A METHOD FOR SOLVING POISSON'S EQUATION FOR SYSTEMS WITH AXIAL SYMMETRY

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In computer experiments on the dynamics of stellar systems special methods are often required for the computation of the forces to keep the problem manageable. For collisionless systems a method based on solving Poisson's equation with Fourier-series expansions has been used with success (Miller *et al.*, 1970; Hohl, 1973). The systems in these studies consist of a large number of particles moving on a rectangular grid.

For three-dimensional systems with axial symmetry a similar method can be used, based on the expansion of density and potential in Legendre polynomials (cf. Prendergast and Tomer, 1970):

$$\varrho(r, \theta) = \sum_{n=0}^{N} a_n(r)\, P_n(\cos\theta), \tag{1}$$

$$\psi(r, \theta) = \sum_{n=0}^{N} b_n(r)\, P_n(\cos\theta). \tag{2}$$

Poisson's equation, $\nabla^2\psi = 4\pi G\varrho$, with appropriate boundary conditions, supplies the relations between a_n and b_n and the potential can be written in the form:

$$\psi(r, \theta) = -4\pi G \sum_{n=0}^{N} \frac{P_n(\cos\theta)}{2n+1} \left[r^n \int_r^\infty s^{1-n} a_n(s)\, \mathrm{d}s + \frac{1}{r^{1+n}} \int_0^r s^{2+n} a_n(s)\, \mathrm{d}s \right], \tag{3}$$

where

$$a_n(r) = \frac{2n+1}{2} \int_{-1}^{1} \varrho(r, \theta)\, P_n(\cos\theta)\, \mathrm{d}\cos\theta. \tag{4}$$

(The boundary conditions $\psi \to 0$ for $r \to \infty$ and $\psi \to$ constant for $r \to 0$ have been used). Given a grid in r and θ, the integrals over r in (3) can be calculated from recurrence relations. The number of operations required to evaluated ψ from (3) and (4) for a given density distribution is therefore proportional to the number of grid divisions in r, the number of divisions in θ and the number of Legendre polynomials.

We have used this scheme to study the evolution of a system of 4000 particles (similar to the systems studied by Gott, 1973); its numerical behavior is quite satisfactory.

References

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DISCUSSION

Miller: A technical question: recurrence-relations such as you mention are usually unstable, and require great care unless you are careful about starting and the direction of recurrence. How do you get stable results?

Van Albada: No direct tests of the behaviour of the recurrence relations have been made. The results of experiments with rotating homogeneous spheres agree well with the 'exact' solutions.

Ipser: Have you tried to use this method to construct self-consistent solutions for equilibrium configurations?

Van Albada: No.

Spitzer: Can you compare the computing time required by your method and the corresponding time required by the method of Dr Gott?

Van Albada: For a system with many particles, moving them takes longer than evaluating the forces. Solving Poisson's equation on a grid of 1000 cells takes about 1 s on a CDC Cyber 74–16.