SUPERNOVA REMNANTS: CAN A FOSSIL H II REGION BE FORMED AS THE RESULT OF A SUPERNOVA EXPLOSION?

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Abstract. An estimate of the amount of ionizing radiation available in a supernova explosion falls far short of the amount needed in the standard model of the Gum nebula. The only possibility left is radiation from the pulsar.

I. Physics of a Fossil H II Region

The Gum nebula is widely thought to be the best example of a fossil H II region. Its properties were discussed at a conference at Greenbelt in 1971 (Maran *et al.*, 1971). The basic data for the model of the nebula were given by Brandt (1971). They are

total number of free electrons	$\approx 2 \times 10^{62}$,
average electron density	$\approx 0.16 \text{ cm}^{-3}$,
energy required to produce the ionization	$\approx 5 \times 10^{51}$ erg.

The Gum nebula is believed to have been formed by the supernova that also produced the Vela pulsar. Its age is therefore estimated to be 10000 yr $\approx 3 \times 10^{11}$ s. Now the recombination rate in the nebula is βn_e, where $\beta = 2 \times 10^{-13} \text{ cm}^3 \text{ s}^{-1}$ is the usual recombination coefficient for ionized hydrogen in interstellar space. With $n_e \doteqdot 0.16$ cm^{-3} the rate becomes $3.2 \times 10^{-14} \text{ s}^{-1}$, and during the lifetime of the nebula only about one per cent of the protons could possibly have recombined. This justifies the estimate for the energy required to ionize the nebula: each H atom needs to be ionized only once, and only negligible energy is required to balance recombinations.

There is another important difference between the physics of fossil H II regions and that of standard H II regions. In the standard case the ionizing radiation comes from early type stars, with surface temperatures around 50000 K. Only a fraction of the photons are beyond the Lyman limit, and only a small proportion of these have energies more than a few eV above the ionization potential of hydrogen. To estimate how many ionizations are produced by the star one then simply works out the rate at which it produces photons in the Lyman continuum.

But the calculation is different for the case of a supernova remnant. Here the radiation comes, for a time at least, from a surface which is much hotter than any stellar photosphere. The ionizing photons are therefore much more energetic, on average. If a photon with, say, 100 eV energy ionizes a hydrogen atom, then the electron so produced has a kinetic energy of about 86 eV. This electron can, and will, produce further ionization by collision. By this argument Brandt derives his estimate for the energy required to ionize the nebula. Almost all the radiant energy absorbed by the gas goes to ionize hydrogen atoms.

But there are some limitations on this argument. The number of protons along the

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line of sight through the nebula is of the order of 10^{20} per cm^2. This is also the number of hydrogen atoms per unit area along the line of sight before ionization. If the ionizing radiation at a given frequency is to be efficiently absorbed then the mixture of elements in the interstellar gas must present a cross-section of at least 10^{-20} cm^2, per H atom, for absorption at that frequency. From the data given by Brown and Gould (1970) it follows that $\nu = 5 \times 10^{16}$ Hz is the largest frequency for effective absorption. This corresponds to a minimum wavelength $\lambda = 60$ Å; the maximum wavelength is, of course, $\lambda = 912$ Å, at the edge of the Lyman continuum. Photons with $\lambda = 60$ Å are typical of radiation with a colour temperature of 500000 K. We are therefore not interested here in the very hard radiation that might be produced when the initial shock from the supernova explosion reaches the surface of the star in which it occurs. The radiation which would possibly produce the ionization must come from the hot remnant, in the early phase of its expansion.

At a later stage of evolution the typical supernova remnant shares its kinetic energy with the ambient interstellar medium via a shock interaction (Woltjer, 1972). Typically, at time t, the speed of sound in the gas behind the shock is a, the radius of the shocked region is of order at and its energy content is of order $\varrho_{is} a^5 t^3$. Here ϱ_{is} is the interstellar gas density before the shock wave overtakes it; in the present case we estimate $\varrho_{is} \sim 2 \times 10^{-25}$ g cm^{-3}. The radiant losses from the shocked region become important only when the temperature has fallen below about 5×10^6 K, or $a \lesssim 2 \times 10^7$ cm s^{-1}. Let E_0 be the energy of the supernova explosion. We must then clearly wait until

$$a \sim \left(\frac{E_0}{\varrho_{is} t^3} \right)^{1/5} \lesssim 2 \times 10^7$$

or

$$t \gtrsim 6 \times 10^{-13} \left(\frac{E_0}{\varrho_{is}} \right)^{1/3} \tag{1}$$

before the kinetic energy given to the supernova remnant is converted back into radiation. In the present case E_0 must exceed 5×10^{51} erg. Therefore the lower limit on the time exceeds 1.8×10^{12} s or 6×10^5 yr. This is very much too late for the Gum nebula. The most likely source of the radiation is therefore the fireball which expands away from the star after the supernova explosion.

II. The Fireball

We now discuss the properties of the expanding fireball. The supernova explosion releases an amount of energy E_0, say 10^{52} or 10^{53} erg, into a mass M_*, say 10^{34} g. Some of this energy is carried to the surface by shock waves, but most of it is trapped in the gas. It cannot be radiated immediately, because the opacity of the gas is too high. Therefore a considerable pressure builds up in the gas, and the fireball accelerates outwards. The acceleration is largest immediately after the explosion.

It is readily shown that radiation pressure will dominate in the fireball. We have, for a fireball of initial radius R_0, that the temperature T is given by

$$E_0 = \frac{3}{2}\frac{M_*}{m} kT + \frac{4\pi}{3} R_0^3 a T^4, \tag{2}$$

where m is the mean molecular weight, about 10^{-24} g, and a is Stefan's constant. The two terms on the right hand side are, respectively, the gas kinetic and the radiant energy content. With $R_0 = 10^{12}$ cm, a typical value, they are in the ratio $1{:}10^3$ (when $E_0 = 10^{53}$ erg) or 1:250 (when $E_0 = 10^{52}$ erg). In either case the radiant energy content and, therefore, radiation pressure dominate. The ratio of radiation pressure to gas pressure remains unchanged as long as the subsequent expansion is adiabatic, that is as, long as radiative losses from the fireball have a negligible effect on its energy content.

We therefore model the radiative loss process of the fireball as follows. A mass M_* is given energy E_0 which is largely converted into kinetic energy. The mass therefore expands with speed $V \equiv (2E_0/M_*)^{1/2}$. At time t we suppose that it is spread over the surface of a sphere with radius $R = (2E_0/M_*)^{1/2}\, t$. The mass per unit area of the sphere is then

$$\sigma = \frac{M_*^2}{8\pi E_0} t^{-2}. \tag{3}$$

If electron scattering is the main cause of opacity in the material, then the optical depth of the shell is

$$\tau \equiv \kappa_e \sigma = \frac{\kappa_e M_*^2}{8\pi E_0} t^{-2}. \tag{4}$$

Here the electron opacity is denoted by κ_e ($= 0.34$ cm^2 g^{-1} in the usual cosmical mixture of elements). Let E be the total radiant energy enclosed by the shell at time t. The rate of change of E is then given by

$$\frac{dE}{dt} = -\frac{E}{R}\frac{dR}{dt} - \frac{4\pi R^2 c}{3\tau}\frac{E}{(4\pi/3)\, R^3}. \tag{5}$$

The first term on the right-hand side gives the adiabatic energy loss rate; the second gives the rate of flow of energy through the surface. On substituting for R and τ in terms of t we get that

$$\frac{dE}{E} = -\frac{dt}{t} - \frac{4\pi\sqrt{2}}{\kappa_e}\left(\frac{E_0}{M_*^3}\right)^{1/2} ct\, dt. \tag{6}$$

On integration

$$E = \frac{\text{const.}}{t} \exp\left\{-\frac{2\pi\sqrt{2}}{\kappa_e}\left(\frac{E_0}{M_*^3}\right)^{1/2} ct^2\right\}. \tag{7}$$

For $E_0 = 10^{52}$ or 10^{53} erg and $M_* = 10^{34}$ g we get a value of 10^{-13} or 3×10^{-13} for the coefficient of t^2 in the exponent. This means that the exponential term does not become small until after time $t = 3\times 10^6$ or 1.7×10^6 s, respectively. Until that time the energy content of the shell varies inversely as the time or the radius of the shell – in other words, its variation is purely adiabatic.

In order to fix the constant in equation (7) we set $E = E_0$ at time $t = t_0$, when the radius of the shell was $R = R_0$. This gives the relation

$$Vt_0 = \left(\frac{2E_0}{M_*}\right)^{1/2} t_0 = R_0,$$

so that t_0 is of the order of a few hundred seconds, much less than the time required for radiation to leak through the shell. At this time the exponential term is very nearly equal to unity, and we find that Equation (7) becomes

$$E = \frac{R_0}{t}\left(\frac{M_* E_0}{2}\right)^{1/2} \exp\left\{-\frac{2\pi\sqrt{2}}{\kappa_e}\left(\frac{E_0}{M_*^3}\right)^{1/2} ct^2\right\}. \tag{8}$$

From (6) and (8) we get that the total radiant energy lost through the shell during its expansion is

$$\begin{aligned} Q &= \int_0^\infty \frac{4\pi\sqrt{2}}{\kappa_e}\left(\frac{E_0}{M_*^3}\right)^{1/2} ct\, E\, \mathrm{d}t \\ &= \frac{4\pi}{\kappa_e}\left(\frac{E_0}{M_*}\right) R_0 c \int_0^\infty \exp\left\{-\frac{2\pi\sqrt{2}}{\kappa_e}\left(\frac{E_0}{M_*^3}\right)^{1/2} ct^2\right\} \mathrm{d}t \\ &= 2^{1/4}\pi\,\frac{R_0 c^{1/2}}{\kappa_e^{1/2}}\,\frac{E_0^{3/4}}{M_*^{1/4}}. \end{aligned} \tag{9}$$

We insert numerical values and find that $Q = 3.5\times 10^{48}$ or 2×10^{49} erg, for $E_0 = 10^{52}$ or 10^{53} erg, and with $M_* = 10^{34}$ g and $R_0 = 10^{12}$ cm. We see that the radiant energy output of the fireball is totally inadequate and cannot possibly result in the formation of a fossil H II region with the parameters that are customarily quoted for the Gum nebula.

In fact the discrepancy is even wider than these arguments suggest. Before radiant losses through the shell become serious, the energy density of the radiation within is, at time t,

$$\frac{E}{\frac{4\pi}{3}R^3} = \frac{3}{16\pi}\,\frac{M_*^2 R_0}{E_0 t^4}, \tag{10}$$

with the help of our formulae. The equivalent temperature of the radiation field within the shell is

$$T = \left(\frac{3}{16\pi}\right)^{1/4} \frac{M_*^{1/2} R_0^{1/4}}{E_0^{1/4} a^{1/4}}\, t^{-1}. \tag{11}$$

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This should also be the colour temperature of the radiation which leaks through the shell.

But in making the estimate for the energy required to form the Gum nebula it is assumed that most of the photons emitted by the fireball are harder than the Lyman limit. This means, effectively, that the colour temperature of the radiation must exceed 60000 K. With the usual values substituted in relation (11) we find that t must therefore be less than 3×10^5 or 1.7×10^5 s, for E_0 equal to 10^{52} or 10^{53} erg. We found earlier that the shell goes on radiating energy, at an almost constant rate, until time 3×10^6 or 1.7×10^6 s in these two cases. But we see now that the photons emitted are hard enough to cause appreciable ionization only during the earliest one-tenth of this time. Therefore, only 3.5×10^{47} or 2×10^{48} erg are available, in these two cases, in the form of ionizing radiation.

III. Discussion

Obviously there is a great discrepancy between our estimate of the amount of ionizing radiation available and the amount that is needed for the standard model of the Gum nebula. The shortfall is so large that it is hardly worthwhile to make such improvements as would replace our approximate calculation by a more exact one; the result would still be qualitatively the same. It seems that only one possibility remains. It is that the ionizing radiation was produced by the Vela pulsar itself. Let us see what this implies.

The likely mass of a pulsar is $M = 10^{33}$ g and its radius $R = 10^6$ cm. The moment of inertia of a neutron star is of the order of 2×10^{44} g cm^2 (Ruderman, 1972), and its limiting angular velocity (to avoid break-up under centrifugal force) is given by

$$\omega_c^2 = \frac{GM}{R^3} = 7 \times 10^7 \text{ s}^{-2}.$$

Hence the maximum kinetic energy of rotation is 7×10^{51} erg. This is also the maximum possible amount of energy available for radiation. Note that ω_c comes out to be about 100 times the present angular velocity of the pulsar (cf. Smith, 1972). In the simplest version of the oblique rotator theory the slow-down time for a pulsar varies like ω^{-2}, so that the bulk of the original energy of the neutron star would have been emitted as radiation over a period of a year. Our estimate for the opacity of the fireball shows that radiation can begin to escape freely once the supernova remnant is more than about 3×10^6 s (or 40 days) old. Therefore the bulk of the pulsar radiation would be available for the production of the fossil H II region.

This seems to be the only possibility. If it fails then there is no way in which one can associate the Gum nebula with the Vela supernova.

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DISCUSSION

Caswell: I would like to point out that the Gum nebula, the experimental prototype of a fossil Strömgren sphere, can be quite satisfactorily interpreted as a normal H II region. A detailed analysis reaching this conclusion was published recently (Beuermann, K. P.: 1973, *Astrophys. Space Sci.* **20**, 27.)

Mathewson: Several weeks ago M. Clearly and I made a detailed H I survey of the Gum nebula using the 18-m reflector at Parkes. High velocity H I was found around the nebula, and the correlation between the start of the H I emission and the optical limits of the nebula was so good that we concluded that the expanding shell of the Gum nebula was producing the high velocity H I. Radial velocities of 70 km s^{-1} were generally recorded although there were areas with velocities as high as 200 km s^{-1}. As there is no known mechanism that can produce such high gas velocities other than SNR we are forced to the conclusion that the Gum nebula is a SNR. The velocity of expansion must be at least several hundred km s^{-1}, which implies a diameter of about 40 pc, which places the Gum nebula at a distance of 60 pc from the Sun, much closer than the Vela 10 SNR which lies in the same direction. The ionization of the Gum nebula would then be due to collisional excitation. (However, optical spectra taken recently by us at five points in the Gum nebula are not characteristic of SNR nor do they show the same radial velocities as the H I, which casts doubt on the conclusion that this nebula is an SNR. It may be that the high-velocity H I belongs to a more distant spiral arm which by chance has an edge coincident with the closer edge of the Gum nebula.)

Baldwin: Can the theoretical models be calculated for expansion into an inhomogeneous medium, and can they account for the observed features, say in Cas A and the Cygnus Loop?

Kahn: The calculation would probably be feasible for small perturbations. But gross inhomogeneities might be very hard to deal with.

Burke: Stringent upper limits have now been placed on the flux from SN 1885 (S And). I wish to report a result obtained by J. H. Spencer with the NRAO interferometer. At 11 cm and 3 cm, a limit of 0.6 mJy was obtained. All the phenomenological theories predict several orders of magnitude more flux. The failure to observe radiation is consistent with the recent theory of Gull, and it appears that S And has not yet reached the 'turn on' stage.