PERRY, P. A., *Scattering theory by the Enss method* (Mathematical Reports Vol. 1, Part 1, Harwood Academic Publishers, New York, 1983) xiv+347 pp., \$82.

In the mathematical theory of quantum mechanical potential scattering, the free and interacting dynamics may be represented by one parameter unitary groups e^{-itH_0} and e^{-itH}, generated respectively by the free Hamiltonian operator H_0 and the total Hamiltonian H. For two particle scattering, the dynamics of the relative motion are described by $H_0=-\frac{1}{2}\Delta$ ($\Delta\equiv$Laplacian operator in $\mathscr{H}=L^2(\mathbb{R}^3)$) and $H=-\frac{1}{2}\Delta+V(r)$ where $V(r_1-r_2)$ is the two particle potential. For a given class of potentials satisfying suitable conditions, further analysis usually depends on providing the following results:

(i) The existence of the so-called wave operators

$$\Omega^{\mp}(H,H_0)=\lim_{t\to\pm\infty} e^{itH}e^{-itH},$$

as strong limits on $\mathscr{H}$. This allows a comparison of the free and interacting dynamics at large positive and negative times. For long range potentials, roughly those decaying at infinity no faster than the Coulomb potential, the existence of these limits requires a modification of the free evolution to take account of the slow decrease of V.

(ii) The equality of the ranges of Ω^+ and Ω^-. This result (asymptotic completeness) means that the scattering operator $S=(\Omega^-)^*\Omega^+$ is unitary, as required for a good physical interpretation. In particular, the range of $\Omega^\pm$ should be the same as $\mathscr{H}_{\text{a.c.}}(H)$, the subspace of absolute continuity for H. In that case one has unitary equivalence of H and H_0 on their respective a.c. subspaces, where $\mathscr{H}_{\text{a.c.}}(H_0)=\mathscr{H}$.

(iii) The absence of singular continuous spectrum for H. The discrete spectrum of H should consist of isolated negative eigenvalues having finite multiplicity.

Substantial progress has been made over the years in proving (i)–(iii) for a wide class of potentials of both short and long range, and corresponding results have even been obtained in a variety of many particle problems. It was, however, the remarkable achievement of V. Enss, in a series of papers initiated in *Comm. Math. Phys.* **61,** 285–291 (1978) and *Ann. Phys.* **119,** 117–132 (1979), to develop a time-dependent method which was, at the same time, physically motivated by a "geometric" study of asymptotic behaviour in phase space, conceptually elegant, and which allowed a proof of (i)–(iii) for (almost) the best class of potentials. Among important refinements by Enss and other workers should be mentioned the introduction by Mourre of the infinitesimal generator of dilations as a tool in reaching a more transparent characterisation of incoming and outgoing states.

The work under review, itself by a notable contributor to the field, is to be warmly welcomed as the first comprehensive treatment of these developments in book form.

Two main chapters deal successively with applications of the Enss method to two body quantum scattering by short and long range potentials respectively. The treatment is thorough and systematic, and largely self-contained. In the long-range case, the added ingredient of dilation analyticity is invoked, with some loss of generality which, however, is justified by the consequent simplification of the analysis. There is a number of relatively minor misprints, but they do not obscure the main arguments.

A final long chapter deals with a variety of applications to problems in both Quantum and Classical physics. Areas covered in the Quantum domain include scattering by oscillating and time-dependent potentials, and by singular potentials in cases where local compactness applies. Asymptotic completeness for N-body systems is not covered (for $N>4$, this remains an open problem!) except for two cluster scattering below the three particle threshold, where essentially two body methods can still be used.

This book, which is the first of a new series of Mathematical Reports, may be strongly recommended to anyone seeking a brief but thorough introduction to this rapidly developing branch of Scattering Theory and its applications. A background in Hilbert Space and in some of the basic notions of spectral and Scattering Theory would be prerequisites, but these are readily accessible in a number of published texts.

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