Some properties of the line of striction of a ruled surface

By RAM BEHARI, M.A., Ph.D.

1. It is known that
(i) the line of striction of a ruled surface is the locus of points at which the geodesic curvatures of the orthogonal trajectories of the generators vanish[1],
(ii) if at each point of a curve C on a surface, a tangent to the surface is drawn, and these tangents generate a ruled surface of which C is the line of striction, then, if each tangent is turned through a constant angle α about its point of contact in the tangent plane, the new set of tangents also form a ruled surface with C as a line of striction[2].

The object of this paper is to obtain the result (i) from purely geometrical considerations and to give independent analytical proofs of the results (i) and (ii) and also to give an easy method for constructing a ruled surface whose line of striction is any given curve.

2. Let the equations of the ruled surface be $x = p + lu$, $y = q + mu$, $z = r + nu$, where p, q, r are the coordinates of a point P on the directrix expressed in terms of the arc v of the directrix measured from a fixed point on it; l, m, n also expressed in terms of v are the direction cosines of the generator through P, and u is the distance measured from P along the generator through P to any point (x, y, z) on it.

If an orthogonal trajectory of the generators be taken as the directrix curve, then $\Sigma\, lp' = 0$. Differentiating this we get $\Sigma\, l'p' = -\Sigma\, lp''$. Where the directrix curve meets the line of striction, we have $\Sigma\, l'p' = 0$, and therefore $\Sigma\, lp'' = 0$. But $\Sigma\, lp''$ is equal to the projection of the curvature vector of the curve on the tangent plane and is thus $1/\gamma$ where $1/\gamma$ is the geodesic curvature of the orthogonal trajectory. Therefore $1/\gamma$ is zero, which proves the result (i).

This result easily follows from geometrical considerations. For if G is the centre of geodesic curvature at P of an orthogonal trajectory,

---

[1] See Forsyth, "Differential Geometry," p. 386.

[2] See Richmond, "A note upon some properties of the curve of striction," *Proceedings of the Edinburgh Math. Soc.*, 1923, p. 95, for a method of obtaining this result from geometrical considerations.

6

then G lies on the generator through P and the tangent planes at P and G are perpendicular. Hence if u and u' be the distances of P and G from the central point, $1/\gamma = PG = u - u' = 0$ where the orthogonal trajectory meets the line of striction.

3. Let l, m, n be the direction cosines of the tangent to a curve C on a surface at a point $P(x, y, z)$ on it; L, M, N those of the normal to C at P that touches the surface, and λ, μ, ν those of the tangent to the surface at P that generates the ruled surface.

The condition that the tangent in the direction λ, μ, ν making an angle θ with l, m, n should generate a ruled surface with C as line of striction is

$$0 = \Sigma x'\lambda' = \Sigma l \,.\, \frac{d}{ds}(l\cos\theta + L\sin\theta) = \sin\theta \,.\, (-\theta' + \Sigma lL').$$

If this condition is satisfied by $\theta = \theta(s) \neq 0$, then it is also satisfied by $\theta = \theta(s) + a$ where a is a constant, which proves the theorem.

Thus there is an infinite number of ruled surfaces having a common line of striction and having the same tangent plane at each point of the line.

In particular, if the constant angle a is a right angle, we arrive at the ruled surface known as the "Strictionsband[1]".

From the above considerations, it easily follows that the Strictionsband of the Strictionsband of a ruled surface is the same ruled surface back again[2].

4. A ruled surface whose line of striction is any given curve C can be easily constructed as follows:—

Draw any developable through the curve C. Develop it into a plane so that the curve C becomes a plane curve C', say. At each point of C' draw a line in an arbitrary fixed direction. When the plane is deformed back into the original developable, these lines will generate a ruled surface with the required property.

St Stephen's College,
University of Delhi.

---

[1] See Study, *Geometrie der dynamen*, p. 93; also Zindler, *Liniengeometrie*, Vol. II, p. 14.

[2] Study, *loc. cit.*, p. 303; Zindler, *loc. cit.*, p. 14.

https://doi.org/10.1017/S0950184300002585 Published online by Cambridge University Press