Chapter 4 consists of a number of miscellaneous topics in the representation theory of the q-Schur algebra. These include the Ringel dual of the q-Schur algebra, truncation to Levi subgroups, components of tensor space, connections with the Hecke algebra, a description of the modules $\nabla(\lambda)$ in terms of bideterminants, Levi subalgebras of q-Schur algebras, quotients of Hecke algebras arising from saturated and cosaturated sets of dominant weights and, finally, global dimension of q-Schur algebras at roots of unity. The appendix gives an exposition of the theory of quasihereditary algebras and their tilting modules.

This is a well-written book which should be accessible to a graduate student with a background in homological algebra. It will be of particular interest to researchers working on the representation theory of the general linear group and quantum groups.

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PIETSCH, A. AND WENZEL, J. *Orthonormal systems and Banach space geometry* (Encyclopedia of Mathematics and its Applications, vol. 70, Cambridge University Press, Cambridge, 1998), ix + 553 pp., 0 521 62462 2 (hardback), £55 (US$85).

The type of 'Banach space geometry' presented in this book is essentially that which can be described in terms of ideal norms. Indeed, 'Orthonormal systems and ideal norms' would have served well as an alternative title. An *ideal norm* is a norm α defined for a suitable class of linear operators which satisfies

$$\alpha(BTA) \leqslant \|B\|\,\alpha(T)\,\|A\|$$

for all bounded operators A, B, where $\|\ \|$ denotes the usual operator norm. Familiar examples of ideal norms are the p-summing norms and the type and cotype norms. If α is an ideal norm and I_X denotes the identity operator in a space X, then it is usually non-trivial to determine $\alpha(I_X)$ (or even to determine whether it is finite): this quantity can therefore be regarded as a parameter describing in some sense the geometry of the space X. The notion of ideal norms runs through the book from beginning to end, and almost every chapter contains the words in its title.

The most basic way in which an ideal norm is derived from orthonormal systems is as follows. Suppose that $a_1, \dots, a_n$ and $b_1, \dots, b_n$ are given orthonormal systems in $L_2(M, \mu)$ and $L_2(N, \nu)$, respectively. Let $T : X \to Y$ be an operator between Banach spaces. The corresponding 'Riemann ideal norm' is the least constant C such that for any $x_1, \dots, x_n \in X$, we have

$$\int_N \left\| \sum_{k=1}^{n} b_k(t)(Tx_k) \right\|^2 \mathrm{d}\nu(t) \leqslant C^2 \int_M \left\| \sum_{k=1}^{n} a_k(s)x_k \right\|^2 \mathrm{d}\mu(s).$$

If (a_k) is replaced by a trivial system, the quantity on the right-hand side becomes simply $\sum_{k=1}^{n} \|x_k\|^2$ and we are left with the 'type' norm for T corresponding to $(b_1, \dots, b_n)$. A similar substitution on the left-hand side gives the 'cotype' norm corresponding to $(a_1, \dots, a_n)$. The powers 2 could of course be replaced by other indices. The classical type and cotype norms are obtained by letting the orthonormal system be either the Rademacher or Gaussian system and allowing n to vary.

A second fundamental family, the 'Dirichlet ideal norms', is described in terms of the action of T on elements of the form

$$\int_M \overline{\alpha_k}(s)\boldsymbol{f}(s)\,\mathrm{d}\mu(s),$$

where $\boldsymbol{f} \in L_2(M,\mu,X)$.

The book gives a very systematic and thorough account of the theory generated by these ideas, first for general orthonormal systems and then for particular ones. Many of the natural orthonormal systems are sets of characters on compact abelian groups, which allows some unification of treatment. Each of the following systems is given an extended treatment: the Rademacher system; the Gauss system; the Fourier system e^{ikt} and the discrete Fourier system; the Walsh system; the Haar system (leading to 'martingale-type ideal norms').

A general theme is the search for equivalence, or asymptotic equivalence, between the norms resulting from these systems. A further variation is to introduce unconditionality by inserting factors $\varepsilon_k \in \{-1, 1\}$ on the right-hand side in the definition quoted above. In this context we meet yet another subspecies, the 'UMD ideal norms'.

This agenda throws up a multitude of hard problems and deep theorems, as well as a certain amount of routine verification. Truly 'geometric' notions such as B-convexity and J-convexity make their appearance at the right time, so that in the end the coverage of Banach space geometry is indeed quite wide.

It must be said that the book does not lend itself to casual dipping. The system of fixed notations is highly logical but not very user-friendly. Each ideal norm, each orthonormal system and each class of operators is accorded its fixed notation; the index lists 65 such classes of operators, each denoted by a group of Gothic characters which most readers will not be able to reproduce in handwriting. Anyone interested in a later topic, say the Rademacher system, will need to start by investing some time in mastering the notation and the earlier, more general sections.

However, the book contains an enormous amount of information, and by taking ideal norms as the unifying thread, the authors have achieved a distinctive and systematic way of organizing this material. For serious students of the subject area it will serve as an invaluable reference book for years to come.

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LAM, T.-Y. *Lectures on modules and rings* (Graduate Texts in Mathematics, vol. 189, Springer, 1999), xxiii + 557 pp., 0 387 98428 3 (hardback), \$46.

The title of this book, and the first sentence 'An effective way to understand the behavior of a ring R is to study the various ways in which R acts on its left and right modules', indicate the approach to Ring Theory to be followed throughout this book. The primary object of interest is the study of rings via their categories of modules. This is in marked contrast with the author's earlier book in the same series *A first course in noncommutative rings* (GTM 131, Springer, 1991), where the rings themselves are firmly in the foreground at all times.

The book under review can be taken either as an adjunct to the earlier book, or as a basis for an independent course of study. There are seven chapters. The first three chapters are devoted to basic theory of modules with a view to homology, the chapter headings being: 1. Free modules, projective modules and injective modules; 2. Flat modules and homological dimension; 3. More theory of modules. This takes one about halfway through the book. There are then two chapters on the localization theory of rings: 4. Rings of quotients, which concentrates on Goldie Theory; 5. More rings of quotients, which considers the maximal ring of quotients and the Martindale ring of quotients. Chapter 6 deals with Frobenius and Quasi-Frobenius rings and Chapter 7 considers Morita Theory.