But from the identity (1),

$$\psi_x'(x_u, y_u, u) \cdot \frac{dx_u}{du} + \psi_y'(x_u, y_u, u)\frac{dy_u}{du} + \psi_u'(x_u, y_u, u) \equiv 0;$$

hence (1) and (2) are equivalent to

$$\psi(x_u, y_u, u) \equiv 0 \text{ and } \psi_u'(x_u, y_u, u) \equiv 0.$$

Any envelope-locus is, therefore, represented in the relation between x and y which is the eliminant of u from the equations

$$\psi(x, y, u) = 0, \ \psi_u'(x, y, u) = 0.$$

The full locus of this eliminant-equation may be geometrically described as

(i) the locus of ultimate points of intersection of "consecutive" curves of the family; and therefore as

(ii) the locus of points in the xy-plane at which the equation $\psi(x, y, u) = 0$ has two roots in u equal;

it obviously includes, on the description (i), the envelopes and multiple-point loci; and the description (ii) of this locus leads to the conclusion that the envelopes are also included in the "p-discriminant" of the differential equation that represents the family.

On the teaching to beginners of such transformations as $-(-a) = +a$.

By D. C. M'Intosh, M.A.