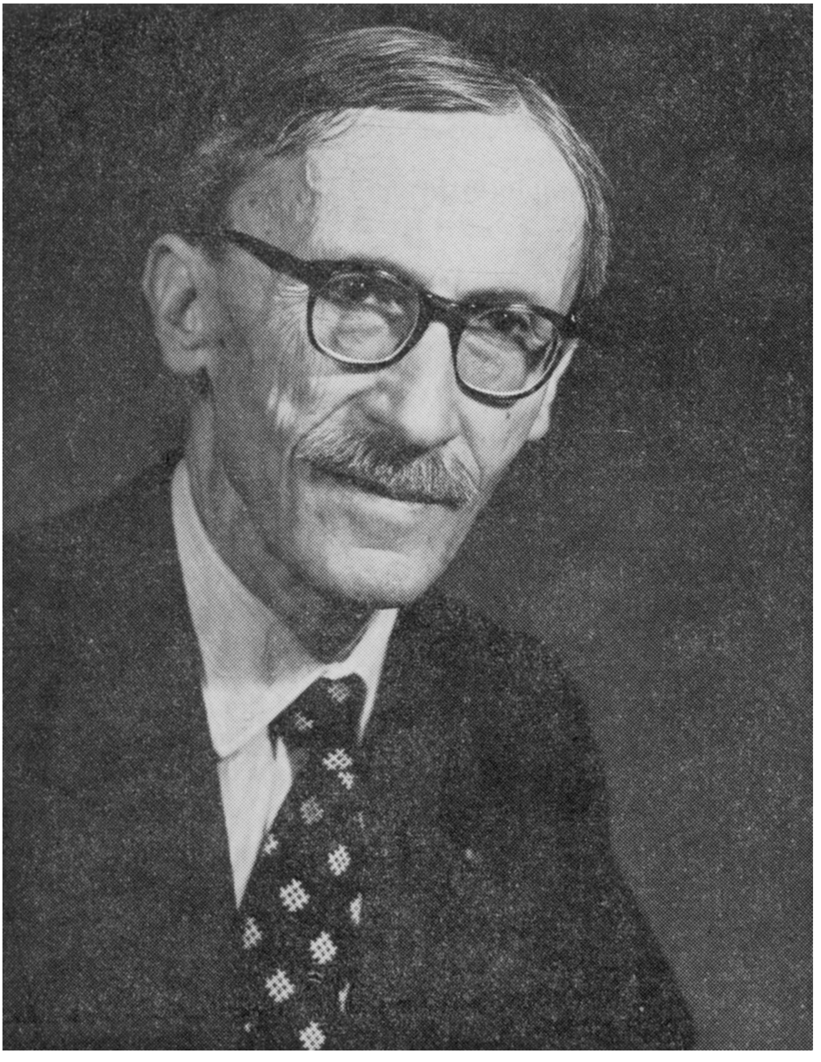

Studio Harcourt, Paris

PAUL LÉVY 1886–1971

Obituary: **PAUL LÉVY**

Paul Lévy's life (15 September 1886–15 December 1971) spanned three periods in the development of probability theory. In the first period, probability theory was accepted as mathematics; it would have seemed pointless at the time to carp at its place within mathematics. It was sufficient that the probability context led to interesting problems, and mathematicians did not look into the meaning of the context except in introductory chapters of textbooks. No one seemed to care that a coin was not more of a mathematical concept then the pair Peter and Paul who tossed it, nor that the term 'tossing' was undefined. In the second period, attempts were made to rationalize the context. But it came to be seen that there was an inner thread (measure theory) in the problems suggested by the context, and that, as far as the pure mathematician was concerned, the context was suggestive but logically unnecessary and mathematically undefinable. In the present third period, pure mathematicians are no longer concerned about the context, and their students are as ignorant of its possibilities as a suggestive background as are algebraic geometry students of the classical geometry of algebraic curves.

Lévy's large body of work reflects these three successive periods. His first probability book (1925) discusses the non-mathematical meaning of probability; his later books discuss mathematical probability as such. Nevertheless he retained throughout his life an extraordinary intuition, operating at the highest levels; his classical background was strong enough for him to visualize members of a family of random variables as defined successively using conditional distributions, rather than all at once as coordinate variables on an infinite dimensional measure space.

Mathematics was a tradition in Lévy's family: his father and grandfather were both mathematics professors. His most influential teacher was Hadamard, under whom he wrote his thesis (1911) in functional analysis in the Volterra tradition. In spite of much contrary evidence, it is sometimes thought that science flourishes only under conditions of absolute freedom, sometimes verging on pure anarchy. In Lévy's case it was fortunate for probability theory that he did not have absolute freedom. In 1919, when he was professor at the École Polytechnique, a post he held until his retirement, the academic dean asked him to give three lectures on probability and the theory of errors, stressing particularly the role of the normal distribution. This command performance made him look seriously at probability, to which he devoted most of the rest of his scientific life. His first important work, in the early twenties, involved characteristic functions. He was one of the first (together with von Mises) to realise the necessity of using distribution functions to handle the most general random variables, rather than treating only (and separately) discrete random variables and those with densities. This insight made possible his fundamental work on characteristic functions.

Lévy's 1937 book, containing his research up to that time, much of it previously unpublished, was the first modern technical book on probability; it was the inspiration of a generation of students. This book was inaccessible to non-probabilists for several reasons. Unlike previous probability books, it was highly technical and stressed the then unfamiliar study of sample functions and sequences. The book was almost entirely *on* probability, and not about problems suggested by probability which any mathematician could understand.

Finally, Lévy's intuition was the despair as well as the inspiration of his readers; the despair was due to the fact that the writing of formal proofs in the modern manner was not his ambition. But more formally inclined mathematicians, who had trouble following his reasoning, frequently found to their discomfiture that when they had devised formal proofs of his results, these were not far from Lévy's. The difficulty of reading his work may explain why he was not fully appreciated until late in his life. It should be added that his feelings toward other writers often reversed their opinion about him; he once remarked that reading the work of other mathematicians caused him actual physical pain.

Lévy's probability work centered around sums of independent random variables and the problems suggested by such sums. He derived the formula for a distribution function in terms of its characteristic function, and the conditions under which convergence of distribution functions corresponds to convergence of characteristic functions, now known as the 'Lévy Continuity Theorem'. He found delicate versions of the central limit theorem, including one for what are now called martingale sequences. He pioneered the study of infinitely divisible and stable laws, and of the corresponding stochastic processes with independent increments. His work on the fine structure of Brownian motion (Wiener process) shows his power and ingenuity at their best.

In summary, Lévy was one of the great classical probabilists, unique not only for the depth of his results but for the style of his thinking; he was the first probabilist to exhibit the possibilities of direct approaches to samples rather than to distributions. He will be much missed by his admiring and respectful colleagues.

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