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The semi-analytical motion theory of the third order in planetary masses for the Sun – Jupiter – Saturn – Uranus – Neptune's system

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Abstract. The averaged four-planetary motion theory is constructed up to the third order in planetary masses. The equations of motion in averaged elements are numerically integrated for the Solar system's giant planets for different initial conditions. The comparison of obtained results with the direct numerical integration of Newtonian equations of motion shows an excellent agreement with them. It suggests that this motion theory is constructed correctly. So, we can use this theory to investigate the dynamical evolution of various extrasolar planetary systems with moderate orbital eccentricities and inclinations.

Keywords. celestial mechanics, methods: analytical, methods: numerical, planets and satellites: individual (Jupiter, Saturn, Uranus, Neptune)

The osculating Hamiltonian of the four-planetary problem is written in Jacobi coordinates. Then it is expanded into the Poisson series in the small parameter and orbital elements of the second Poincaré system. This algorithm is described in (Perminov, Kuznetsov (2015)).

The averaged Hamiltonian is constructed by the Hori–Deprit method. The implementation of the Hori–Deprit algorithm is considered in (Perminov, Kuznetsov (2016) and Perminov, Kuznetsov (2020)). The first-order terms of the averaged Hamiltonian are constructed up to 6^{th} degree in eccentric and oblique Poincaré elements, the second-order and third-order terms are constructed up to 4^{th} and 2^{nd} degrees correspondingly.

The equations of motion in averaged elements are integrated by the Everhart method of 15^{th} order (Everhart (1974)) with a time step of 1000 years to modeling of the orbital evolution of the Solar system's giant planets for different initial conditions (according to DE 432 ephemeris). Also, the same simulation is performed by Wisdom–Holman integrator with symplectic corrector of 11^{th} order (Rein, Tamayo (2015)) and a time step of 4 days. The Time interval of the integration is 100 Myr for all cases. The limits of the change of osculating orbital eccentricities (e_{min}, e_{max}) and inclinations (I_{min}, I_{max}) in barycentric frame giving by semi-analytical (SA) motion theory and Wisdom–Holman (WH) methods are presented in Table 1 for four initial dates of the integration (at the moment $00^{\mathrm{h}}00^{\mathrm{m}}00^{\mathrm{s}}$ UTC). The transition from averaged elements to osculating ones is carried out using the variables change functions of the first order. Table 2 presents the periods of the change of averaged orbital elements and the MEGNO indicator, which is computed in the process of the numerical simulation for each initial date.

The differences between periods of the change of orbital inclinations obtained by numerical methods and semi-analytical motion theory do not exceed 0.2% for all planets and all initial conditions. These discrepancies between periods of the change of orbital

Table 1. The range of osculating orbital eccentricities and inclinations.

Date	Theory	e_{min}	e_{max}	I_{min}, °	I_{max}, °	e_{min}	e_{max}	I_{min}, °	I_{max}, °
		Jupiter				Saturn			
31.01.2016	SA	0.025428	0.061603	1.093929	2.065336	0.012644	0.086018	0.557715	2.600120
	WH	0.025074	0.061959	1.093609	2.062945	0.009449	0.087273	0.561106	2.595098
29.02.2016	SA	0.025663	0.061743	1.093711	2.065397	0.012919	0.086132	0.559167	2.597615
	WH	0.025252	0.062026	1.095261	2.063808	0.009323	0.087498	0.561043	2.596079
31.01.2020	SA	0.026164	0.061377	1.089352	2.066233	0.012506	0.085095	0.559413	2.599554
	WH	0.025670	0.061914	1.094836	2.065492	0.009516	0.087377	0.560680	2.595056
29.02.2020	SA	0.025727	0.061232	1.094927	2.065816	0.012196	0.085157	0.563841	2.595928
	WH	0.025180	0.061789	1.094095	2.062955	0.009289	0.087229	0.565407	2.594549
		Uranus				Neptune			
31.01.2016	SA	0.006304	0.071602	0.391638	2.766937	0.003247	0.015126	0.775906	2.376785
	WH	0.003386	0.073773	0.378761	2.777150	0.002383	0.016046	0.773544	2.373315
29.02.2016	SA	0.006084	0.071265	0.425230	2.740155	0.003322	0.015133	0.780070	2.374410
	WH	0.003572	0.073201	0.427249	2.734788	0.002635	0.015978	0.783357	2.373475
31.01.2020	SA	0.005017	0.070079	0.446119	2.718907	0.003483	0.014972	0.769256	2.390563
	WH	0.002174	0.071974	0.444978	2.728521	0.002727	0.015824	0.777828	2.379745
29.02.2020	SA	0.005501	0.070569	0.414775	2.735945	0.003487	0.014914	0.776324	2.380880
	WH	0.002735	0.072512	0.430827	2.730216	0.002553	0.015786	0.779464	2.379941

Table 2. The periods of osculating orbital elements and the MEGNO indicator.

		Jupiter	Saturn	Uranus	Neptune	Jupiter	Saturn	Uranus	Neptune	
Date	Theory	Periods of orbital eccentricities, years				Periods of orbital inclinations, years				MEGNO
31.01.2016	SA	54 675	54 675	1 136 375	537 640, 364 967	49 213	49 213	432 905	1 886 811	2.28
	WH	54 735	54 735	1 136 375	534 765, 363 640	49 141	49 141	432 905	1 886 811	
29.02.2016	SA	53 764	53 764	1 136 375	537 640, 364 967	49 262	49 262	432 905	1 886 811	6.69
	WH	54 055	54 055	1 123 607	534 765, 362 323	49 189	49 189	432 905	1 886 811	
31.01.2020	SA	51 949	51 949	1 111 122	540 546, 363 640	49 432	49 432	432 905	1 886 811	103.63
	WH	51 841	51 841	1 098 912	537 640, 362 323	49 335	49 335	432 905	1 886 811	
29.02.2020	SA	54 113	54 113	1 123 607	540 546, 364 967	49 335	49 335	432 905	1 886 811	1.99
	WH	53 677	53 677	1 111 122	537 640, 362 323	49 237	49 237	432 905	1 886 811	

eccentricities of Jupiter and Saturn are in the range $0.1\% - 0.8\%$. These differences do not exceed 1.1% and 0.7% correspondingly for Uranus and Neptune. The discrepancies for minimal and maximum values of the orbital eccentricities and inclinations do not exceed a few percent, except minimal orbital eccentricities of Saturn, Uranus, and Neptune.

The constructed semi-analytical motion theory can be used to study the orbital evolution and stability of extrasolar planetary systems with moderate orbital eccentricities and inclinations. The orbital elements of extrasolar systems are known from observations with highly uncertain, and some elements are not determined due to the specificity of the observation methods. We can vary unknown and known with errors orbital elements within allowable limits to determine the set of various initial conditions for modeling the orbital evolution. The limits of the change of orbital elements can be determined depending on the specific initial conditions. The assumption about the stability of observed planetary systems allows us to exclude the initial conditions leading to extreme values of the orbital eccentricities and inclinations that identify those under which these elements conserve small or moderate values over the whole modeling interval. Thus, it is possible to narrow the allowable range of unknown orbital elements and determine their most probable values in terms of stability.

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