BULL. AUSTRAL. MATH. SOC. 18A30, 18C10, 08A25
VOL. 19 (1978), 157-158.

Duality in topological algebra: Addendum

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It has been observed by Banaschewski [1], Proposition 1, that, in the notation of the author's paper [2], $\overline{P} = P' = \mathit{ProP}$ if $P \subseteq FU$ is hereditary and finitely productive. This fact does *not* require the use of injectives as in [2].

Thus, under the preceding hypotheses on P, we have the following:

PROPOSITION 1. *The inclusion* $P \subset \mathit{ProP}$ *is codense* (*that is,* $A \cong \int_P \{U(A, P), P\}$ *for all* $A \in \mathit{ProP}$).

Proof. We have $P \subset \mathit{ProP} \subset U$. Let E denote the subcategory of U whose objects are those of U and whose morphisms are the regular epimorphisms (equals coequalisers) in U. Let $H = E \cap P$. Then, because $A \in \mathit{ProP}$, we have $A \cong \int_{P \in H} \{E(A, P), P\}$. The canonical map

$\int^{Q \in H} E(A, Q) \times U(Q, P) \to U(A, P)$ is an epimorphism for all $A \in \mathit{ProP}$ and $P \in P$, since each map $f : A \to P$ factors as $A \twoheadrightarrow Q \rightarrowtail P$, $Q \in P$, as P is hereditary. Thus there is a monomorphism

$$\begin{aligned}
\int_P \{U(A, P), P\} &\rightarrowtail \int_P \left\{\int^{Q \in H} E(A, Q) \times U(Q, P), P\right\} \\
&\cong \int_{Q \in H} \left\{E(A, Q), \int_P \{U(Q, P), P\}\right\} \\
&\cong \int_{Q \in H} \{E(A, Q), Q\} \quad \text{by the representation theorem,} \\
&\cong A .
\end{aligned}$$

Received 13 June 1978.

https://doi.org/10.1017/S0004972700008613 Published online by Cambridge University Press

By considering the appropriate diagram (see [3], Theorem 2.3) one has that this monomorphism is left inverse to the canonical morphism
$A \to \int_P \{U(A, P), P\}$; hence is an isomorphism. //

PROPOSITION 2. *There is a duality between* $\mathcal{P}rop$ *and the* G*-copresentable algebras from* $\mathcal{P}$ *to* $\mathcal{E}ns$ *where* $G : \mathcal{P} \to \mathcal{E}ns$ *is the forgetful functor.*

References

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