The

Mathematical Gazette

A JOURNAL OF THE MATHEMATICAL ASSOCIATION

Vol. 59 June 1975 No. 408

The unacceptable face of modern mathematics

There are plenty of historical precedents for reforms which have had results different from those intended by their protagonists. The changes in mathematics teaching over the past 20 years had their roots in several different but complementary causes of dissatisfaction, and it is hardly surprising if there is currently some confusion amongst teachers (and examiners) about 'modern mathematics'. The years pass, however, and there is an increasing danger that the trivial, the irrelevant and the plain wrong will become permanent features in our mathematics syllabuses—at least until the reformers of the next millenium try once again to restore sanity and balance.

Of course, the use of the word "syllabus" begs the whole question. It was inevitable that the secondary 'projects' of the 1960s should have been developed in the context of the GCE examinations. (CSE, it must be remembered, had not even started then—and when it did, many of its assumptions could be traced back to O level traditions.) Even the new freedom which found its way into primary teaching about the same time needed to justify itself against the criteria for selection at 11+ then obtaining. But perhaps the most disturbing lesson to emerge from this experience is the ease with which codification of a course—however stimulating for both teachers and pupils—into a list of 'topics for examination' can distort the teaching of the subject.

It is widely agreed, for example, that the interpretation of various parts of mathematics—whether probability, proportion or point symmetry—in terms of the basic concepts of 'set' and 'function' can help to give a more balanced, unified view of mathematics. But there is no more justification for asking, in what is for many candidates their last ever examination in mathematics, questions such as "if $R = \{1, 2, 3\}$, $S = \{4, 5, 6)$, $\mathscr{E} = \{1, 2, 3, 4, 5, 6\}$, what is $R' \cap S$?" than there was in times past for asking them to "factorise $a^2 - b^2 - 4a + 4b$". (Indeed, there is even less, since the latter question might at least be of some use to the minority who would choose to go on with mathematics.) This is bad enough in the examination

https://doi.org/10.2307/3616638 Published online by Cambridge University Press

itself, but far worse is the thought that, in schools throughout the country, fifth year pupils the next year, and the year after that, and the next, and the next, ... will have their time wasted repeating the same trivial exercises in 'practice papers'. Can we not use the idea of a set in teaching mathematics without elevating 'sets' to the status of an 'examination topic'?

Let us take another example. Experience of writing numbers to various bases, supported by activities using appropriate structured apparatus, has been found helpful in junior schools for developing understanding of place value notation. At a later stage, occasional work in unconventional number bases offers scope for some worthwhile mathematical explorations, and some children will enjoy their applications to games or to computers. It can sometimes even make a good starting-point for an examination question. But it is a far cry from this to the following, taken from a recent 16+ examination: "Give two bases less than eleven for which the answers to the addition sum $103 + 24 + 1011$ are both 1138. Find, in base ten, *the actual difference* between these two last answers." (Our italics.) What possible mathematical significance can be claimed for this?

It could of course be argued that this is a criticism of external examinations, not of modern mathematics. After all, it is easy enough to make mock of examination questions, and there are plenty of candidates (ancient *and* modern) for such treatment. But here we are concerned with a deeper problem—a serious failure of communication between the originators of 'new mathematics' and many practising teachers and examiners. For what was being attempted ten years ago was not just a refurbishing of the content of school mathematics (e.g. the replacement of "harder factors" by "arithmetic to different number bases") but a fundamental re-appraisal of the aims of mathematics teaching, in the context of modern insights into both the nature of mathematics and our understanding of children's learning processes. So long as "skill in factorising" was an objective of O level mathematics, it made good sense for children to sit down to a test in which they were given a list of algebraic expressions to write in different forms; and there are topics introduced under the heading of 'modern mathematics' which may also fit into this category. But "understanding place value and the rôle of the base in numerical expressions" is not one of these; it is a more subtle objective, and one which is in danger of being totally devalued by insensitive examining. And there are a number of similar examples to be found in modern mathematics teaching.

Then there is the pedantry which some teachers have mistaken for the essence of 'modern mathematics'. It was admittedly helpful, when reviewing the mathematics curriculum, to remark that pupils' errors and misunderstandings sometimes stem from our own lax use of technical terms, and to try to do something about it. "Two minuses make a plus" is ambiguous and unhelpful, and it is just as easy and more correct to talk about "the area inside a circle" rather than "the area of a circle". But every practising mathematician gets along very well most of the time by referring to "the

https://doi.org/10.2307/3616638 Published online by Cambridge University Press

line $2x + 3y = 7$" and "the function $f(x) = x^2 - 3x + 1$"; creative mathematics would soon grind to a halt if we insisted on circumlocutions such as "the line $\{(x,y):2x + 3y = 7\}$ and "the function $f:\mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2 - 3x + 1 \ \forall \ x \in \mathbb{R}$". And worse, one has seen this notation mangled into meaningless forms such as "$\{2x + 3y = 7\}$" and "$f \rightarrow x^2 - 3x + 1$" by people who were convinced that, by writing their mathematics in this way, they were doing 'modern mathematics'. "The letter killeth ..."

It is to excesses of this kind that we must attribute the beginnings of the mistrust of modern mathematics which has become prevalent amongst science teachers and in industry and further education at the present time. The unfortunate result has been for all the shortcomings, real and imagined, of mathematical competence of our pupils to be laid at the door of 'modern mathematics'. There is little hard evidence for this view; a working party following up criticisms which had been made of 'modern' courses in Southampton schools, for example, reported that experiments conducted by a local employer and college of technology could detect no significant difference over a range of basic computational skills between school leavers who had followed respectively 'modern' and 'traditional' courses at school. But on this point, one must admit that the more sanguine hopes entertained in the early days of reform have proved illusory in the testing-ground of the classroom. One would not find a conference of experienced teachers writing in 1975, as they did in 1961: "Greater emphasis should be placed on an appreciation of the structure of algebra—stressing the commutative, distributive and other similar properties—rather than on the acquisition of techniques. A greater understanding of the structure will *inevitably* enable this acquisition of techniques to be made." The recently published pamphlet from the SMP on *Manipulative skills in school mathematics* strikes a better balance—and is to be welcomed, along with the conference held at Nottingham last summer and reported by A. R. Tammadge in the January issue of *Mathematics in School*, as a discussion-starter with the long-term aim of defining a position acceptable on all sides.

This is not to suggest that the reforms of the 1960s have failed; far from it, most of our current problems stem from their extraordinary success. Many teachers who initially eyed the new programmes of SMP, MEI and MME with suspicion are now finding a place for a more 'open' approach in their own teaching. This year almost 80% of O level candidates from the schools served by one GCE Board are being examined on one of the modern syllabuses. But it is time for us all to stand back and review the situation. The Association's report *Mathematics: 11 to 16* has set the content of school mathematics in a framework which finds no place for a dichotomy between 'modern' and 'traditional'. It seems increasingly inappropriate for CSE candidates to be offered a choice between options entitled "General Mathematics" and "Modern Mathematics". Could we not consign these names once for all to the educational junk-heap—with their excrescences, but not of course the best of what each stands for? There are many teachers

https://doi.org/10.2307/3616638 Published online by Cambridge University Press

in both camps who recognise that they now have more pressing problems on their hands.

Experiments in the past two years associated with proposals for a common examination system at 16+ have already brought GCE and CSE Boards together to plan new syllabuses covering a wider ability range; and whatever the ultimate fate of these proposals, there are clearly prospects in the immediate future for continued collaboration of this kind. May we hope that the syllabuses that emerge will not just be the mixture as before, but that they will reflect a fresh appraisal of the aims of mathematics teaching and the needs of our pupils in the last quarter of the 20th century? And may we hope that, this time, a way may be found of laying down syllabuses which opens the gate to examinations (or other forms of assessment) in the spirit in which they were conceived? This is not a plea for a common national syllabus, or for the stifling of experiment—quite the reverse—but for us all to use our cherished flexibility of curriculum more responsibly.

D.A.Q.

Inductio ad absurdum?

D. R. WOODALL

Suppose that P_n is some proposition about the integer n, which we want to prove for all $n \geqslant n_0$ (usually $n_0 = 0$ or 1). The form of inductive argument most commonly taught in schools is the following:

> A. *Simple induction.* If P_{n_0} is true, and $P_n \Rightarrow P_{n+1}$ for each $n \geqslant n_0$, then P_n is true for all $n \geqslant n_0$.

This, indeed, is the only form of mathematical induction that many students ever meet. Many descriptions of induction, including whole books and films devoted to the subject, concern themselves almost exclusively with this simple case. The only other form of induction at all commonly taught is:

> B. *Strong induction.* If P_{n_0} is true, and $(P_{n_0} \& P_{n_0+1} \& \ldots \& P_n) \Rightarrow P_{n+1}$ for each $n \geqslant n_0$, then P_n is true for all $n \geqslant n_0$.

This form of induction is easily reduced to form A by the substitution $Q_n = (P_{n_0} \& P_{n_0+1} \& \ldots \& P_n)$ for each n. The proof of A and B is essentially:

> C. *Method of descent.* If P_n is not true, for some $n \geqslant n_0$, choose the smallest $n \geqslant n_0$ for which it is not true. If we can deduce the existence of an m, $n_0 \leqslant m < n$, for which P_m is not true, or can obtain a contradiction in some other way, then this contradiction will establish the fact that P_n is true for all $n \geqslant n_0$.