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Nonlinear Waves in Force-Free Fibrils

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Abstract. Korteweg-de Vries equations for slow body and torsional weakly nonlinear Alfvén waves in twisted magnetic flux tubes are derived. Slow body solitons appear as a narrowing of the tube in a low β plasma and widening of the tube, when $\beta \gg 1$. Alfvén torsional solitons appear as a widening ($\beta > 1$) and narrowing ($\beta < 1$) of the tube, where there is a local increase of tube twisting. Two scenarios of nonlinear dissipation of strongly nonlinear waves in twisted flux tubes are proposed.

1. Introduction

A treatment of prominence dynamics has to account for their fine structure and currents. Linear and nonlinear waves in structured magnetized plasma with currents differ fundamentally from MHD waves in a uniform magnetic field. The theory of waves in structured atmospheres is based on the theory of waves in thin flux tubes, which is well developed for linear waves in flux tubes without currents (see Roberts (1991) and references therein). The first steps in the theory of linear waves in force-free flux tubes have been done by Zhugzhda (1996, 1998, these proceedings). Nonlinear waves in current-free flux tubes have been considered by Roberts (1991) and Zhugzhda and Nakariakov (1997a, b). We present here preliminary results on nonlinear waves in force-free flux tubes, while details of the treatment are in Zhugzhda and Nakariakov (1997).

2. Basic Equations

The general set of equations for force-free flux tubes (see Eqs. (1-8) in Zhugzhda (1998, these proceedings – hereafter Paper 1) is reduced to the following KdV equation by the method, which is described by Zhugzhda and Nakariakov (1997a, b). The KdV equation is obtained

$$\frac{\partial B}{\partial \tau} - \delta \frac{\partial^3 B}{\partial \xi^3} - \varepsilon B \frac{\partial B}{\partial \xi} = 0, \qquad (1)$$

where the coefficients δ and ε are rather cumbersome functions of modified Alfvén and sound velocities $C_{\pm}$ (Eq. (13) of Paper 1). The explicit expressions for $\beta \ll 1$ are written below. The KdV equation (1) is valid for long wavelength slow body and torsional weakly nonlinear Alfvén waves in twisted flux tubes. Besides, the KdV equation (1) is valid for torsional waves of arbitrary wavelength in weakly twisted tubes.

3. Slow Body Solitons

The KdV equation (1) describes nonlinear body waves, when the phase velocity, C_+ (Eq. (13) of Paper 1), is substituted in δ and ε. In the limiting case of an untwisted flux tube, $K = 0$ (Eq. (15) of Paper 1) reduces to the KdV equation, which have been derived by Zhugzhda and Nakariakov (1997a, b). If $K \ll 1$ and $\beta \ll 1$, the coefficients of the KdV equation for slow body waves are

$$\delta = \frac{\mathcal{A}_0}{8\pi}(1+4K)\beta^{1.5}C_A, \quad \varepsilon = \frac{(\gamma+1)(1-3K)C_A}{2B_0\beta^{0.5}} \tag{2}$$

and the soliton solution of the KdV equation (1) is

$$B = B_a cosh^{-2}\left(\frac{\xi + V\tau}{L}\right), \tag{3}$$

where

$$L = 2(3\delta/B_a\varepsilon)^{1/2}, \quad V = \varepsilon B_a/3, \tag{4}$$

is the wavelength and speed of the soliton in the moving frame. The amplitude of soliton $B_a > 0$. The soliton is retarded, because its speed in the laboratory frame is less then the speed of the linear waves, C_-. According to expression (3), the soliton involves a narrowing of the magnetic tube, which is accompanied by an increase of the magnetic field, a decrease of the gas pressure and density, and an acceleration of the plasma in the soliton throat. The velocity and wavelength are not noticeably affected by a weak twisting, $K \ll 1$ in the case of strong magnetic field $\beta \ll 1$. In the case of sufficient twisting, the nonlinear coefficient ε could change sign. This occurs for slight twisting in the case of weak magnetic fields, when $\beta \gg 1$. For example, the sign of ε changes for $K = 0.1$ ($\alpha\mathcal{R}_0 = 0.28$), if $\beta = 15$, and $K = 0.001$ ($\alpha\mathcal{R}_0 = 0.09$), if $\beta = 50$. When the coefficient of nonlinearity is negative, the solution (3) has to be rewritten as

$$B = -B_a cosh^{-2}\left(\frac{\xi - V\tau}{L}\right), \tag{5}$$

and the absolute value of ε has to be used in expressions (4) for the velocity and wavelength of the soliton. In this case nonlinear slow body waves widen the tube, which runs in the laboratory frame with a velocity larger than the modified tube velocity, C_-. The plasma is compressed and heated in the tube.

4. Alfvén Torsional Solitons

In the case of $\beta < 1$, the KdV equation for torsional Alfvén waves is

$$\frac{\partial B}{\partial \tau} - \delta\frac{\partial^3 B}{\partial \xi^3} + \varepsilon B\frac{\partial B}{\partial \xi} = 0, \tag{6}$$

where parameters δ and ε are defined in the case of a weakly twisted tube ($K \ll 1$) by

$$\delta \approx \frac{A_0 C_A \beta^2 K^2}{2\pi}[1-\beta+K(3-4\beta)(1-2\beta)(1+\beta)] \tag{7}$$

$$\varepsilon \approx \frac{C_A}{(1-\beta)B_0}\left[1+\beta K\left(3\beta+\gamma-\frac{2}{1-\beta}\right)\right] \tag{8}$$

The function (5) is the solution of the KdV equation (6). In the case of a weakly twisted tube, the tube wavelength and phase velocity of the Alfvén torsional soliton equal approximately

$$L \approx \sqrt{6}\mathcal{R}_0 K\beta(1-\beta)\left(\frac{B_0}{B_a}\right)^{1/2}, V \approx \frac{C_A B_a}{3(1-\beta)B_0} \tag{9}$$

Thus, the Alfvén soliton involves a widening of the tube. The velocity of the soliton in the laboratory frame exceeds the modified Alfvén velocity, C_+. The Alfvén soliton could produce strong twisting in a weakly twisted flux tube, when the full ϕ-component of magnetic field on the surface of the tube equals

$$B_\phi \approx B_\phi^{(0)} - \frac{4BB_0}{(1-\beta)B_\phi^{(0)}}, \tag{10}$$

where $B_\phi^{(0)}$ is the ϕ-component on the surface of an undisturbed tube and B is defined by solution (5) of the KdV equation (6). If the twisting of the tube is very small, $B_\phi^{(0)} \ll B_0$ or $\beta \approx 1$, the soliton produces strong twisting of the tube. The twisting is accompanied by fast rotation of the plasma in the soliton. The azimuthal component of the plasma velocity on the surface of a weakly twisted tube equals approximately

$$V_\phi \approx -\frac{4C_A}{1-\beta}\frac{B}{B_\phi^{(0)}}. \tag{11}$$

The temperature, density and gas pressure drop in the soliton.

Both parameters δ and ε become negative when $\beta > 1$. In this case the solution of the KdV equation (6) is (3), where absolute values of parameters (7) and (8) have to be used for calculations of the wavelength and velocity of the soliton. In this case, the Alfvén soliton appears to involve narrowing, where twisting increases in accordance with (10) and rotation appears in accordance with (11), while pressure and density drop. However, the change of sign of δ and ε do not happen exactly at $\beta = 1$ and, moreover, do not happen at the same values of β. This means that there is a rather small range of parameters, β and K, when the parameters, δ and ε, of the KdV equation (6) for Alfvén torsional waves have different signs. In this case, the hyperbolic functions in the "soliton" solution of the KdV equation should be replaced by trigonometric functions, which have singularities. It seems to us that the stationary solution should contain discontinuities. At least it is clear that in the case of nonlinear wave propagation along the tube with varying parameters of β and K, the crossing of the region, where $\beta \approx 1$ has to be accompanied by some special effects. This is the region where twisting due to solitons could increase strongly and a narrowing wave transfers to a widening wave, or vice versa.

5. Discussion

From the point of view of the dynamics of prominences, it is important to understand what happens with nonlinear waves of rather large amplitudes, when dispersive effects cannot balance nonlinear effects and nonlinear dissipation has to appear. The scenario of nonlinear dissipation of slow body waves in untwisted flux tube has been proposed by Zhugzhda and Nakariakov (1997a, b). The main point of this scenario is the transfer from subsonic to supersonic flow in the throat of a narrowing tube, as happens in a de Laval nozzle. The twisting of the tube does not change significantly the temperature of the plasma and flow velocity in the narrowing throat. There is some relative decrease of flow velocity and increase of temperature in a twisted tube compared to an untwisted one, if the amplitudes of solitons are the same. Thus, the scenario of nonlinear dissipation of slow body solitons in twisted and untwisted tube is the same.

The Alfvén torsional soliton causes a drop of pressure and temperature, when it appears as tube narrowing for $\beta < 1$. Consequently, the transfer from subsonic to supersonic flow in the narrowing throat is possible without shock formation. A shock has to be appear at the end of the jet because the tube does not widen infinitely. However, the alternative scenario of nonlinear dissipation of Alfvén torsional waves is possible. The drop of density and temperature in the Alfvén soliton decreases the number of electric charge carriers, while the increase of twisting follows the increase of the induction electric field. If the velocity of electrons in the Alfvén soliton are at about the ion temperature or higher, plasma instabilities or acceleration of energetic particles can occur. This scenario may be considered as a discharge in a flux tube with current. In this case, dissipation of magnetic energy takes place while dissipation of kinetic energy of plasma flow occurs in the alternative scenario.

Dissipation of nonlinear slow body and Alfvén torsional waves could both be responsible for an appearance of supersonic flows in prominences. The peculiarities of these flows is that they started from the regions of cool plasma.

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