New Perspectives on Solar Prominences, IAU Colloquium 167
ASP Conference Series, Vol. 150, 1998
David Webb, David Rust, and Brigitte Schmieder (eds.)

Coronal Plasmoid Dynamics

C. Delannée and S. Koutchmy

Institut d'Astrophysique de Paris, CNRS, 98 bis, bd Arago, 75014 Paris, France

A. Zhukov and I. Veselovsky

Institute of Nuclear Physics, Moscow State University, Moscow 119899, Russia

Abstract. A possible acceleration mechanism by the magnetic force is suggested for a plasmoid, modeled as a spherical body with a magnetic dipole situated in its center of mass. The governing equations of the motion of a dipole in an inhomogeneous magnetic field are solved analytically and numerically. Both methods show the possibility for the plasmoid to be accelerated in the direction of the external field gradient.

1. Introduction

Magnetically isolated clouds of ionized gas, or coronal plasmoids, have been theoretically described by many authors, for example, by Priest (1982), Cargill and Pneuman (1986), Pneuman (1983) and Mullan (1990). A possibility to test these theories experimentally appeared after the observation of the coronal plasmoid by the Canada–France–Hawaii Telescope (CFHT) during the July 11, 1991 total solar eclipse (Vial et al. 1992, Koutchmy et al. 1994). The dynamics of the plasmoid were described in Delannée and Koutchmy (1996), and it was reported in this paper that the plasmoid seen during 230 s at 90,000 km above the solar limb moved at a high velocity (about 75 km s^{-1}). Its dimension was around 1 Mm; it was denser (3×10^9 cm^{-3}) and colder (2×10^4 K) than the surrounding corona (3×10^8 cm^{-3}, 2×10^6 K). To support the external gas pressure the plasmoid had to be more magnetized than the corona; estimates give $B_{plasmoid} = 2$ G and $B_{corona} = 1$ G.

In the present paper we discuss a possible acceleration mechanism – the acceleration due to the magnetic force, which is probably dominant in the plasmoid dynamics (Delannée et al. 1998).

2. Outline of the Problem

The plasmoid is assumed to be solid, i.e., incompressible. Inside the magnetic field can be produced by a ring current, which is equivalent to a magnetic dipole if the radius of the ring current is small enough versus the radius of the plasmoid. Outside the magnetic field is produced at the center of the sun or in an active region; both kinds of magnetic field can be modeled by a magnetic dipole

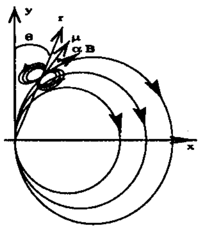

Figure 1. Diagram of the problem.

but of different sizes and strengths. Additionally, we consider the dipole field frozen inside the plasma contained in the plasmoid. Finally, we fully neglect the influence of the external plasma. So we solve the problem of the motion of a magnetic dipole inside an inhomogeneous magnetic field, see Figure 1. The equation of motion is:

$$m\dot{\vec{v}} = \vec{\nabla}(\vec{\mu}\cdot\vec{B}), \tag{1}$$

where m is the mass of the plasmoid, v its velocity, μ the magnetic dipole moment inside the plasmoid and $\vec{B}$ the magnetic field outside the plasmoid. The magnetic dipole moment can oscillate around the external magnetic field vector. The equation of conservation of energy is:

$$\frac{mv_0^2}{2} + \frac{1}{2}I_{ik}\theta_{i0}\theta_{k0} - \mu B\cos\alpha_0 = \frac{mv_1^2}{2} + \frac{1}{2}I_{ik}\theta_{i1}\theta_{k1} - \mu B\cos\alpha_1, \tag{2}$$

where I is the inertia momentum of the dipole, θ the oscillation angle of the dipole in the spherical coordinates of the reference frame of the external magnetic field, α is the angle between the external magnetic field vector and the magnetic dipole moment vector, the subscripts, 0 and 1, correspond to the values of the quantities taken at two different times, t_1 and t_2, and the subscripts, i and k, correspond to the two angular coordinates of the spherical coordinates of the problem.

3. Equilibrium, Oscillation and Rotation

The equilibrium of the dipole versus its oscillation in the external magnetic field is reached when its potential energy of the oscillations ($E_p = -\vec{\mu}\cdot\vec{B}$) is maximal or minimal. There are two configurations, see Figure 2. The stable configuration is such that the force is directed toward the stronger magnetic field.

When the dipole moves in an inhomogeneous magnetic field, it feels the change in the magnetic field direction and starts to oscillate around the magnetic field direction. We solve the equations of conservation of energy in the reference frame of the center of the dipole. The problem is axisymmetric. The change in direction of the magnetic dipole moment due to the change of the magnetic field

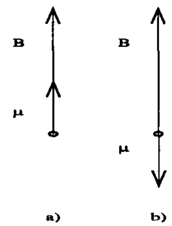

Figure 2. Equilibrium configurations of the magnetic dipole in the external magnetic field. a) is the stable equilibrium and b) is the unstable equilibrium.

direction is considered very small. So, the two components of the oscillation are equal to the angle α. The solution is:

$$t = \pm\frac{2}{\dot{\alpha}_0}F(|\alpha/2|\,,k) \tag{3}$$

where $k^2 = 4\Omega^2/\dot{\alpha}_0^2$; $\Omega = \sqrt{\mu B/I}$ is the characteristic proper frequency of the plasmoid; $F(|\alpha/2|\,,k)$ is the incomplete Legendre elliptic integral of the first kind. The two different regimes of the oscillations are described in this solution: if $|\ k\ | > 1$, the function $F(|\alpha/2|\,,k)$ is complex and the motion is a rotation, and if $|\ k\ | < 1$, the function $F(|\alpha/2|\,,k)$ is real and the motion is an oscillation

4. Motion

We develop the gradient of the potential energy. We see in the solution of the problem given in the previous section that $\alpha = \alpha(t)$. This leads to: $\vec{\nabla}\cos\alpha = 0$. Projecting the equation of motion on the axes parallel and perpendicular to the magnetic field gradient direction, we obtain that the velocity perpendicular to the external magnetic field gradient is constant during the motion and only the velocity parallel to it can be accelerated with the equation:

$$m\dot{v} = \mu\cos\alpha\nabla B \tag{4}$$

If $\cos\alpha < 0$ (i.e., $\pi/2 < |\alpha| \leq \pi$), the velocity of the plasmoid increases.

In the case of oscillations, we integrate the equation of motion over a period of oscillation, T. The solution is:

$$v(T) = v_0 + \frac{2\sqrt{2}\mu\nabla B}{m\Omega}\psi(\alpha_{\max}), \tag{5}$$

with

$$\psi(\alpha_{\max}) = \sqrt{2}\left\{2E(\sqrt{\frac{1-\cos\alpha_{\max}}{2}}) - K(\sqrt{\frac{1-\cos\alpha_{\max}}{2}})\right\}, \tag{6}$$

where E and K are respectively the complete Legendre elliptic integrals of the second and first kind.

$\alpha_{\max}$ is the maximal angle reached during an oscillation. Using the equation of conservation of the energy, we find $\cos\alpha_{\max} = 1 - \frac{\dot{\alpha}_0^2}{2\Omega^2}$. We define $\alpha^*_{\max}$, the critical value of the maximal angle of the oscillations, such as $\psi(\alpha^*_{\max}) = 0$. In the case of the plasmoid seen in during the 1991 total solar eclipse, $\alpha^*_{\max} \approx 131°$ (or $\alpha^*_{\max} \approx 2.29$). If $\alpha_{\max} > \alpha^*_{\max}$, then $\psi(\alpha_{\max})$ is negative, so the motion is accelerated. If $\alpha_{\max} < \alpha^*_{\max}$, then $\psi(\alpha_{\max})$ is positive, so the motion is decelerated.

In case of fast rotation, the function of the angle versus time can be written as: $\alpha = \dot{\alpha}_0 t$. We average the equation of motion over a period of rotation ($T = 2\pi/|\dot{\alpha}_0|$). The solution is:

$$v(T) = v_0 - \frac{\pi\mu}{|\dot{\alpha}_0|\, m}\left(\frac{\Omega}{\dot{\alpha}_0}\right)^2 \nabla B \qquad (7)$$

5. Numerical Simulation

We solved the equations of the motion in the case of an external magnetic field produced by a magnetic dipole situated just under the photosphere, i.e., at 90 Mm. The equations useful to describe the motion are:

$$\begin{cases} m\left(\ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\dot{\varphi}^2\right) = -\frac{3\mu M\cos\alpha}{r^4}\sqrt{1+3\cos^2\theta} \\ m\left(2\dot{r}\dot{\theta} + r\ddot{\theta} - r\sin\theta\cos\theta\dot{\varphi}^2\right) = -\frac{3\mu M\cos\alpha}{r^4}\frac{\cos\theta\sin\theta}{\sqrt{1+3\cos^2\theta}} \\ m\left(2\dot{r}\sin\theta\dot{\varphi} + 2r\cos\theta\dot{\theta}\dot{\varphi} + r\sin\theta\ddot{\varphi}\right) = 0 \\ I\ddot{\alpha} = \frac{\mu M\sin\alpha}{r^3}\sqrt{1+3\cos^2\theta} \end{cases}$$

where $M = Br_0^3/\sqrt{1+3\cos^2\theta_0} \approx 3.1 \times 10^{32}$ G cm^3 is the value of the external magnetic moment. We solve this system using the forth order Runge-Kutta method, including the correction due to the changes of the magnetic field direction during the plasmoid center of mass motion. We computed the equations using physical quantities close to the ones of the observed plasmoid (Delannée et al. 1998).

The results give the two regimes of oscillations and rotation. The dipole can be accelerated, in this case its motion is such that θ increases. In the case of a decelerated motion, θ decreases. In all initial cases, if there is a speed high enough, the oscillation moves to a rotation and the motion is accelerated.

6. Conclusion

This simple model of a dipole inside a plasmoid shows that the plasmoid can be accelerated out of the corona. The main remarks are about the distribution of the current inside the plasmoid. This current is more complicated and some current surface can exist which modifies this internal force. Furthermore, we have to think about the way that the current ring can support the oscillations or the rotations of their axis of symmetry. Finally, we have to detail the differences between the two theories and compare the two forces computed in these two theories.

References

Cargill P.J. and Pneuman G.W. 1986, ApJ, 307, 820

Delannée C. and Koutchmy S. 1996, C. R. Acad. Sci. Paris, 322, Série II b, 79

Delannée C., Koutchmy S., Veselovsky I.S. and Zhukov A.N., 1998, A&A, 329, 1111

Koutchmy S. et al., 1994, Solar Dynamic Phenomena and Solar Wind Consequences, ESA SP-373, p. 139

Mullan D.J. 1990, A&A, 232, 520

Pneuman G.W. 1983, ApJ, 265, 408

Priest E.R. 1982, Solar Magnetohydrodynamics, Reidel, Dordrecht, Holland

Vial J.-C., Koutchmy S. and CFH Team 1992, in ESA Workshop on Solar Physics and Astrophysics at Interferometric Resolution, Paris, p. 87