A MODEL FOR A TWO COMPONENT GALAXY.

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As most of today approaches to the problem of galactic structure are based on the consideration of the two main components of a galaxy, halo and disk, it was thought that even simple models of such kinds of double structures, based only on the application of general theorems, could be of some use, at least for simplified outlooks concerning galactic evolution. With this aim in mind, we have undertaken the treatment of bodies constituted by two different axially deformed polytropic structures, homocentrical and coaxial, with arbitrary values for the two masses M_I and M_{II}, the two radii a_I and a_{II} of the undeformed objects ($a_I < a_{II}$) and polytropic indices n_I and n_{II}, interacting with each other only gravitationally. The flattening of the spheroids is assumed to be due to solid body rotation and tidal interaction, and the treatment follows closely the method used for such single structures by Chandrasekhar and Lebovitz.

The main lines are then the following: the equations of equilibrium (in polar coordinates r, $\mu=\cos\theta$) to be satisfied by the two spheroids are:

$$\frac{\partial p_i}{\partial r} = \rho_i \frac{\partial V}{\partial r} + \rho_i \omega_i^2 r(1-\mu^2) \qquad i=I,II \qquad (1)$$
$$\frac{\partial p_i}{\partial \mu} = \rho_i \frac{\partial V}{\partial \mu} + \rho_i \omega_i^2 r^2\mu$$

with ρ_i=density, p_i=pressure and ω_i=angular velocity of the ith spheroid, while the gravitational potential obeys Poisson equation:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \mu}\left[(1-\mu^2)\frac{\partial V}{\partial \mu}\right] = -4\pi G(\rho_I+\rho_{II}). \qquad (2)$$

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Assuming the usual polytropic definitions:

$$\rho_i=\lambda_i\ \Theta_i^{n_i}(r,\mu) \qquad p_i=\lambda_i^{1+1/n_i}K_i\Theta_i^{n_i+1}(r,\mu) \qquad i=I,II \tag{3}$$

one arrives in the usual way to the main Emden type equations:

$$\frac{1}{\xi_i^2}\frac{\partial}{\partial\xi_i}\left(\xi_i^2\frac{\partial\Theta_i}{\partial\xi_i}\right)+\frac{1}{\xi_i^2}\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial\Theta_i}{\partial\mu}\right]=-\left[\frac{\lambda_I}{\lambda}\Theta_I^{n_I}+\frac{\lambda_{II}}{\lambda}\Theta_{II}^{n_{II}}\right]+v_i \qquad i=I,II \tag{4}$$

with

$$\lambda=\lambda_I+\lambda_{II};\quad \alpha_i=\left[\frac{\lambda_i^{1/n_i}}{\lambda}\frac{K_i(n_i+1)}{4\pi G}\right]^{\frac{1}{2}};\quad v_i=\frac{\omega_i^2}{2\pi G\lambda};\quad r=\alpha_i\xi_i;\quad i=I,II \tag{5}$$

One can easily deduce that Θ_I and Θ_{II} are connected by a linear relation, which allows to eliminate Θ_I in the second of the equations (4), thus leading to a differential equation with the single unknown function Θ_{II}: $(\xi=\xi_{II})$

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\left(\xi^2\frac{\partial\Theta_{II}}{\partial\xi}\right)+\frac{1}{\xi^2}\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial\Theta_{II}}{\partial\mu}\right]=-\left[(1-\delta)(\gamma^2\Theta_{II}-\gamma^2+1)^{n_I}+\right.$$
$$\left.+\delta\Theta_{II}^{n_{II}}\right]-\frac{1}{4}n_I(1-\delta)\gamma^2(v_I-v_{II})\xi^2(1-\mu^2)(\gamma^2\Theta_{II}-\gamma^2+1)^{n_I-1}+v_{II} \tag{6}$$

with $\gamma=\dfrac{\alpha_{II}}{\alpha_I}$ $\quad \delta=\dfrac{\lambda_{II}}{\lambda}$.

Following closely the technique used by Chandrasekhar and Lebovitz, we assume for the solution of (6):

$$\Theta_{II}(\xi,\mu)=\Theta_{oII}(\xi)+v_{II}\left[\Psi_{oII}(\xi)+\sum_1^\infty A_\ell^i\Psi_{\ell II}(\xi)P_\ell(\mu)\right]+$$
$$+\ w\left[X_{oII}(\xi)+\sum_1^\infty B_\ell^i X_{\ell II}(\xi)P_\ell(\mu)\right] \tag{7}$$

where $w = v_I-v_{II}$.
The border of the inner spheroid I is then described by:

$$\Xi^i(\mu) = \xi_r+v_{II}\sum_0^\infty q_\ell^i P_\ell(\mu)+w\sum_0^\infty p_\ell^i P_\ell(\mu) \quad ; \tag{8}$$

on this border we must have: $\Theta_I(\Xi^i)\equiv 0$ $\quad(\xi=\xi_r$ for $r=a_I)$. (9)

Outside this border, the main equation (6) reduces to the deformed Chandrasekhar equation for a single rotating spheroid:

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\left(\xi^2\frac{\partial\Theta_{II}^e}{\partial\xi}\right)+\frac{1}{\xi^2}\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial\Theta_{II}^e}{\partial\mu}\right] = -\Theta_{II}^e+v_{II}^e \ . \tag{10}$$

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For its solution we set:

$$\Theta^{e}_{II}(\xi,\mu)=\frac{1}{\delta^{1/n_{II}}}\Big[\theta(\xi)+v_{II}\Big(\psi_o(\xi)\ \sum_1^{\infty} A^{e}_{\ell}\psi_{\ell}(\xi)P_{\ell}(\mu)\Big)+$$
$$+\ w\ \Big(\chi_o(\xi)+\sum_1^{\infty} B^{e}_{\ell}\chi_{\ell}(\xi)P_{\ell}(\mu)\Big)\Big]\ . \qquad (11)$$

The outer border of spheroid II will be similarly expressed by:

$$\Xi^{e}(\mu)\ =\ \xi_1+v_{II}\ \sum_0^{\infty} q^{e}_{\ell}P_{\ell}(\mu)+w\ \sum_0^{\infty} p^{e}_{\ell}P_{\ell}(\mu) \qquad (12)$$

and again, on this border we must impose:

$$\Theta^{e}_{II}(\Xi^{e})\equiv 0\quad .\quad (\xi=\xi_1\ \text{for}\ r=a_{II}) \qquad (13)$$

By substituting (7) and (11) respectively into (6) and (10), by neglecting all powers of v_{II} and w above the first, and equating separately to zero the coefficients of v_{II}, w, and the different $P_{\ell}(\mu)$, one obtains a series of differential equations which allow to determine all the different functions present in (7) and (11). Further, by applying conditions (9) and (13), by expressing on the inner surface boundary (8) the continuity of the density, and on both the inner and the outer boundary (12) the continuity of the potential and of its first radial derivative, one obtains enough conditions in order to determine all the coefficients present in (7), (8), (11) and (12). The problem is thus fully solved from the purely formal point of view.

Masses and angular momenta of both polytropes are obviously given by the formulae:

$$M_i\ =\ 2\pi\alpha_i^3\ \lambda_i\int_0^{\xi_1}\xi^2 d\xi\int_{-1}^{+1}\Theta_i^{n_i}(\xi,\mu)d\mu \qquad i=I,II \qquad (14)$$

$$J_i=2\omega_i\ I_{11i}\quad \text{where}\ I_{11i}=\pi\alpha_i^5\lambda_i\int_0^{\xi_1}\xi^4 d\xi\int_{-1}^{+1}\Theta_i^{n_i}(\xi,\mu)(1-\mu^2)d\mu$$
$$i=I,II \qquad (15)$$

If all these four quantities are supposed to be assigned, it is easily seen that for any given value of a_I (once a_{II} is fixed) one obtains a single set of values λ_I, λ_{II}, v_I, v_{II}, whose knowledge entirely determines the corresponding physical configuration. Thus by keeping fixed the two masses and the two angular momenta, one obtains a succession of possible configurations increasingly collapsed.

As a preliminary application, identifying the spheroid II with the halo star component and spheroid I with the gaseous disk, and starting from an initial configuration where the two spheroids are assumed to have same radii, flattenings, and angular velocities, we have worked out the behaviour of a few cases with different values for the masses and angular momenta. Cases 1, 2, 3 are chosen in such a way that total mass and total angular momentum are kept constant while the ratio M_I/M_{II} changes; cases 2, 4, 5 are such that total mass and ratio M_I/M_{II} are constant while total angular momentum is varied. Further we have taken $n_I = n_{II} = 1$.

For each case a_I is decreased up to the point where centrifugal velocity is reached at the equatorial edge of the disk spheroid. The main points to be stressed from the results are:

When centrifugal equilibrium is reached:

- For the first series (models 1, 2, 3):

1) the radius for the disk component is smallest when the mass of the disk component is largest;
2) the flatness is independent from the mass ratio of the two components;
3) the increase of the central density λ_I of the disk is the largest for the largest disk component;
4) the same is true for the angular velocity, ω_I largest for the largest gas component.

- For the second series (models 2, 4, 5):

1) the radius of the disk is smallest for the smallest angular momentum;
2) the flatness is again independent from the value of the angular momentum;
3) the central density of the disk component is the largest for the smaller angular momentum;
4) the angular velocity of the disk is again largest for the smallest angular momentum.

More details will be published elsewhere.

References:

Chandrasekhar, S. and Lebovitz, N.R. 1962, Ap.J. 136, 1082.

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