significant amounts of He. Fourth, the observed absence of any radial pulsations implies that perhaps the surface conditions on the DA stars are such that the energy leakage from potential radial modes is sufficient to dissipate any driving present (see Hansen *et al.* 1985 for a discussion of this.) Finally, the temperature of the observed blue edge implies that if the mixing length theory is used to model convection, it must be very efficient.

The theoretical blue edge of the helium instability strip is also extremely sensitive to convective efficiency. Here again, in order to match the blue edge, very efficient convection is required (Winget *et al.* 1983, Koester *et al.* 1985). Thus we have calibrated the mixing-length theory of convection, as applied to the white dwarf stars, by using the locations of the blue edges of the DBV and DAV instability strips, with the wistful hope that such a calibration is physically meaningful.

The compact pulsators are relatively rich in periods but not nearly so rich as the nonradial g-mode spectrum. This argues that there is some sort of selection mechanism which causes modes with particular values of k to be preferentially excited over other modes, perhaps even with adjacent values of k. The work of Winget *et al.* (1981) and Dolez and Vauclair (1981) has shown that resonances of the eigenfunction of certain g-modes with the thicknesses of the surface composition layers decreases the kinetic energy of the modes by orders of magnitude and decreases their amplitude in the damping region, making them preferentially easier to excite. This effect (referred to as mode-trapping and discussed in detail by Winget and Fontaine 1982) is sensitive to the composition transition zone thickness: the thinner the composition transition zone the larger the difference in the kinetic energies of the trapped modes relative to the modes with larger penetration into the core. The clear presence of a selection mechanism in the DAV stars then provides direct evidence for the operation of element diffusion significantly below the photosphere. Also, because the frequencies of the resonances are determined by the mass of the surface layer, comparison of the observed periods with theoretical models may ultimately yield layer masses and temperatures for individual objects.

As we mentioned before, S. Kawaler and collaborators (1986a, Kawaler et al. 1986) have carried out theoretical nonadiabatic pulsation calculations of the pulsation properties of evolutionary models representing PNN and DO stars with active shell burning sources. They have computed models appropriate to H-rich and He-rich PNN. Briefly, they find that *all* models with active nuclear burning shells (H or He-shells or both)—no matter how you build them, or whose input physics you use—pulsate. Since pulsation is clearly the exception rather than the rule in the planetary nebula nuclei and hot DO stars, this implies that these objects, contrary to our previous beliefs, *do not* have active nuclear shell burning sources.

This conclusion demands that the mass loss process for the DA and DB progenitors in the PN stage must have proceeded down to a level where the shell burning sources were extinguished. This result may explain the observed presence of significant amounts of O at the surface of the hot DO and many PNN stars. It may also explain the reason for the extremely thin H layer on the DA stars. The layer is thin because the mass loss proceeded to the point where the H-shell was extinguished and hence the residual hydrogen mass fraction was much lower than $10^{-4}M_{\odot}$.

Kawaler is responsible for another fascinating "seismological" result (Kawaler 1986a, and these proceedings). His analysis of the observed periods in the DOV star PG 1159-035 indicate that while they do not represent consecutive values of k, it is possible to derive the period spacing of consecutive k modes from a statistical analysis of the 8 periods present in the light curve. Since the k value appropriate to the observed periods is quite large (cf. Kawaler *et al.* 1985a), the spacing is just the asymptotic value to a very good approximation. The spacing so derived from the observations can then be compared with the asymptotic period spacings for a grid of theoretical models in which the only free parameters are ℓ and the total stellar mass; because the region of period formation is largely below the surface driving layers, and all models look essentially the same in the degenerate interior, the period spacing is independent of most uncertainties in the input physics and the envelope composition structure (*cf.* Kawaler *et al.* 1985a). *The observed periods match with only ℓ of 1 or 3.* If we then adjust the mass until we get an exact match, we find that the total stellar mass is $0.60 \pm 0.01M_{\odot}$, a mass measurement of unprecedented precision.

Arguably the most interesting observable seismological quantity is the rate of period change, because it provides a direct measure of the rate of evolution of the star. We can see this, qualitatively, by differentiating the simple asymptotic expression for the periods of the nonradial g-modes. For the compact pulsators, this gives (Winget *et al.* 1983, Kawaler *et al.* 1985a):

$$\frac{1}{\Pi}\frac{d\Pi}{dt} = -\frac{a}{T}\frac{dT}{dt} + \frac{b}{R}\frac{dR_{\star}}{dt} \qquad (1)$$

This expression is extremely crude; clearly there are additional terms, such as the effect of rotation (potentially important in the DOV and PNNV stars, for a discussion see Kawaler *et al.* 1985b), but these only modify the coefficients a and b which are of order unity.

Equation (1) is sufficient to demonstrate that the magnitude of the rate of period change is inversely proportional to the evolutionary timescale. It also illustrates that the sign of $d\Pi/dt$ for the DBV and DAV stars, where $dR_\star/dt \sim 0$, is expected to be positive (since $dT_c/dt < 0$). In contrast, the sign for $d\Pi/dt$ can be either negative or positive for the DOV and PNNV stars since the radius term is not negligible and the overall sign will depend on the relative magnitudes of a and b.

Doubtless all readers are now sitting on the edge of their seats wondering if such a measurement, no matter how interesting, is possible only in the wildest dreams of the most Π-eyed theorist. Surprisingly, considering the complexity of even the simplest of the compact objects, the answer is *no*, we can, and have, actually measured this quantity in the compact pulsators—but the measurement is extremely difficult.

Let's examine briefly how this might be done for a particular star. The first step is to resolve the light curve completely into its component frequencies. In order to proceed, we must find at least one period which is rigorously stable in amplitude and frequency. We must then measure this period accurately enough to eliminate the possibility of any cycle count error; in order to do this we typically have not only 24-hr aliases of the period of interest to contend with but the other periods in the light curve and their aliases as well. At this point we continue to add data, from season to season, for example, constantly improving the value for our period and initial phase. The errors in this process then provide an upper limit on any period change. At some point then adding more data does not improve the fit and we are forced to consider the next simplest model for the period which incorporates a term allowing for a secular change in the period. Having done all this there are two related ways to determine the rate of this period change. In practice both are used in order to provide an additional check on the solution.

The most direct method for computing the rate of period change is to substitute a frequency plus a rate of change term for the constant frequency term in the fit to the data. Then a non-linear least-squares or equivalent procedure can be used to solve for the new term. Here again, there is a new set of one-cycle aliases to be worried about, so you might say that alias space becomes two-dimensional in this analysis, accounting for why it is so difficult and requires so much data.

The second method is somewhat easier to visualize and gives us a better feel for the nature of the aliases. This method uses the (O-C) diagram, where the difference between the observed time of maximum, O, and the time computed from an ephemeris constructed from previous observations, C, is plotted against the cycle number (computed using the best period from previous data), E. If there is no period change, then the result will be a straight line; any deviation from the straight line is then due to a period change. We can see this if we write the expression for the time of maximum of a pulsation as a series expansion in E, and keep only terms to order E^2.

$$t_{max} = t_\circ + \Pi_\circ E + \frac{\Pi_\circ}{2}\frac{d\Pi_\circ}{dt}E^2 \tag{2}$$

where $\Pi_\circ$ and $t_\circ$ refer to the period and time of maximum with respect to some arbitrarily chosen reference epoch. We can now see that an alias arises anytime the accumulated error in Equation (2), due to error in $\Pi_\circ$, or $d\Pi/dt$, or both, adds up to one-cycle. Note that a one cycle error in $t_\circ$ does not effect the determination of either $\Pi_\circ$ or its derivative, but the combination of the other two sources of cycle count error is more than enough to keep us busy. Equation (2) also illustrates the reason measurements of period change are possible at all: the precision of a limit or measurement improves with the *square* of the time represented by the baseline.

The results we have to date on period changes in the compact pulsators are summarized in Table 1. We have limits on four objects, a measurement on one, and are already in a position to draw some interesting conclusions about the input physics of the theoretical models. First, because the measured timescale for the DOV star is in excellent agreement with the theory (Winget, Van Horn, and Hansen 1983, and Kawaler *et al.* 1985a), and the theoretical timescale is dominated by the energy loss rate through plasmon neutrinos, we can infer that the theoretical energy loss rate, ϵ_ν(plasmon), is essentially correct. This is the first direct test of this rate. Second, the new limit for the the DAV star G117-B15A (Kepler *et al.* 1986) is now within a factor of 5 of the expected value for C/O core white dwarfs, and eliminates any core composition significantly heavier than carbon or oxygen.

TABLE 1

Period Changes in Compact Pulsators

Name	Class	Π (s)	$\lvert\Pi/\dot{\Pi}\rvert_{obs}$ (yr)	$\lvert\Pi/\dot{\Pi}\rvert_{theor}$ (yr)	Ref.
PG1159–035	DOV	516	$(1.4 \pm 0.1) \times 10^6$	$\sim 10^6$	1
PG1351+489	DBV	489	$> 3 \times 10^5$	$\sim 3 \times 10^5$	2
G117–B15A	DAV	215	$> 6.9 \times 10^8$	$\sim 4 \times 10^9$	3
R548	DAV	213	$> 3 \times 10^7$	$\sim 4 \times 10^9$	4
L19–2	DAV	114	$> 9 \times 10^6$	$\sim 4 \times 10^9$	5

1. Winget *et al.* (1985); 2. J. A. Hill (1986, private communication); 3. Kepler *et al.* (1987); 4. Stover *et al.* (1980); 5. O'Donoghue *et al.* (1982).

The direct measurement of evolutionary timescales allows us to calibrate the theoretical cooling sequences. This has surprisingly far reaching implications, even beyond the exploration of the high density, high temperature properties of matter. The reason stems from the work of Liebert and collaborators (Liebert 1980, and references therein, Liebert, Dahn, and Sion 1983) on the luminosity function of the white dwarf stars. They have conclusively shown that there is a dramatic shortfall in the white dwarf luminosity function, below $\log(L/L_\odot) \approx -4.5$, and that there is no pile-up of white dwarfs prior to the luminosity of the shortfall. This implies that the cause of the shortfall is the finite age for the earliest epoch of star formation in the disk, τ_{SFD}.

Recently, Van Horn *et al.* (1986, preprint) have demonstrated that the age of disk determined in this way is $\tau_{SFD} = 10.5 \pm 1.5$ Gyr, comparable to the age derived by nucleocosmochronoligical methods by Fowler and Meisl (1986) of $\tau_{SFD} = 10.7 \pm 2.5$ Gyr. Their technique is illustrated in Figure 5. Here, I have sketched the theoretical luminosity function representing C/O core white dwarfs, with compositionally stratified envelopes. This is adapted from the pure carbon sequence of Van Horn *et al.*) by multiplying by their estimated corrections for C/O interior composition and compositional stratification. Note that the stellar mass distribution determined by Weidemann and Koester (1984) for the DA's was used. The resultant age of the disk, however, is largely independent of the shape of the mass distribution; only the number of white dwarfs below the shortfall, not the location of the shortfall, is sensitive to the mass distribution. For this reason it is also insensitive to the initial mass function for the main sequence. In addition, since the progenitors of the white dwarfs are anything with $M_\star/M_\odot < 5 - 8$, we can safely neglect the main sequence lifetimes.

Fowler and Meisl (1986) point out that given an age for the disk one need only add about 1 Gyr—the time for galaxy formation—to arrive at the age of the universe. Thus the white dwarfs yield an age of the universe of $\tau_u = 11.5 \pm 2.0$ Gyr (I've added 0.5 Gyr to the error budget to account for uncertainties in the pre-galactic evolution), and we haven't

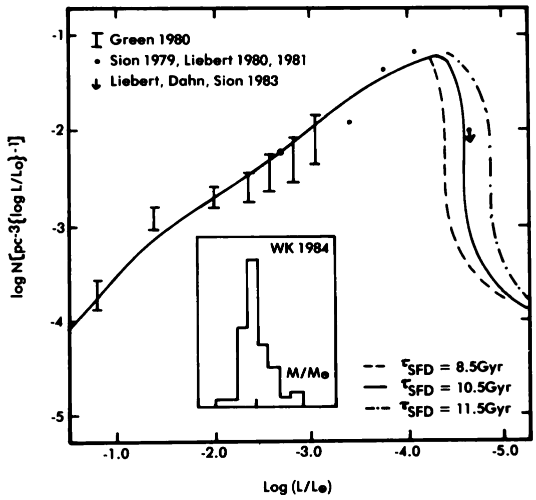

Figure 5. The observed white dwarf luminosity function. Comparison with theory (solid line) gives (~10.5 ± 1.5) for the age of the disk.

measured a single redshift, or climbed a single distance pyramid.

The error in this technique is already relatively small, and will improve. It depends only on two things: the location of the luminosity shortfall, and the cooling times for the white dwarf. The shortfall is already statistically quite significant (Liebert 1980), and its precise location will improve steadily as we get more and better temperature and luminosity determinations for more cool white dwarfs. The cooling theory will be continually refined by comparison with the cooling times measured for the compact pulsators. In summary the error is small, relative to other measures of the age of the universe, for one principle reason: the technique depends only on the theory and observation of an *extremely* homogeneous class of objects, the white dwarfs.

5. SUMMARY AND FUTURE PROSPECTS

In the study of the compact pulsators we are extremely fortunate that we already have a basic framework for interpreting the observations. By this I mean that we have established, at a high confidence level, what kind of modes we are observing and we also have a pretty good idea of how they are excited: the oscillations are due to nonradial gravity modes driven by surface partial ionization zones. Some parts of this basic model may need to be modified for some individual objects, but it gives us a firm point of departure.

5.1. Where do we stand?

To summarize our progress to date:

- We have frequencies accurate to one part in 10^9 for comparing with the theory.
- We have calibrated the mixing length theory as applied to the white dwarf stars.
- We have set stringent limits on the surface layer masses of the white dwarf stars and demonstrated the prospect for the measurement of the masses of individual stars.
- We have shown how the observational decomposition of the light curve a DOV variable star can provide us with an accurate measure of the object's total stellar mass.
- We have used the rate of period change measurement in a DOV to show that plasmon neutrinos have roughly the expected properties.

- **We have demonstrated that the measurement of the evolutionary timescales in the compact pulsators can be used to calibrate the white dwarf cooling sequence, which can in turn be used to obtain an accurate measure of the age of the Galactic disk.**

5.2. Where to from here?

Our understanding of the compact pulsators is far from complete and a number of questions and problems in need of investigation immediately come to mind. We can usefully sort these into two categories: the short term, specific problems; and the longer term, more general problems.

Two of the most pressing specific problems relate to the nature of the driving mechanism in the DOV and PNNV stars. If partial ionization of surface elements is responsible for the driving, then why are there a number of objects which are spectroscopically identical to the DOV and PNNV stars, right down to the oxygen lines, yet *do not* pulsate (Cox 1986). In particular, why does the object H1504+65 not pulsate (from recent observations at McDonald Observatory, and also A. Grauer, private communication)? It has a significant oxygen abundance and much less helium than the DOV and PNNV stars and is otherwise spectroscopically similar (Nousek *et al.* 1986). The reduction in the amount of the "pulsation poisoning" from He should make this an ideal candidate for pulsations.

The second problem is that if it is not partial ionization which is completely responsible for the driving, then where are the 200 s oscillations which seem to be a signature of a nuclear shell-burning source? A search for ~200 s pulsations in a large sample of PNN stars is needed.

Another worthy goal is the solution of the light curves of the two remaining (after the solution of PG 1159-035) DOV stars and the PNNV K1-16. This would provide us with a crucial test of the applicability of Kawaler's (1986b) technique for using period spacings to constrain masses and identify the ℓ values of the observed modes. In addition it would be an important first step towards ultimately measuring the value of $d\Pi/dt$ for these objects. It would also be of interest to try to apply Kawaler's technique for searching for period spacings to the DAV and DBV stars; Some of the stars pulsating with $\Pi \sim 1,000$ s may be sufficiently close to the asymptotic limit of high k for this technique to be applicable.

In addition, there are a number of important problems that are either long term in nature, or require a large scale effort, or both. One such is to improve the statistics of the PNNV, DOV, and DBV stars. Attempts to define class properties and instability regions are seriously frustrated by the small number of examples of these stars. Recall that most of these objects originally came from the Palomar-Green survey (Green, Schmidt and Liebert 1986); hence, a similar survey would be ideal if it were extended well below the magnitude limit of the original and carried out in both hemispheres.

On the theoretical side, the period ratios observed in several of the compact pulsators have no simple explanation in terms of linear analysis of non-radial g-modes and cry out for nonlinear nonradial calculations. Also, as more of the light curves of these stars are resolved, the observation of average pulse shapes become meaningful and nonlinear calculations are needed to exploit these. Such calculations are complex, of course, but with the increasing availability of super-computers perhaps the prospects for progress may be good.

Similarly, all of these objects apparently have significant surface convection zones. In order to improve our understanding of the red edge of the instability strips and the nature of possible mode-selection mechanisms we must have a reliable theory of pulsation/convection interaction. Since this assumes an understanding of convection, the "Holy Grail" of hydrodynamics, as a necessary first step, the prospects for progress here may not be as good. Although it might be easier to teach donkeys to fly, the widespread importance of this problem in *all* of Helio- and asteroseismology demands that we continue to work on it.

The kind and quality of seismological data on the structure and evolution of compact pulsators has increased dramatically in recent years. Interpreting this information is importance not only in understanding the physics of matter at high densities and temperatures, but in measuring the age of the universe. These improved constraints demand a new generation of evolutionary models of the compact objects incorporating the recent major advances in time dependent diffusion theory and the equation of state for dense plasmas. Again, thanks to the supercomputers, the prospects for these efforts are good.

The single most exciting prospect for the future is the extended coverage of large numbers of compact pulsators currently in the works. Ed Nather and his collaborators in a host of countries, with observing sites well distributed in longitude in both hemispheres, have launched a major long-term international campaign which will establish a network of observers around the globe. This network is now funded and should be in place within a year. Each observing site will be equipped with identical state-of-the-art multi-star photometers. These new photometers will provide accurate continuous extinction coefficients for determining amplitudes to unprecedented accuracy and, when coupled with the extended time base, will allow us to look at variations in the compact pulsators on timescales never before possible—up to several hours, and perhaps longer. We can expect also that these observations will help us to resolve many light curves previously impossible due the forest of aliases inherent in observations at a single site. This will allow us to measure rates of period change in a much larger sample of objects than is currently possible. In addition the total time-base needed to detect period change can be expected to decrease because the extended coverage will dramatically increase the timing accuracies for individual seasons.

The above represents a sampling of what we can confidently *expect* to find, but if the history of this field has taught us anything it has taught us that we will find little that we do expect and much that we would never expect. Either way, extended coverage will produce a qualitative revolution in the study of the compact pulsators that can only be compared with the introduction of the two-star photometer. It is likely that after the data begin to come in, the observations (and probably most of the theory) described in this review will be relegated to the status of the quaint pre-history of the seismology of compact pulsators.

MORAL: We should all be in *busi*ness for an extended period of time squared to come. But before we get back to the future, let us always keep an eye out for the mistakes of the past, lest the *ass* (Figure 6) creep back into our field.

I am deeply grateful to H. M. Van Horn and C. J. Hansen for teaching me how to think about the compact pulsators, and E. L. Robinson and R. E. Nather for teaching me also to look. I thank S. O. Kepler, S. D. Kawaler, B. P. Hine, J. A. Hill, and especially M. A. Wood, for many useful discussions on this subject and also for their tireless help in preparing this manuscript.

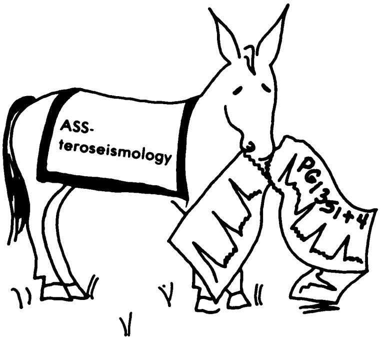

Figure 6. That which we strive to eliminate: The ass standing in the field of asteroseismology.

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