

On dynamical analysis of hydrostatic self-gravitating sphere: application to galaxy clusters

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Consider an hydrostatic self-gravitating sphere . The gravitational field is generated by 2 components: a visible (o) and an unseen (x). The resolution of the equations yields global quantities such as the mass (observed, unseen or total) of matter in the configuration $M_{o,x,t}$, the ratio of unseen mass to visible one $R = M_x/M_o$, the mean quadratic velocity computed on the configuration $V_{o,x}^2$, $C_v = V_x^2/V_o^2$ the relative concentration indicator of kinetic energy, and the relative concentration indicators of x-matter and x-potential energy C_i and C_x ;

If the dynamical analysis is performed, using visible matter only, one can derive dynamical quantities and compare them to those of the model. With an appropriate virial theorem we may write :

$$\mu = \frac{1+2C_iR+C_xR^2}{1+C_vR} \qquad R_{dyn} = \frac{(2C_i-C_v)R+C_xR^2}{1+C_vR}$$

μ is the ratio of the dynamical MLR deduced from the virial theorem to the individual one . Note that the indicators vary simultaneously. R_{dyn} is the relative amount of dynamical unseen mass to the visible one.

Clearly, μ and R_{dyn} are non-linear with respect to R and on to the relative concentrations of the components.

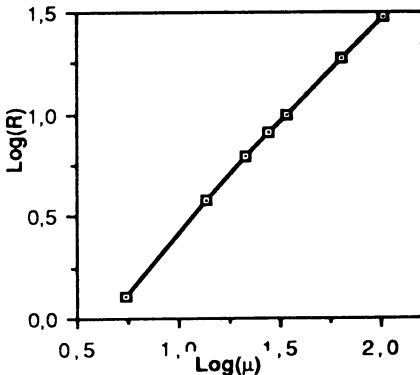


Fig1-Typical true mass ratio versus dynamical one (large concentration)

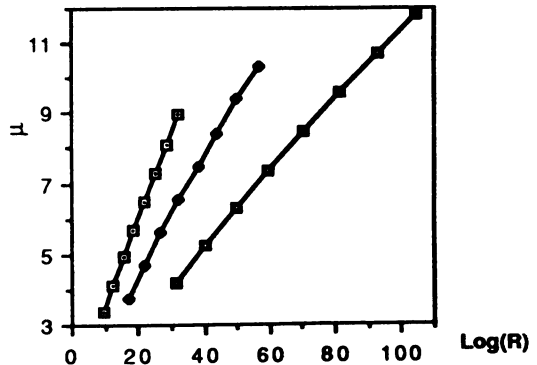


Fig2- R_{dyn} versus true ratio (various weak relative concentrations)

As a consequence we can see from fig1, that very large MLR-overestimation, does not necessary infer a great amount of dynamical unseen mass, but also one can show from fig2 that a very large amount of unseen mass may lead to a small MLR-overestimation. It depends crucially on relative concentrations.

As an example an MLR of ≈ 250 , leads to a mass of unseen matter (X-ray gas?) only three times greater (if more concentrated) than the luminous one (fig 1) or a contrario, to masses of unseen matter (following various weak concentrations) $\approx 35, 55, 90, \dots$ times the visible one !(fig2)

However the application of the virial theorem to clusters is not obvious:

- i) Optical data, X-ray surface brightness etc , constrain unknown relative concentrations
- ii) The clusters are projected on the sky.

In summary, the interpretation of the results of virial analysis is not straightforward. So it is necessary to built up and analyse improved multicomponent models to test effects such as suggested by the above formulae

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