PART VII

RELEVANT INPUTS FROM OTHER FIELDS

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RECONSTRUCTION FROM PROJECTIONS IN MEDICINE AND ASTRONOMY

(Invited paper)

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It is a great pleasure for me to address this colloquium of radio astronomers on reconstruction from projections, especially since this opportunity allows me to combine an early interest in astronomy with my current work.

Some of you have been applying methods of reconstruction since the mid-1950's. Radio astronomers formed a cohesive society then and the methods reached fruition and matured in a natural progression. Reconstruction from strip scans and then interferometer data has been so fundamental to the formation of images in radio astronomy that one may well say that the field could not have progressed without them.

The situation in medicine has been quite different. Shortly after the discovery of x-rays in 1895 they were used in medicine. By the 1920's "tomographic" methods were discovered for obtaining cross sections of the body by blurring out images of unwanted planes. The human body contains high contrast objects (bones) which appear in exquisite detail in either ordinary radiographs or the "newer" tomograms. High contrast is also obtained by injecting radio-opaque substances into the blood stream (angiography) and the intestinal tract. Thus inferences can also be made about soft tissues. Perhaps because of the success of tomography and the lesser mathematical training of radiologists, the reconstruction problem never got formulated within that field in a mathematical way. The math was, of course, second nature to astronomers.

Reconstruction from projections was primarily thrust upon medicine from without. The earliest published attempt was a complete one: a group of mathematicians, programmers and engineers (Kalos, et al., 1961), trying to find defects in atomic reactors, rediscovered the mathematics of Johannes Radon (1917), put together a prototype, and designed an instrument in 1961 which was essentially identical to the first EMI scanner marketed in 1972 (Hounsfield, 1973). Unfortunately, although this group realized the potential impact on medicine, they could not gain the cooperation of physicians. Similar

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sad tales were to be repeated for a number of other early starters, including myself, during the 1960's. Many manufacturers were approached with the idea and uniformly turned it down, including major manufacturers of medical x-ray equipment. It took EMI, which is better known for its involvement in military electronics and phonograph records, to bring the idea to commercial fruition.

The x-ray scanners, now marketed by roughly 20 companies, have gone through five distinct "generations":

1) Two motion, single detector instrument using a pencil beam which is translated parallel to itself, and then rotated slightly (usually 1 degree) for the next scan.

2) Two motion, multiple detector instrument using a narrow fan beam. The rotation is made in larger angular increments, corresponding to the angle of the fan beam.

3) Single motion, multiple detector instrument using a wide fan beam which is simply rotated about the patient with the detectors.

4) Single motion, wide fan beam instrument, with a full circle of stationary detectors. Only the x-ray source rotates about the patient.

5) No motion instrument with a number of stationary fan beam x-ray sources and corresponding detectors.

The only fundamental distinction in this sequence of instruments is that each generation is faster in its data collection than the previous one. The last generation, which is just now being prototyped, uses a burst of x-rays to stop the motion of the beating heart (cf. Gordon, et al., 1975). The convolution algorithm (based on Radon's formula) is almost exclusively used in commercial scanners because it is computationally fast and easy to design into special purpose and parallel computers (cf. Tasto, 1977). This algorithm is not optimal in terms of giving the best image for the least dose (Gordon, 1976).

Computed tomography, as reconstruction from projections has come to be known in medicine, has had a large and immediate impact because it allows soft tissues to be seen with clarity. Classical tomography could do little more than image bones. Computed tomography may be used for nearly any medical problem whose diagnosis would be aided by peering inside the body.

Classical tomography has not been completely displaced, however, because computed tomography, as now practiced, cannot achieve the same spatial resolution without giving the patient dangerous amounts of x-radiation (Gordon, 1976, 1978). This anomaly deserves the attention of astonomers, for they too do not have as much radiation available as they would like. By careful design of the reconstruction algorithm, better images can be obtained from the available radiation.

In order to illustrate this point, I would like to give an example of optimal use of the radiation available in a problem in medical reconstruction, which may have application in x-ray astronomy. I will demonstrate an algorithm (called ARTIST) which attempts to make maximal use of gamma ray coincidence pairs in positron tomography (Gordon, 1975). In this diagnostic method a patient is administered a positron emitting radionuclide. Each positron emitted wanders a short distance within the patient (1 to 6 mm) before meeting an electron. The two annihilate with production of a pair of gamma rays which travel in opposite directions along a common straight line. If this line is in the plane of a ring of detectors, the two gamma rays will arrive nearly simultaneously ("coincidentally") at two of the detectors.

When a coincidence pair is detected, we know that the annihilation event occurred somewhere along a strip between two particular detectors. The reconstruction problem is to estimate the most likely point of origin of the photon pair. I will describe the ARTIST algorithm as it operates on each consecutively received coincidence pair, by assuming that it has already been operating for some time. Thus, imagine that we have an ordinary storage cathode ray tube, with a point displayed for each annihilation event whose location has already been estimated. We observe the screen through a slit, which is aligned with the coincidence line and opened slowly. The amount of light coming through the slit will increase in steps. We stop opening the slit either when we have reached a preset threshold count, or when we have reached some maximum slit width. The light coming through the slit is then scanned one dimensionally, resulting in a function which is regarded as a probability distribution. A random number is used to locate a single point along the coincidence line, chosen by this probability distribution. This point on the screen is then made to glow, and we are ready for the next coincidence event.

The ordinary approach to this reconstruction problem is to bin the coincidence counts in parallel strips between pairs of detectors. One may then use any ordinary reconstruction algorithm which handles data in parallel strips. However, fluctuations in the number of counts in each bin become noise in the data which is carried into the reconstructed image. In order to reduce this noise, the bin widths are typically set at 1 cm (Phelps, et al., 1977), which can be ten times the resolution limit set by the wandering distance of the positron! The algorithm I have presented, on the other hand, makes use of every single event and can do so without throwing away resolution during data collection. It thus suggests a different design of positron scanners, one with a large number of very small detectors.

The ARTIST algorithm may be a good replacement for the ART algorithm (Gordon, et al., 1970) now in use in x-ray astronomy (Charles and Culhane, 1977). There are two points that its existence raises which may be germane to radio astronomers: 1) a reconstruction algorithm should be designed around the physics of the problem being investigated; 2) instrument design should come after such an algorithm is designed and tested. This apparently logical sequence of events is almost never followed in medicine.

I would like to point out some other areas in which astronomers are or could be using reconstruction from projections:

1) A number of gamma ray bursts of cosmic origin have been discovered recently. An attempt is being made to obtain their precise celestial positions and reconstruct images of these sources from data collected by a satellite camera with collimators consisting of multiple slits oriented in a number of directions (Gorenstein, Helmken and Gursky, 1975, 1976). Here we have another case in which reconstruction must be done from very few, individually recorded photons.

2) The solar corona is an x-ray and light emitting plasma whose structure may be reconstructed in three dimensions. The major difficulty is to obtain more than one view in a short enough time. This may best be done by having a number of satellites circling the sun simultaneously. Lacking such multiple views, solar astronomers make the assumption that the structure of the corona does not vary much from day to day as the sun rotates (Altschuler and Newkirk, 1969; Levinson, et al., 1975). Thus we may obtain a number of views of the corona from the earth.

3) When a star explodes in a supernova, it throws off a shell of gas and filaments which glow by emitted or reflected light. If the shell is rotating relative to our line of sight, it may be possible to reconstruct inhomogeneities in it from images recorded a few decades apart. This would be the first example of reconstructing the three dimensional structure of an object outside our solar system.

4) When a planet occults a star, we obtain an integral of the absorption of light at different wavelengths along the refracted path through the planet's atmosphere. Such data has been used to reconstruct atmospheres, assuming radial symmetry (cf. Colin, 1972 and Veverka et al., 1974). Ionospheres may also be reconstructed from occultations of radio sources (Fomalont, personal communication).

5) There is a new method for exploring inhomogeneities in the surface layers of the earth that could be used on the moon and the planets on which we can land a craft. First one drills two parallel holes into the ground. Then by placing transmitting and receiving antennas at various depths, the attenuation and phase of the signal may be measured for a number of paths between the two holes (Lager and Lytle, 1975). A reconstruction of the material between the two holes may then be calculated. A similar method has been used for reconstructing

an atmospheric profile using a pair of satellites (Liu, 1976).

6) Reconstruction from projections is just beginning to have an impact on nondestructive testing, and thus may find many applications in the testing of components of telescopes of all kinds.

7) The construction of large parabolic mirrors for visual astronomy is a major undertaking. In order to obtain larger apertures than currently available, a "strip telescope" has been proposed (Murata and Baba, 1975). The strip telescope would scan an object whose fine structure is desired, at many angles, and then reconstruct its image (Baba and Murata, 1975, 1977).

8) For the extraterrestrial and science fiction enthusiasts reconstruction from projections provides many exciting possibilities. For instance, a proposal has been made to image the interior of planets by observing the neutrinos passing through them on their way from the sun (Clement, 1970; cf. Saenz, et al., 1977). Certainly the rotation of the planet would provide multiple views for reconstruction. In reports of encounters of the third kind people sometimes tell of an "eye" which rotates about their body (J. Musgrave, personal communication). Perhaps it is a form of scanner. Science fiction cartoons have suggested that multiple "views" of a person be taken for transmitting a set of projections and materially reconstructing their bodies. In any case, since transmission of the projections of an image may be more efficient than transmission of a whole picture (Gordon and Herman, 1971; Wee and Hsieh, 1976), we might expect that we'll have to reconstruct extraterrestrial signals to make them intelligible!

I would like to end by discussing reconstruction from a mathematical and perhaps philosophical point of view. Reconstruction from projections is as fundamental to science as calculus. It is a battery of methods for unfolding a structure from observations. The data is an intricate convolution of our means of measurement with the object's structure. Indeed, this is generally true of all physical measurements. We are rarely interested in direct measurements of a system, but rather use them to infer an underlying structure.

A fundamental limitation has been discovered in reconstruction using parallel x-ray beams. This is the Indeterminacy Theorem (Smith, Solmon and Wagner, 1977) which states that "finitely many views give no information about an object in its interior". As I will show, this theorem has important implications for radio astronomers. Its proof is complex, requiring use of a fundamental theorem in differential equations which was only proved in 1956. I will give a heuristic "proof" which will make the result seem reasonable, even though it is counter-intuitive.

Let us suppose the object is contained within a connected two dimensional region A and consider any subset B of A. If we have N views or sets of parallel projections of the object in A then we may construct a 2N-sided polygon anywhere within A whose sides are parallel to the projections (Gordon and Herman, 1971). If we add some density at one vertex of the polygon, the same amount may be subtracted from an adjacent vertex, so that the ray sum along their shared edge is still the same as before. Alternate additions and subtractions around the whole polygon may be made, so that all N projections are left undisturbed. Now if one vertex of the polygon is in subset B and the rest are in its complement A-B, we may make an arbitrary change at a point in B and compensate for this change at points in A-B. By repeating the operation an uncountable number of times the reconstruction can be changed into any picture whatsoever in the region B (Gordon, 1979). (This is the mathematically weak step of this alternate "proof".) Since the size of B can be nearly the whole of A, we may say loosely that any picture is compatible with any finite set of projection data.

The Indeterminacy Theorem is directly applicable to radio astronomy because interference data is a subset of parallel projection data. This may be seen by taking the Fourier transform of a single visibility reading, which represents a component of the Fourier expansion for some projection.

The Indeterminacy Theorem is perhaps the most important result of pure mathematics in this area since Radon's original inversion formula. No matter how much data we collect, and no matter how precisely it is measured, we cannot narrow down what the reconstruction should look like. This is a sorry state of affairs, forcing us to re-examine the whole operation of reconstruction. The authors of the theorem draw one positive lesson, which perhaps indicates the way we must proceed: in light of the Indeterminacy Theorem it is not merely an improvement, but rather it is essential that we use a priori information to restrict the range of solutions to a given reconstruction problem. We must take a hard and long look at our sources of a priori information, ranging from the positivity constraint to the maximization of certain functionals, such as entropy, and to the use of "human judgement" (cf. Gordon, 1973). These are, of course, exactly the approaches being discussed in much of this conference. However, we have a limited understanding of the effects of a priori information. Thus we have not as yet mastered the fundamentals of reconstruction from projections.

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