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FINITISTIC DIMENSIONS AND GOOD FILTRATION DIMENSIONS OF STRATIFIED ALGEGRAS

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$\Delta-$finitistic dimensions of standardly stratified algebras are defined similarly to properly stratified algebras. It is proved that the finitistic dimension for any standardly stratified algebra is bounded by the sum of the $\Delta-$finitistic dimension and the $\overline{\nabla}$-good filtration dimension. Finally, the $\overline{\nabla}-$good filtration dimension of standardly stratified algebras is equal to the $\overline{\Delta}-$good filtration dimension of their Ringel duals.

1. INTRODUCTION

As generalisations of quasi-hereditary algebras, properly stratified algebras and standardly stratified algebras have been introduced by Cline, Parshall, Scott [**3**] and Dlab [**4**]. They appear in the work of Futorny, König and Mazorchuk on a generalisation of the category O ([**7**]). Analogously to quasi-hereditary algebras ([**13**]), given a standardly stratified algebra (A, Λ), of central importance are the modules filtered by respectively standard modules, costandard modules, proper standard modules or proper costandard modules (the precise meaning will be given in Section 2) [**1, 2, 12, 14, 15**].

Recently, in order to calculate the global dimension of the Schur algebra for GL_2 and GL_3, Parker [**10, 11**] introduced the notion of ∇- (or Δ-)good filtration dimension for a quasi-hereditary algebra. In [**15**] we generalise this notion to standardly stratified algebras, and observe that the $\overline{\nabla}-$good filtration dimension of standardly stratified algebras is the projective dimension of their characterisation module. By using this, we gave an upper bound of the finitistic dimensions for certain class of proper stratified algebras, whose characteristic tilting modules coincide with characteristic cotilting modules. This result was soon generalised and strengthened. On the one hand, this upper bound is proved in [**8**] to be true for all proper stratified algebras; on the other hand, adding the condition that there is a duality preserving simple modules on this class of algebras, the bound is exactly the finitistic dimension ([**9**]). For a given standardly stratified algebra $(A \leqslant)$, the Ringel dual $(A', \leqslant^{op})$ is not standardly stratified, but its opposite algebra is standardly stratified ([**1**]) (note that if A is quasi-hereditary, then A' is also

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quasi-hereditary [**13**]). In [**6**], the precise relation between good filtration dimensions of quasi-hereditary algebras and of their Ringel duals is given.

The aim of this note is firstly, to generalise the result in [**8, 15**] on the upper bound of finitistic dimension of properly stratified algebras to standardly stratified algebras; and secondly, to prove the two good filtration dimensions for standardly stratified algebras and their Ringel duals are equal to each other. The later generalises the corresponding result on quasi-hereditary algebras in [**6**].

2. Preliminaries

Let R be a commutative Artin ring and A a basic Artin algebra over R. We shall consider finitely generated left A-modules; maps between A-modules will be written on the righthand side of the argument, and the composition of maps $f: M_1 \to M_2$, $g: M_2 \to M_3$ will be denoted by fg. The category of left A-modules will be denoted by A-mod. All subcategories considered will be full and closed under isomorphisms.

Given a class Θ of A-modules, we denote by $\mathcal{F}(\Theta)$ the full subcategory of all A-modules which have a Θ-filtration, that is, a filtration

$$0 = M_t \subset M_{t-1} \subset \cdots \subset M_1 \subset M_0 = M$$

such that each factor M_{i-1}/M_i is isomorphic to an object of Θ for $1 \leqslant i \leqslant t$. The modules in $\mathcal{F}(\Theta)$ are called Θ-good modules, and the category $\mathcal{F}(\Theta)$ is called the Θ-good module category.

In the following, $(A, \leqslant)$ will denote the algebra A together with a fixed ordering on a complete set $\{e_1, \ldots, e_n\}$ of primitive orthogonal idempotents (given by the natural ordering of indices). For $1 \leqslant i \leqslant n$, let $E(i)$ be the simple A-module, which is the simple top of the indecomposable projective $P(i) = Ae_i$. The standard module $\Delta(i)$ is by definition the maximal factor module of $P(i)$ without composition factors $E(j)$ with $j > i$. $\overline{\Delta(i)}$ will be the notation for proper standard module, which is the maximal factor module of $\Delta(i)$ such that the multiplicity condition

$$\left[\overline{\Delta(i)} : E(i)\right] = 1$$

holds. Dually for $1 \leqslant \lambda \leqslant n$, we have costandard modules $\nabla(\lambda)$ and proper costandard modules $\overline{\nabla(\lambda)}$.

Let Δ be the full subcategory consisting of all $\Delta(\lambda)$, $\lambda \in \Lambda$. In a similar way, we introduce ∇, and so on.

The pair $(A, \leqslant)$ is called standardly stratified if ${}_AA \in \mathcal{F}(\Delta)$ (compare [**1, 2, 3, 12**]). $(A, \leqslant)$ is called properly stratified if ${}_AA \in \mathcal{F}(\Delta)$ and ${}_AA \in \mathcal{F}(\overline{\Delta})$ (compare [**4**]). Note that these properties generalise the concept of quasi-hereditary algebras where we require the additional condition that the standard modules are Schur modules. For a standardly stratified algebra $(A, \leqslant)$, the opposite algebra $(A^{op}, \leqslant)$ is not standardly stratified in general. In contrast $(A, \leqslant)$ is properly stratified if and only if so is $(A^{op}, \leqslant)$. We denote by $D(-)$ the usual duality from A–mod to A^{op}–mod.

We say that A has a duality if there exists an exact involutive, contravariant equivalence $(-)^\circ$: A-mod $\to$ A-mod preserving simple modules, that is $\big(E(i)\big)^\circ = E(i)$ for any i.

Firstly we note that standardly stratified algebras $(A, \leqslant)$ with a duality $(-)^\circ$ preserving simple modules must be properly.

Proposition 2.1. *Let $(A, \leqslant)$ be a standardly stratified algebra with a duality preserving simple modules. Then $(A, \leqslant)$ is properly stratified.*

Proof: Let $(-)^\circ$ be the duality preserving simple A–modules. It is easy to see that $\big(\Delta(i)\big)^\circ = \nabla(i)$ and $\big(\overline{\Delta}(i)\big)^\circ = \big(\overline{\nabla}(i)\big)$ for any i. Since ${}_AA \in \mathcal{F}(\Delta)$, the injective generator $D(A_A) \in \mathcal{F}(\overline{\nabla})$ ([**1**]). It follows that $\big(D(A_A)\big)^\circ \in \mathcal{F}\big((\overline{\nabla})^\circ\big)$. Therefore ${}_AA \in \mathcal{F}(\overline{\Delta})$, that is $(A, \leqslant)$ is a properly stratified algebra. ∎

3. Good filtration dimensions

Given a standardly stratified algebra (or a properly stratified algebra) $(A, \leqslant)$. Let Θ be any class of A–modules. Let M be a left A–module. Now we recall the notation of good filtration dimensions from [**10, 15, 8**].

We say M has a Θ-resolution of (possibly infinite) length l if we have a resolution

$$0 \to X_l \to X_{l-1} \to \cdots \to X_1 \to X_0 \to M \to 0$$

with all $X_i \in \Theta$.

We define the Θ–filtration dimension of M, denoted by $\dim_\Theta(M)$ to be the minimal length of a finite Θ–resolution of M, should one exist and $\dim_\Theta(M) = \infty$ otherwise.

Dually we say M has a Θ–co-resolution of (possible infinite) length l if we have a resolution

$$0 \to M \to X_0 \to X_1 \to \cdots \to X_{l-1} \to X_l \to 0$$

with all $X_i \in \Theta$.

We similarly define Θ–filtration codimension of M, denoted by $\operatorname{codim}_\Theta(M)$ to be the minimal length of a finite Θ–co-resolution of M, should one exist and $\operatorname{codim}_\Theta(M) = \infty$ otherwise.

It is easy to see that for any $M \in A$–mod, $\operatorname{codim}_{\mathcal{F}(\overline{\nabla})}(M) < \infty$, and if A is properly stratified, then $\dim_{\mathcal{F}(\overline{\Delta})}(M) < \infty$ ([**10, 15, 8**]). Hence we can define $\operatorname{codim}_{\mathcal{F}(\overline{\nabla})}(A)$ and $\dim_{\mathcal{F}(\overline{\Delta})}(A)$ if A is properly stratified as the maximal value of $\operatorname{codim}_{\mathcal{F}(\overline{\nabla})}(M)$ and $\dim_{\mathcal{F}(\overline{\Delta})}(M)$ for $M \in A$–mod respectively ([**10, 15**]).

We also can define the Δ-finitistic dimension for the standardly stratified algebra A:

$$\text{fin}\,.\,\dim_{\mathcal{F}(\Delta)}(A) = \sup\{\dim_{\mathcal{F}(\Delta)}(M) \mid M \in A - \text{mod with } \dim_{\mathcal{F}(\Delta)}(M) < \infty\};$$

If A is properly stratified, we can define the ∇-finitistic codimension for A.

$$\text{fin}\,.\,\text{codim}_{\mathcal{F}(\nabla)}(A) = \sup\{\text{codim}_{\mathcal{F}(\nabla)}(M) \mid M \in A - \text{mod with } \text{codim}_{\mathcal{F}(\nabla)}(M) < \infty\}.$$

We recall the finitistic dimension for an algebra:

$$\text{fin}\,.\,\dim\,.(A) = \sup\{\text{proj}\,.\,\dim\,.(M) \mid M \in A - \text{mod with } \text{proj}\,.\,\dim\,.(M) < \infty\}.$$
$$\text{fin}'\,.\,\dim\,.(A) = \sup\{\text{inj}\,.\,\dim\,.(M) \mid M \in A - \text{mod with } \text{inj}\,.\,\dim\,.(M) < \infty\}.$$

Lemma 3.1. *Let $(A, \leqslant)$ be a standardly stratified algebra. Then* $\text{fin}\,.\dim_{\mathcal{F}(\Delta)}(A) < \infty$. *Moreover if $(A, \leqslant)$ is properly stratified, then* $\text{fin}\,.\,\text{codim}_{\mathcal{F}(\nabla)}(A) < \infty$.

Proof: Let X be a left A-module with $\dim_{\mathcal{F}(\Delta)}(M) = n < \infty$. Then there exists an exact sequence

$$0 \to X_l \to X_{l-1} \to \cdots \to X_1 \to X_0 \to X \to 0$$

with all $X_i \in \mathcal{F}(\Delta)$. It follows from [**1**, Proposition 1.8.] that $\text{proj}\,.\dim X_i \leqslant n-1$ for all i. Then $\text{proj}\,.\dim\,.X < \infty$. Therefore $\text{fin}\,.\,\dim_{\mathcal{F}(\Delta)}(A) \leqslant \text{fin}\,.\,\dim\,.(A)$. It is well-known that the finitistic dimension of A is finite ([**2**]). Then $\text{fin}\,.\,\dim_{\mathcal{F}(\Delta)}(A) < \infty$. The second conclusion follows from [**8**, Lemma 2], but we present another proof here for the completeness. Let $(A, \leqslant)$ be a properly stratified algebra. Then $(A^{op}, \leqslant)$ is also a properly stratified algebra. It is easy to see that $\text{fin}\,.\,\text{codim}_{\mathcal{F}(\nabla)}(A) = \text{fin}\,.\,\dim_{\mathcal{F}(\Delta)}(A^{op})$ and the final term is finite which is proved above. Then $\text{fin}\,.\,\text{codim}_{\mathcal{F}(\nabla)}(A) < \infty$. ∎

Theorem 3.2. *Let $(A, \leqslant)$ be a standardly stratified algebra. Then the finitistic dimension of A is bounded by the sum of* $\text{fin}\,.\dim_{\mathcal{F}(\Delta)}(A)$ *and* $\text{codim}_{\mathcal{F}(\overline{\nabla})}(A)$, *that is,* $\text{fin}\,.\,\dim(A) \leqslant \text{fin}\,.\,\dim_{\mathcal{F}(\Delta)}(A) + \text{codim}_{\mathcal{F}(\overline{\nabla})}(A)$.

Proof: Let M be a left A-module with finite projective dimension. Then M has a finite Δ-filtration dimension t, where $t \leqslant \text{fin}\,.\,\dim_{\mathcal{F}(\Delta)}(A)$. Let

$$0 \to X_t \to X_{t-1} \to \cdots \to X_1 \to X_0 \to M \to 0$$

with all $X_i \in \mathcal{F}(\Delta)$ be a resolution of M. Then $\text{proj}\,.\,\dim\,.X_t \leqslant \sup\{\text{proj}\,.\,\dim\,.\Delta(i) \mid i \in \Lambda\} = \text{proj}\,.\,\dim\,.T$, where T is the characteristic module of $(A, \leqslant)$. Therefore we have that $\text{proj}\,.\,\dim\,.X_t \leqslant \text{codim}_{\mathcal{F}(\overline{\nabla})}(A)$ as $\text{proj}\,.\,\dim\,.T = \text{codim}_{\mathcal{F}(\overline{\nabla})}$ ([**15**]). It follows that $\text{proj}\,.\,\dim\,.M \leqslant t + \text{codim}_{\mathcal{F}(\overline{\nabla})} \leqslant \text{fin}\,.\,\dim_{\mathcal{F}(\Delta)}(A) + \text{codim}_{\mathcal{F}(\overline{\nabla})}(A)$. ∎

Corollary 3.3. *Let $(A, \leqslant)$ be a standardly stratified algebra with a duality preserving simple modules. Then* $\text{fin}\,.\,\dim_{\mathcal{F}(\Delta)}(A) = \text{codim}_{\mathcal{F}(\overline{\nabla})}(A) = \text{proj}\,.\,\dim\,.(T)$; *and*

$\text{fin.dim}(A) \leqslant 2\,\text{codim}_{\mathcal{F}(\overline{\nabla})}(A) = 2\,\text{proj.dim.}(T)$, *where T is the characterisation tilting module of $(A, \leqslant)$.*

PROOF: Let $(A, \leqslant)$ be a standardly stratified algebra with a duality. It follows from Proposition 2.1. that $(A, \leqslant)$ is properly stratified. Then using the duality, we have $\dim_{\mathcal{F}(\overline{\Delta})}(A) = \text{codim}_{\mathcal{F}(\overline{\nabla})}(A) = \text{proj.dim.}(T)$. It follows from [**8**, Lemma 2] that $\text{fin.dim}_{\mathcal{F}(\Delta)}(A) = \dim_{\mathcal{F}(\overline{\Delta})}(A) = \text{codim}_{\mathcal{F}(\overline{\nabla})}(A) = \text{proj.dim.}(T)$. ∎

If $(A, \leqslant)$ is a properly stratified algebra, then there are characteristic tilting module T and cotilting module S such that $\text{add}\,T = \mathcal{F}(\Delta) \sqcap \mathcal{F}(\overline{\nabla})$ and $\text{add}\,S = \mathcal{F}(\nabla) \sqcap \mathcal{F}(\overline{\Delta})$ ([**1, 12, 14, 15**]). It was proved in [**15, 8**] that

$$\text{fin.codim}_{\mathcal{F}(\nabla)}(A) = \text{codim}_{\mathcal{F}(\overline{\nabla})}(A) = \text{proj.dim}(T)$$
$$\text{fin.dim}_{\mathcal{F}(\Delta)}(A) = \dim_{\mathcal{F}(\overline{\Delta})}(A) = \text{inj.dim}(S).$$

With the results above, we have the following [**8**, Theorem 1].

COROLLARY 3.4. *Let $(A, \leqslant)$ be a properly stratified algebra. Then*

$$\text{fin.dim.}(A) \leqslant \text{proj.dim.}(T) + \text{inj.dim.}(S)$$
$$\text{fin}'\text{.dim.}(A) \leqslant \text{proj.dim.}(T) + \text{inj.dim.}(S).$$

PROOF: The first inequality follows from Theorem 2.3. The proof for the second one is dual to the first. We give a proof for the sake of completeness. Since $(A, \leqslant)$ is a properly standardly stratified algebra, $(A^{op}, \leqslant)$ is also a properly standardly stratified algebra with $\nabla_{A^{op}}(i) = D(\Delta_A(i))$, $\Delta_{A^{op}}(i) = D(\nabla_A(i))$, $\overline{\nabla}_{A^{op}}(i) = D(\overline{\Delta}_A(i))$ and $\overline{\Delta}_{A^{op}}(i) = D(\overline{\nabla}_A(i))$. Therefore the characteristic tilting module and cotilting module for $(A^{op}, \leqslant)$ are $D(S)$ and $D(T)$. By the first part of the Corollary, we have

$$\begin{aligned} \text{fin.dim.}(A^{op}) &\leqslant \text{proj.dim.}(D(S)) + \text{inj.dim.}(D(T)) \\ &= \text{inj.dim.}(S) + \text{proj.dim.}(T). \end{aligned}$$

Therefore $\text{fin}'\text{.dim.}(A) = \text{fin.dim.}(A^{op}) \leqslant \text{inj.dim.}(S) + \text{proj.dim.}(T)$. ∎

REMARK 3.5. For properly stratified algebras $(A, \leqslant)$, $\text{fin.dim.}(A) \neq \text{fin}'\text{.dim.}(A)$ in general. See the following example.

EXAMPLE 3.6. Let A be the algebra with the quiver and relations:

$$\overset{2}{\circ} \xleftarrow{\ \alpha\ } \overset{1}{\circ} \circlearrowleft \beta \qquad \alpha\beta = 0 = \beta^2$$

It is easy to see that $(A, \leqslant)$ is a properly stratified algebra, and $\text{fin.dim.}(A) = 1$, but $\text{fin}'\text{.dim.}(A) = 0$.

REMARK 3.7. There are upper bounds for quasi-hereditary algebras in [**5**] and for standardly stratified algebras in [**2**]. These bounds are known as "$2n - 2$" where n is the number of pairwise non-isomorphic simple modules. However many examples show these

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bounds are not exact [**2, 5, 8**]. The algebra $(A, \leqslant)$ in Example 3.6. above also illustrates the upper bound in Theorem 3.2. can be more effective than that given in [**2**]: in fact, the finitistic dimension of A is 1, the projective of characteristic tilting module T of A is 1 and the characteristic cotitling module S of A is the injective A–module. Then $\operatorname{fin.dim}.A = 1 = \operatorname{proj.dim}.T + \operatorname{inj.dim}.S \leqslant 2n - 2 = 2$, where the final term is the bound given in [**2**].

REMARK 3.8. If $(A, \leqslant)$ is a properly stratified algebra with a duality such that its characteristic tilting module and cotilting module coincide, then the finitistic dimension of A is proved in [**9**] to be twice of the projective dimension of its characteristic tilting module.

Let $(A, \leqslant)$ be a standardly stratified algebra with characteristic tilting module T and $B = \operatorname{End}_A T$ the Ringel dual. We denote by F the functor $\operatorname{Hom}_A(T, -)$ and by G the functor $D\operatorname{Hom}_A(-, T)$, both mapping A–mod to B–mod. The left projective B–module $F\big(T(n-i+1)\big)$ is denoted by $B(i)$. Then $(B, \leqslant)$ denotes the algebra B equipped with natural order coming from the indices of projective modules $B(i)$. It is the opposite order inherited from A and T. It was proved in [**1**] that $\overline{\Delta}_B(i) \simeq F\big(\overline{\nabla}_A(n-i+1)\big)$ and $\nabla_B(i) \simeq G\big(\Delta_A(n-i+1)\big)$; and ${}_BB \in \mathcal{F}(\overline{\Delta}_B)$, that is $(B^{op}, \leqslant)$ is a standardly stratified algebra. The following result is proved in [**6**] for quasi-hereditary algebras.

THEOREM 3.9. *Let $(A, \leqslant)$ be a standardly stratified algebra and $(B, \leqslant)$ the Ringel dual as above. Then* $\operatorname{codim}_{\mathcal{F}(\overline{\nabla})}(A) = \dim_{\mathcal{F}(\overline{\Delta})}(B)$.

PROOF: Let $\Delta(i)$ be the standard modules for A and $\Delta(A) = \bigoplus_{i=1}^{n} \Delta(i)$. It follows from [**15**, Proposition 2.1] that $\operatorname{codim}_{\mathcal{F}(\overline{\nabla})}(A) = \sup\Big\{d, \mid \operatorname{Ext}^d\big(T, \Delta(A)\big) \neq 0\Big\}$. It is easy to see the number equals to $\sup\{d, \mid \operatorname{Ext}^d(T, A) \neq 0\}$. Then it follows from [**15**, Proposition 2.3] that the both numbers above equal to $\operatorname{proj.dim}.T$. So we have that $\operatorname{codim}_{\mathcal{F}(\overline{\nabla})}(A) = \operatorname{proj.dim}.T = \operatorname{codim}_{\mathcal{F}(\overline{\nabla})}\big(\Delta(A)\big)$. Now let $0 \to \Delta(A) \to T_0 \to T_1 \to \cdots \to T_s \to 0$ be the minimal $\mathcal{F}(\overline{\nabla})$–co-resolution of $\Delta(A)$. Then all T_i are in $\operatorname{add} T$ and the cokernels of all maps in the resolution above are in $\mathcal{F}(\Delta)$. By applying $\operatorname{Hom}_A(-, T)$ to the resolution above, we get a minimal projective resolution of $\Delta(B^{op})$:

$$0 \to \operatorname*{Hom}_A(T_s, T) \to \operatorname*{Hom}_A(T_{s-1}, T) \to \cdots \to \operatorname*{Hom}_A(T_0, T) \to \operatorname*{Hom}_A(\Delta(A), T) \to 0.$$

It follows that $\operatorname{proj.dim}.\operatorname{Hom}_A(\Delta(A), T) = s = \operatorname{proj.dim}.T$. Therefore $\operatorname{codim}_{\mathcal{F}(\overline{\nabla})}(A)$ $= \operatorname{codim}_{\mathcal{F}(\overline{\nabla})}(B^{op}) = \dim_{\mathcal{F}(\overline{\Delta})}(B)$. ∎

We note that if $(A, \leqslant)$ is a quasi-hereditary algebra, then the Ringel dual $(B, \leqslant)$ of A is also quasi-hereditary, the Ringel dual of B comes back to A. Using these facts and Theorem 3.9., we have that [**6**, Corollary 2.1.5].

COROLLARY 3.10. *Let $(A, \leqslant)$ be a quasi-hereditary algebra and $(B, \leqslant)$ its Ringel dual. Then* $\operatorname{codim}_{\mathcal{F}(\nabla)}(A) = \dim_{\mathcal{F}(\Delta)}(B)$ *and* $\dim_{\mathcal{F}(\Delta)}(A) = \operatorname{codim}_{\mathcal{F}(\nabla)}(B)$.

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