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Abstracts of Australasian Ph.D. theses

On some aspects of finite soluble groups

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Let $\underline{\underline{F}}$ denote a saturated formation, $\underline{\underline{H}}$ a Fischer class, G a finite soluble group, Σ a Sylow system of G, and V the $\underline{\underline{H}}$-injector of G into which Σ reduces. Moreover, let D and W denote, respectively, the $\underline{\underline{F}}$-normalizer and the Prefrattini subgroup of G corresponding to Σ. Let L stand for the sublattice of the subgroup lattice of G generated by V, D, and W. It is shown that

(i) the lattice L is distributive;

(ii) any two elements of L are permutable subgroups of G;

(iii) each element of L has an explicitly specified covering/avoidance property with respect to the chief factors of G;

(iv) the Sylow system Σ reduces into each element of L;

(v) for each element A of L, the conjugates of A in L form a characteristic class of subgroups of G;

(vi) the group G can be so chosen that L is a free distributive lattice on the three generators, so it has eighteen distinct elements. This extends the results in [2].

The second part of the thesis gives an upper bound for the Fitting length $h(G)$ of an arbitrary finite soluble group G in terms of the number $\nu(G)$ of the conjugacy classes of maximal nilpotent subgroups of G. Namely, it is proved that

$$h(G) \leq \begin{cases} 1 & \text{if } \nu(G) = 1 , \\ 2\left(1 + \log_3 \frac{18\nu(G)-19}{2}\right) & \text{if } \nu(G) > 1 . \end{cases}$$

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It is also shown that if $\nu(G) = 2$ then $h(G) \leq 3$ and if $\nu(G) = 3$ then $h(G) \leq 4$; examples show that for these two cases the bounds given are best possible. Further, a family of examples is constructed, consisting of one group G_n to each integer n greater than 1 , with $h(G_n) = n \geq \log_3 \log_3 \nu(G_n)$. These results extend and improve the results in [1]. They will appear in [3].

The final chapter gives a bound on $h(G)$ in terms of the number of conjugacy classes of maximal metanilpotent subgroups of G . In fact, a much more general result is proved there, but its full statement would require too much technical detail.

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