NUMERICAL STUDIES OF THE LUNAR ORBIT AT CERGA

O. Calame
Centre d'Etudes et de Recherches Géodynamiques et Astronomiques
Grasse, France

I - INTRODUCTION

Until very recently the only available source of accurate positions for the Moon and planets were the american export ephemerides from JPL in particular DE86/LE40 which expired in May 1979 and then DE96/LE44 and DE102/LE51. Unfortunately, these ephemerides were constructed for various applications, but are not necessarily adapted for a purpose such as the EROLD program. Furthermore, these ephemerides are badly documented, which prevents having good homogeneity with the computations for the representation of the various motions of a lunar laser reflector with respect to a terrestrial station. Thus, in this framework, some other studies were useful.

A few years ago, we decided that it would be desirable to support the Lunar Laser data reduction with the construction of a new ephemeris by numerical integration. This was considered valuable not only from the scientific point of view, but also as an operational tool for the EROLD and MERIT programs. Early in 1980, we adopted this ephemeris, called ECT 18, in the EROLD computations for the Annual Reports of BIH and for the needs of MERIT Operating Center and Analysis Center at CERGA.

II - DESCRIPTION OF BASIC MODEL

The anachronym ECT means "Ephéméride CERGA-Texas" in order to underline that the studies were under the responsibility of the author at CERGA, but collaborating with the University of Texas for the integrator model, of which a version (Lawson 1965) was already operational in Austin and was thus modified and adapted to our computer. Since then, there has been a considerable evolution in the improvements of the acceleration requirements of the laser data.

The basic integrator is a classical Adams-Moulton predictor-corrector, in rectangular coordinates, used for simultaneous integration of eleven

O. Calame (ed.), High-Precision Earth Rotation and Earth-Moon Dynamics, 233–243.

bodies. For lunar integration, the method uses differences at 12th-order, at a step of 0.25 day.

The predictor-corrector mode is used for the "critical" bodies of the solar system : Moon, Earth, Venus while the predictor only mode is applied to the other bodies.

The starting procedure is made by application of the Gauss f and g series, with the closure to a limit chosen by the user, currently 10^{-25}. The computations of integration are performed in the full CDC double precision, i.e. 28 significant figures.

III - MODELLING OF ACCELERATIONS

The computation of accelerations for the Moon and the planets are performed in several steps : the point-mass force model, including relativistic aspects, the effects of Earth's zonal harmonics, the effects of the lunar gravity harmonics, the effects of tidal friction on the lunar orbit.

1/ Point-mass force model : The differential equations governing the motion of the centers of masses are written and integrated in the inertial Cartesian coordinate system, referred to the mean equinox and equator of B1950.0. The origin is the solar system barycenter.

The principal force terms represent the Newtonian attractions in $1/r^2$ of each body, treated as point-masses. The effect of the mass of the body i on its own acceleration is contained in its contribution to the Newtonian potential at each perturbing body j and in its contribution to the Newtonian acceleration of each body j.

Obviously a pure Newtonian theory is not sufficient to obtain the required accuracy for the lunar laser data analyses. On the other hand, a full process of the post-Newtonian expansion is very complicated and very expensive as computing time, so that truncatures of the series have often been performed by several authors in selecting only the greatest terms which seem to be the most significant ones. For example, Brouwer and Clemence (1961) considered only the terms of second-order that would exist if the Sun and the considered body were the only with non-zero masses in the system. A few other authors considered some additional terms for the three body effects in the Earth-Moon system (Devine,1967; O'Handley et al.,1969), essentially for the lunar applications. However, this approach may be very dangerous and these approximations, even if they are quite adequate to the Einsteinian general relativity motion of the Earth-Moon barycenter and of the other major planets, represent only a few percents of the relavistic contribution to the geocentric motion of the Moon.

Consequently, we have based our model on the full 2nd-Order post-Newtonian general relavistic accelerations of the eleven point-masses.

The expressions used for the acceleration of each body i with respect to the barycenter of the solar system, in the rectangular inertial system are adopted from the equation 35 of T. D. Moyer (1971).

2/ Tidal acceleration : This effect represents essentially an acceleratio in the lunar orbital longitude due to the tidal energy dissipation in the Earth which causes a deceleration of the Earth rotation so that, in the hypothesis of the conservation of angular momentum in the Earth-Moon system, it is transferred to the lunar orbit.

Two types of procedures have been studied for the modelling, with the goal of optimizing the computations. Details of both models may be found in Mulholland and Calame (1982).

3/ Effects of Earth's zonal harmonics : Due to the significant mass and non-sphericity of the Earth and the Moon, the geocentric motion of the Moon is strongly disturbed by the gravitational figures of both.

The direct acceleration due to the oblateness of a body is derived from the generalized potential function :

$$U = \frac{\mu}{r}\left[1 + \sum_{n=1}^{\infty}\sum_{m=o}^{n}\left(\frac{a_p}{r}\right)^n P_n^m(\sin\phi) \times (C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)\right]$$

where : μ = gravitational constant of the body

r,ϕ,λ = radius, latitude and longitude relative to the body

a_p = mean equatorial radius of the body

P_n^m = associated Legendre functions

C_{nm}, S_{nm} = harmonic coefficients

For the effects relative to the Earth, we consider only the zonal terms of second, third and fourth-degree.

The orientation of the Earth's figure, then the orientation of the axis with respect to the adopted inertial system, is computed from the principal terms of precession. The nutation effect is not yet taken into consideration.

4/ Effects of the lunar gravity field : Similarly, the orbital motion of the Moon is influenced by its own figure. The modelling of the orientation of the figure axis is a little more delicate here because its motion is less uniform. Thus, it is necessary to take account of the physical librations, since the principal axis of inertia is shifted from the adopted frame, with significant oscillations.

https://doi.org/10.1017/S0252921100002657 Published online by Cambridge University Press

In our modelling, the effects of the second and third-order and degree of the lunar gravity field are introduced. The necessity to include so many terms may not be evident, even for the high accuracy of the laser data, but in fact the effects of third-order are not completely negligible and also, due to the spin-orbit interactions, these introductions were done in the preparation of future achievements to integrate simultaneously the orbital and rotational motions of the Moon, in order to ensure consistency between them. Indeed, the offset of the C_{22} coefficient is particularly critical and may cause a fictitious secular acceleration of the Moon along its orbit.

5/ Current omissions : In the present model, there are some significant omissions which are under study but of which the effects are not yet introduced. In particular, the lunar elasticity and dissipation are not considered. Also, most of the constant values are those for the 1976 IAU system of Astronomical Constants, excepting for the precession, nutations, lunar and planetary masses and lunar harmonics.

IV - RESULTS

This modelling of the positions of the solar system bodies was used to provide the ephemeris ECT18, early in 1980. For the Moon, the initial conditions of the motion were adjusted to the lunar laser data available from McDonald Observatory from 1969 to December 1979. The planetary initial conditions were obtained from DE96. The result was used for the purposes of the EROLD program and the MERIT campaign, as it is described by Calame (1982). This ephemeris may possibly be exportable, but this has not yet been done.

Furthermore, to estimate the quality of this product, we have performed some comparisons with recent JPL export ephemerides (DE96,DE102). Samples of differences for the geocentric position of the Moon are shown in figures 1 and 2. These diagrams are corrected for the differences in basic constants to show the discrepancies inherent only to the modellings themselves and to the initial conditions. It appears that the ECT18 ephemeris presents smaller discrepancies with DE96 than with DE102, but this may be explained by the choice of the initial conditions for the planets, which perturb largely the Moon motion. Among the differences, there are essentially annual, monthly and long-term (19 years) effects. After global solutions from the Lunar Laser data on 11 years (1969-1980), for 50 parameters, without fitting of the short-term variations in the Earth rotation, the post-fit r.m.s. residuals, from these three ephemerides (ECT18, DE96, DE102), are 3.92 ns, 3.98 ns and 3.85 ns, respectively. Obviously, this ephemeris ECT18 is still experimental; some new studies are under investigation for improved versions.

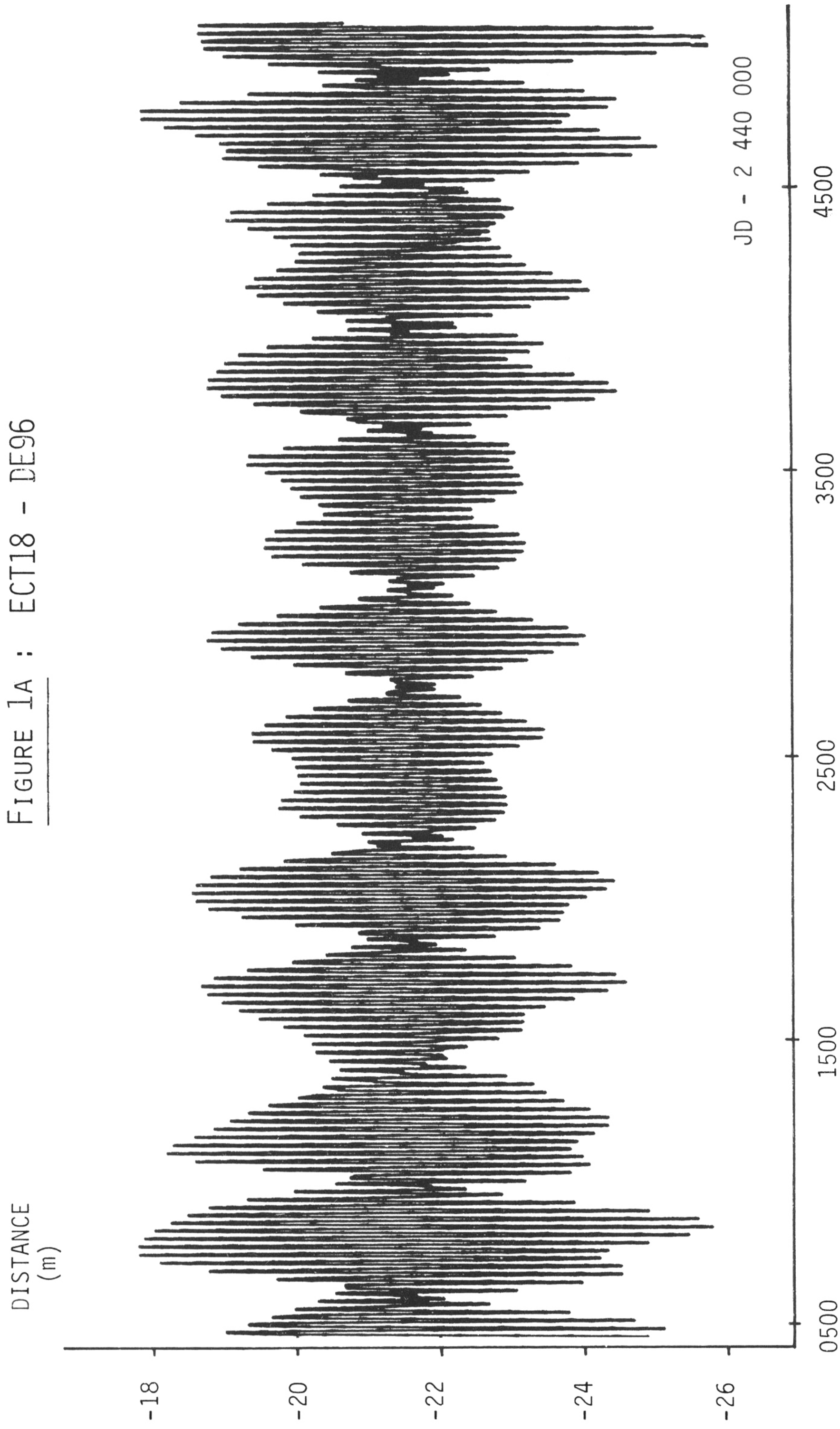


FIGURE 1A : ECT18 - DE96

https://doi.org/10.1017/S0252921100002657 Published online by Cambridge University Press

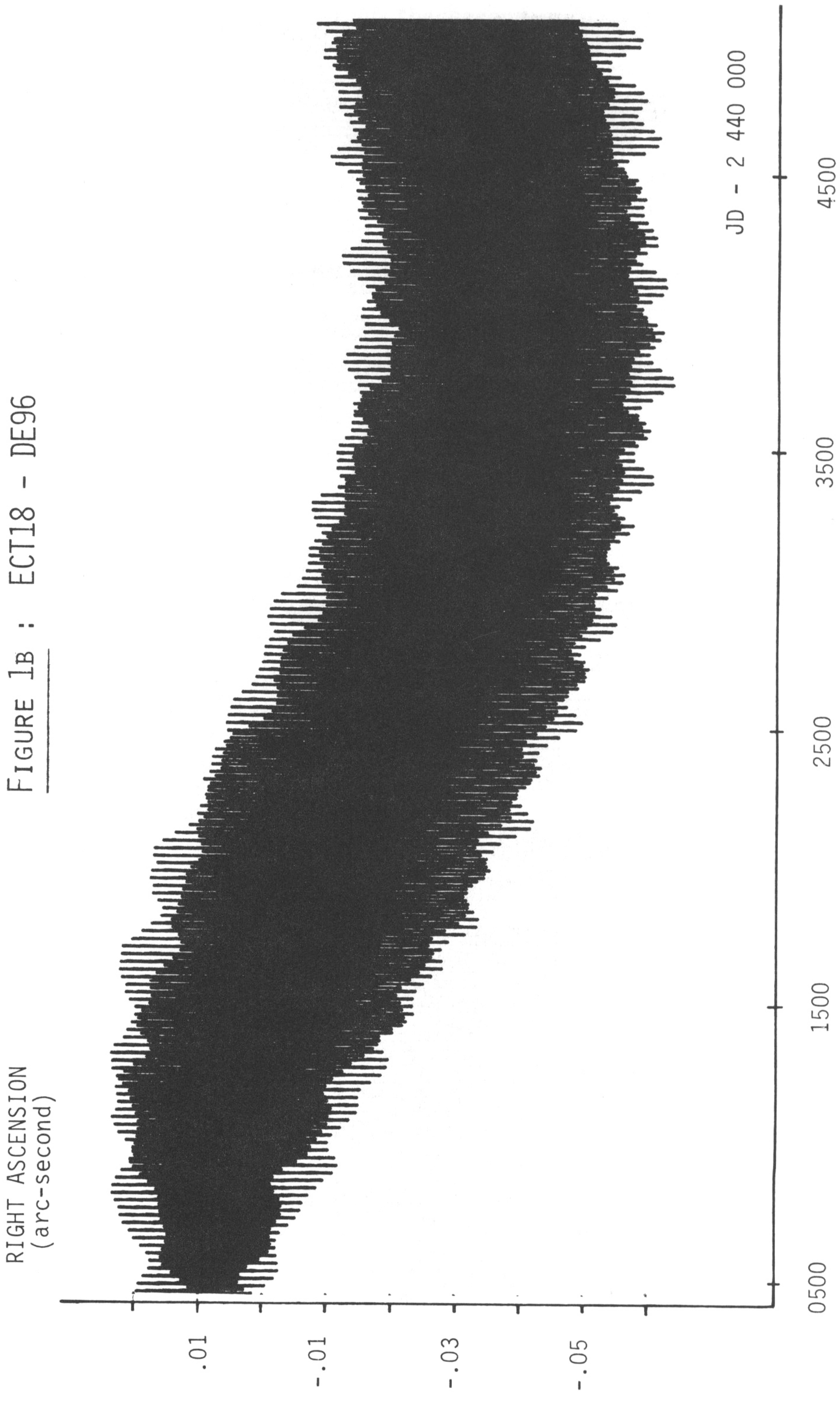


FIGURE 1B : ECT18 - DE96
RIGHT ASCENSION
(arc-second)
.01
-.01
-.03
-.05
0500
1500
2500
3500
4500
JD - 2 440 000

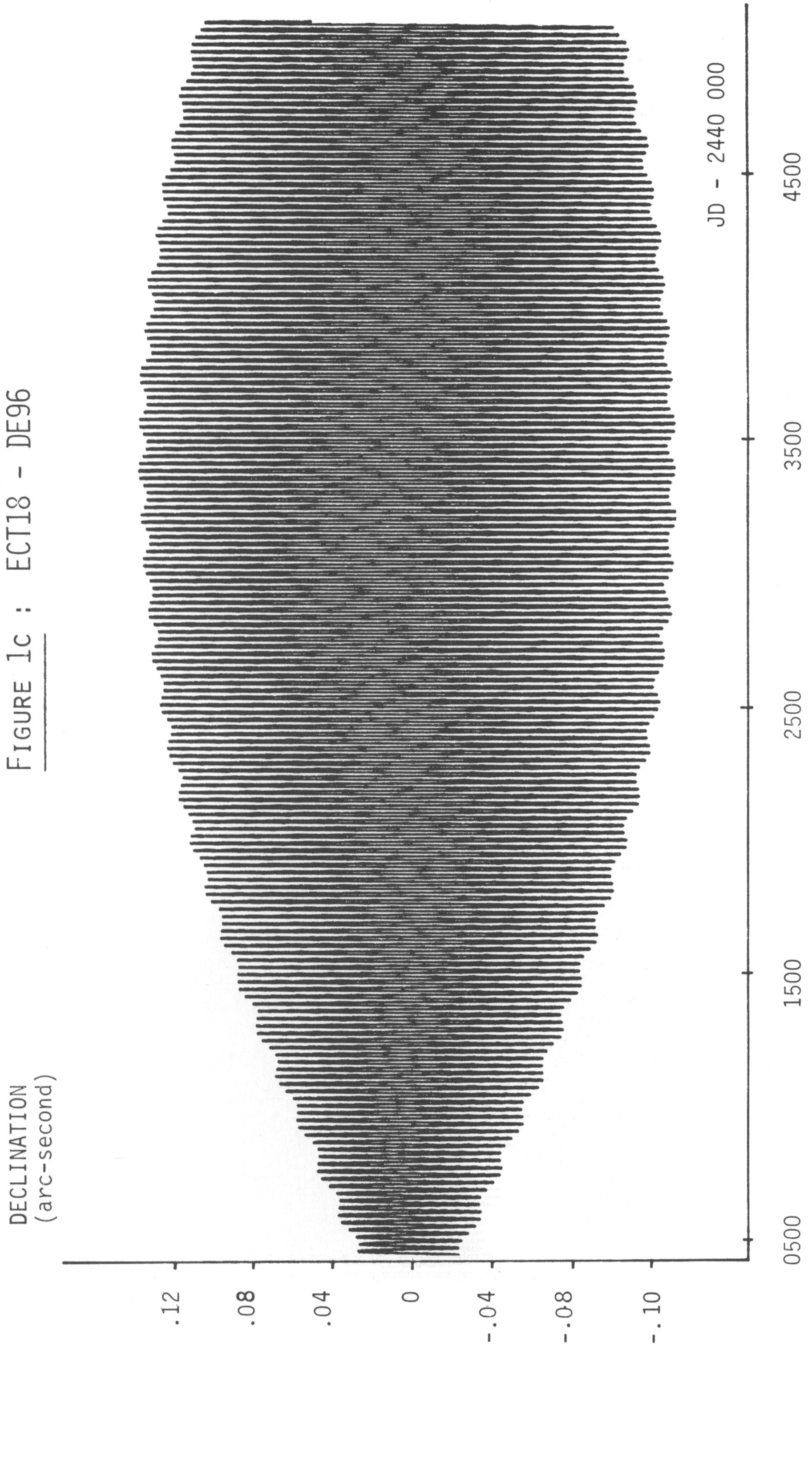
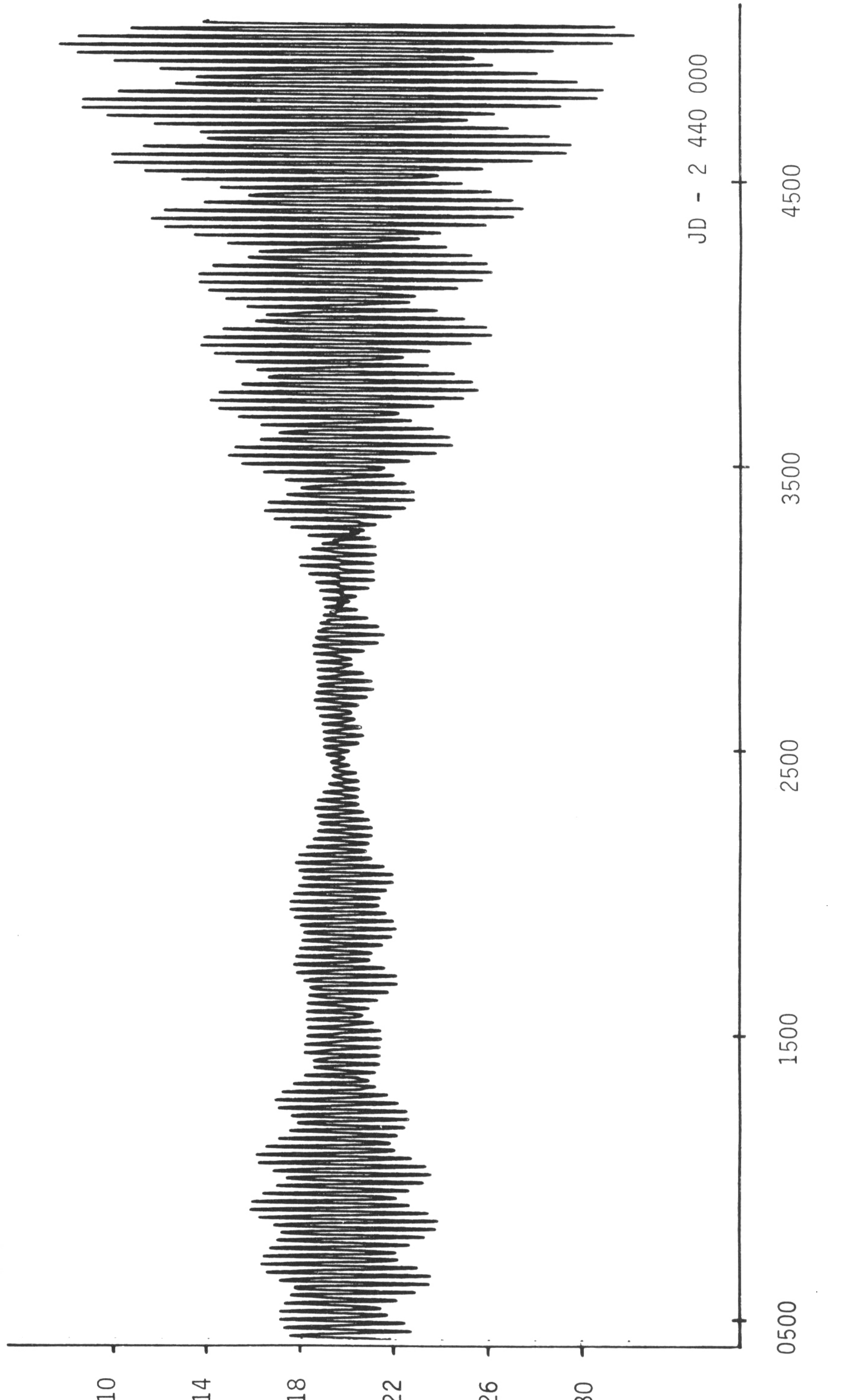


FIGURE 1c : ECT18 - DE96

https://doi.org/10.1017/S0252921100002657 Published online by Cambridge University Press

FIGURE 2A : ECT18 - DE102



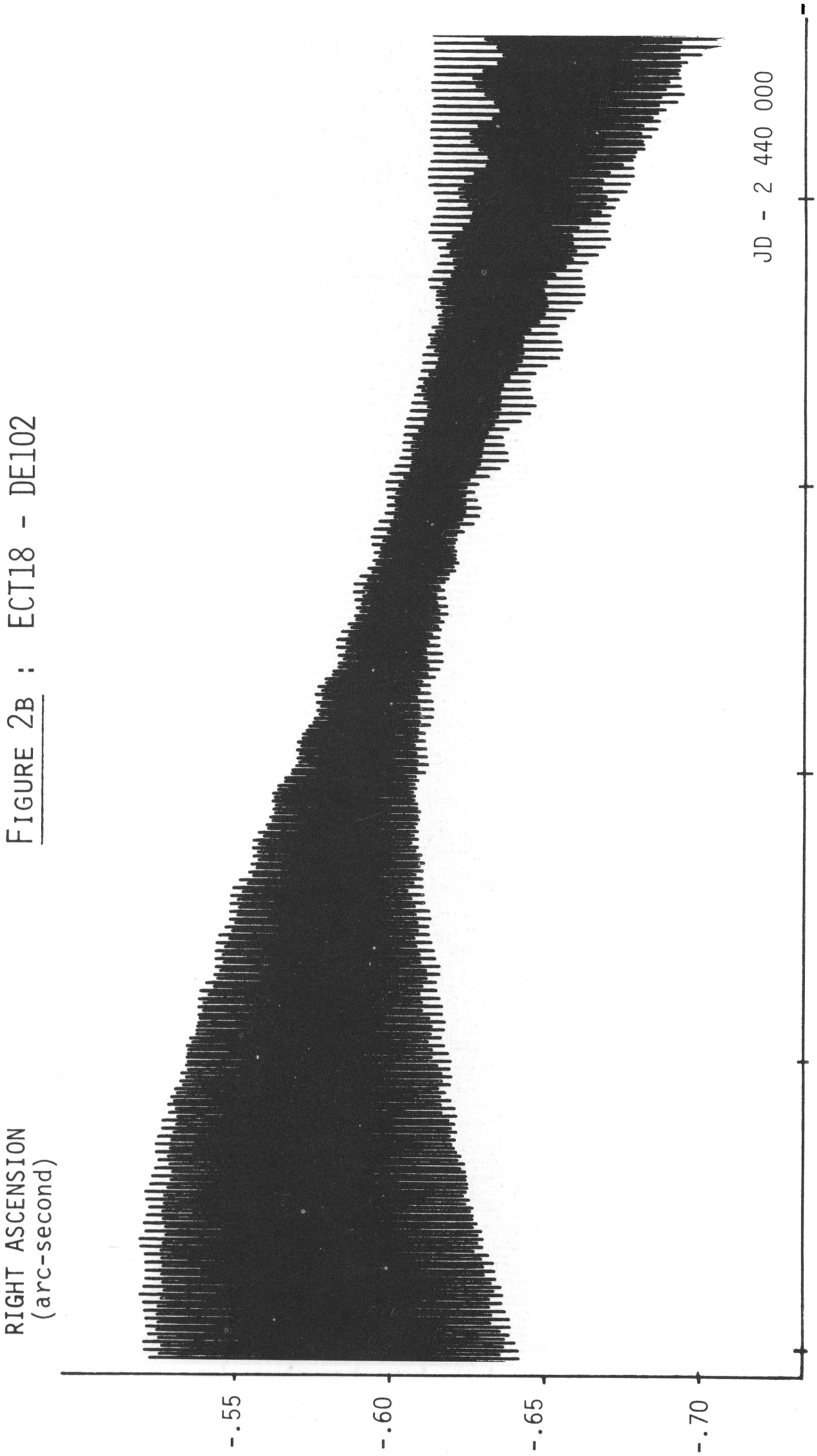


FIGURE 2B : ECT18 - DE102

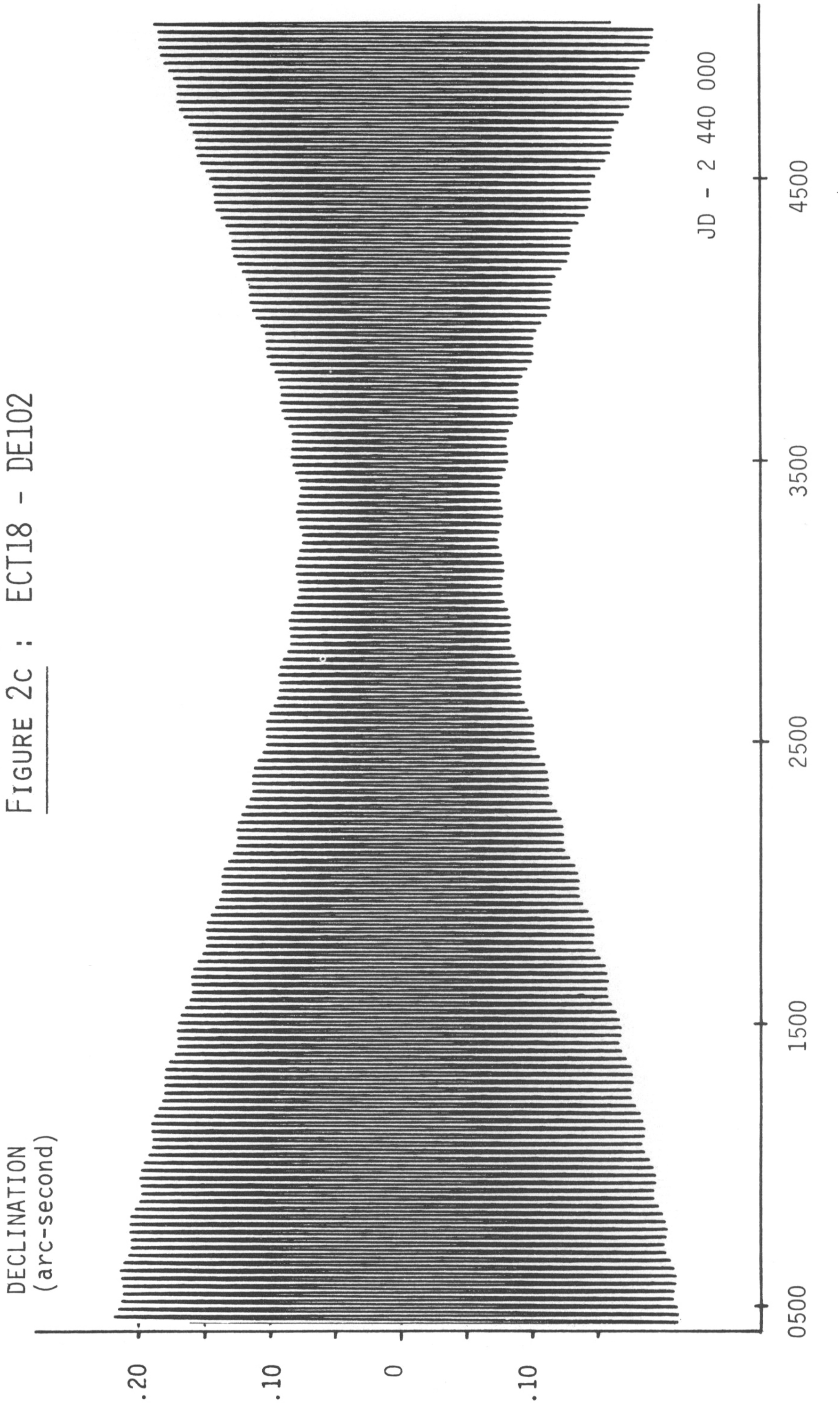


FIGURE 2c : ECT18 - DE102

https://doi.org/10.1017/S0252921100002657 Published online by Cambridge University Press

REFERENCES

Brouwer D. and G. M. Clemence : 1961, *Orbits and masses of planets and satellites*, in "Planets and Satellites", edited by G. P. Kuiper and B. M. Middlehurst, 43, University of Chicago Press, Chicago
Calame O. : 1982, these proceedings, 41, 233.
Devine C. J. : 1967, *JPL Development Ephemeris Number 19*, Report TR 32-1181, Jet Propulsion Laboratory, Pasadena
Moyer T. D. : 1971, *Mathematical Formulation of the Double-Precision Orbit Determination Program* (DPODP), NASA Technical Report 32-1527, Jet Propulsion Laboratory, Pasadena
Mulholland J. D. and O. Calame : 1982, these proceedings, 199.
O'Handley D. A., D. B. Holdridge, W. G. Melbourne and J. D. Mulholland : 1969, *JPL Development Ephemeris Number 69*, Report TR 32-1465, Jet Propulsion Laboratory, Pasadena

DISCUSSION

Chapront : What initial conditions did you introduce in ECT18 for the planetary perturbations ?

Calame : Those obtained from the DE96.

Lestrade : Does the relativistic model from Moyer, used in ECT18, is the same as Estabrook's formula in the solar system barycentric coordinates ?

Calame : I do not have the Estabrook's formula, so that no comparison has been done, but the formulation would probably be compatible.

https://doi.org/10.1017/S0252921100002657 Published online by Cambridge University Press