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Representations of classical groups on the homology of their split buildings

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In this thesis we define the notion of the split building $\mathcal{S}(G)$ for the group G of K-rational points of a reductive K-group. This is essentially the poset of pairs of opposite parabolic subgroups of G. It is a covering of the Tits building and the main theorem states that it is Cohen-Macaulay if G is a finite classical group.

The proof proceeds by identifying $\mathcal{S}(G)$ with the split building $\mathcal{S}_V^{\perp}$ of a vector space V with a σ-sesquilinear reflexive form and then using algebraic topological methods to prove that $\mathcal{S}_V^{\perp}$ is Cohen-Macaulay.

These methods involve the investigation of certain subposets of the Tits building $\mathcal{T}_V$ which are shown to be Cohen-Macaulay en route to the main result. The inductive argument employed in the proof also shows that certain natural subposets of $\mathcal{S}(G)$ are Cohen-Macaulay.

It follows that G has a representation on the top homology of $\mathcal{S}(G)$ whose character may be computed using the Hopf trace formula. We carry this out in Chapter 7 and in the case of type A we give a formula for the dimension of $H_t(\mathcal{S}(G))$ (where $t = \dim \mathcal{S}(G)$) which is a consequence of the proof of the main theorem.

The representation of G on $H_t(\mathcal{S}(G))$ is further studied in the case of type A in Chapter 7. In particular we show that if $G = GL(V)$ then the multiplicity of the reflection representation in the representation on $H_t(\mathcal{S}(G))$ is two.

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