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## CONTENTS

**Articles**

The Steiner-Lehmus angle-bisector theorem	<i>John Conway and Alex Ryba</i>	193
Runs, strings and alphabets	<i>Barry Lewis</i>	204
Inequalities for the perimeter of an ellipse	<i>G. J. O. Jameson</i>	227
A fresh look at Euler's limit formula for the gamma function	<i>G. J. O. Jameson</i>	235
Two girls – the value of information	<i>Keith Parramore and Joan Stephens</i>	243
Quotient polynomials with positive coefficients	<i>Mark B. Villarino</i>	250
Graph-theoretic models for the Fibonacci family	<i>T. Koshy</i>	256
Conics and convexity	<i>K. Robin McLean</i>	266
Wind over tide	<i>John D. Mahony</i>	273
A cubic surface of revolution	<i>Mark B. Villarino</i>	281
Cryptic crossnumber puzzles: a setter's perspective	<i>Alastair Cuthbertson (Oyler)</i>	291
Ordering 15 marbles with a three-way scale	<i>Charles Audet</i>	304
<b>Notes 98.09 to 98.19</b>		
Nint reciprocity	<i>Stan Dolan</i>	317
Associativity of symmetric set difference	<i>Des MacHale</i>	319
Rearranging the signs of the alternating harmonic series	<i>J. R. Nurcombe</i>	321
A convergent series with zero sum	<i>J. A. Scott</i>	325
Some double series related to $\zeta(3)$	<i>Tim Jameson</i>	327
An unexpected closed form	<i>J. A. Scott</i>	331
A class of dense subsets on the real line	<i>István Mező</i>	332

**Notes 98.09 to 98.19 (continued)**

A variation of Miquel's theorem and its generalisation	<i>Michael de Villiers</i>	334
The return of the returning explorer	<i>Jeremy D. King and Nick Lord</i>	339
Another look at the orthocentroidal disc	<i>J. A. Scott</i>	343
A new proof that the rationals are countable	<i>K. B. Subramaniam</i>	345

**Teaching Notes**

A non-calculator challenge: show that $\ln 2 < \frac{1}{\sqrt{2}}$	<i>Nick Lord</i>	346
Evaluating $\sum_1^n r^2$ and $\sum_1^n r(r+1)$ using moments	<i>Nick Lord</i>	346
Discrete random variables having equal mean and variance	<i>Nick Lord</i>	347
Cubics whose vertical translates factorise	<i>Nick Lord</i>	349
Approximate constructions of 1 radian	<i>Jim Dickson and Nick Lord</i>	350
Orbits made easy by complex numbers	<i>Tim Jameson</i>	353
<b>Feedback</b>		356
<b>Problem Corner</b>	<i>Nick Lord</i>	358
<b>Student Problems</b>	<i>Stan Dolan</i>	367
<b>Reviews</b>		370

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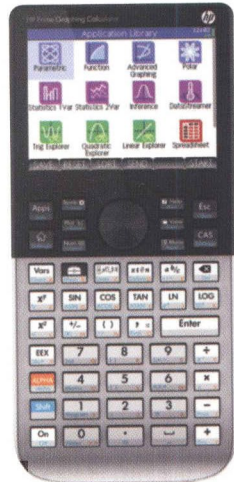


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