theoretical physics who wished to pick up some more of the mathematics presently needed in their subject. Different parts of the book can largely be read independently and with its clarity of presentation it could very readily act as a reference volume.

Inevitably in a book which covers such a wide range of topics one may expect to have minor quibbles in areas where one feels particularly competent. Also, some may view the absence of bundles and forms as a notable omission. Nevertheless this seems to me a very valuable book, well written at a good mathematical level; I recommend it highly to anyone who needs a clear introduction to any of the topics it covers.

D. J. WALLACE

BEINEKE, L. W. and WILSON, R. J. (eds.), *Selected topics in graph theory* (Academic Press, 1979), pp. 451, £34·40.

Books with two editors and a title as vague as this tend to be collections of invited (or uninvited) conference lectures hastily gathered together in book form. The result is often disappointing for one of the following reasons: it is unreadable except for the person who knows it all already, each chapter uses different conventions of notation and terminology, and the topics are of such specialised interest that few will want to read about them anyway. Happily, this present volume is from a different stable. The editors have got together a group of well-known expositors who together survey some of the most important areas in modern "pure" graph theory. The result is a beautifully produced volume which deserves a place in every mathematics library, although most of the material can, of course, be found elsewhere. Topics covered include the four colour theorem, topological graph theory, hamiltonian graphs, tournaments, the reconstruction problem, minimax theorems, strongly regular graphs, enumeration and Ramsey theory, line graphs and edge colourings and contributors include P. J. Cameron, C. St. J. A. Nash-Williams, A. T. White and D. R. Woodall among others, as well as the two editors. A final chapter by Ronald Reid considers the impact computers have made and can make on graph theory research.

For those more interested in applications of graph theory, a companion volume entitled *Applications of Graph Theory* has also appeared. Finally, for those interested not so much in existence theorems but more in how to actually find a Hamiltonian cycle, for instance, the recent book *Graphs and Networks* by B. Carré (O.U.P. 1979) is recommended

I. ANDERSON

BILLINGSLEY, P., *Probability and measure* (Wiley, 1979), pp. 532, £28·95.

First and foremost the game-plan is excellent. Probabilistic intuition and measure-theoretic competence are developed side by side. For example, in the first chapter coin-tossing is used to show the need for events more complicated than finite unions of intervals and to lead rather quickly to Lebesgue measure. The calculus of infinite sequences of events (Borel-Cantelli lemmas, Zero-One Law etc.) is developed without recourse to integration. That was probability—but quite a deal of measure-theoretic intuition was picked up on the way. The second chapter takes up general measures in earnest, develops integration, Fubini's theorem, etc. That's measure theory—so we go on to random variables, expected values....

All of this is admirable and there's a healthy redundancy about it too. For example the treatment of differentiation lingers long enough on the line to make the power and efficiency of the abstract formulation of absolute continuity entirely convincing; then conditional probability, conditional expectation are introduced with great care and with many worked examples. In fact these topics and the most basic facts about martingales occupy a solid chapter of 176 pages.

That means I strongly approve the treatment and the book itself. Of course there are problems. There are many students for whom the main motivation of Lebesgue integration should be that it offers efficient no-nonsense convergence theorems which are simple to apply. The present treatment is not well-adapted to their needs, nor will they find the ideas surrounding the Riesz representation theorem and the links between measure and topology. (Rudin's "Real and Complex Analysis" and Hewitt and Stromberg's "Real and Abstract Analysis" are favourites of mine which cover some of that ground.)

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Where does the book fit? It's fine for a U.S. graduate program but harder to place in the U.K. or Australia. Much of it would be accessible to a good third or fourth year undergraduate and I would love to see all mathematicians learning measure theory in this way. In practice, however, the case for *any* measure theory (as opposed to integration theory) can be hard to argue particularly when statisticians often settle for discrete random variables at undergraduate level. A graduate student in analysis or mathematical statistics would find "Probability and Measure" suitable for self-study and highly rewarding. I recommend it strongly. (The text itself is horribly disfigured by broken formulae on page after page but appears remarkably free from misprints and errors.)

GAVIN BROWN

SAH, C.-H., *Hilbert's third problem: scissors congruence* (Pitman, 1979), pp. 240, £9·95.

If two planar polygons have the same area then one of them can be cut into triangular pieces which can be rearranged to cover the other polygon exactly. This fact, which is the two dimensional version of Hilbert's third problem, is usually attributed to F. Bolyai (1832) and to P. Gerwein (1833). It is pointed out in this book that the problem was solved about twenty years earlier by William Wallace (who was later a Professor at Edinburgh). The above problem was posed by Wallace as Question 269 in volume 3 of the new series of Leybourn's Mathematical Repository in 1814 and the printed solution is by Lowry. The solution given is simple and elegant. Gauss considered the three dimensional version of the problem and pointed out that to prove that two prisms with the same base and equal altitudes have the same volume it seems essential to use an infinite process of some kind. Hilbert's third problem asked for a rigorous justification of Gauss's assertion. An attempt at such a proof had already been made by R. Bricard in 1896 but Hilbert's publicity of the problem gave rise to the first correct proof—that by M. Dehn appeared within a few months. The third problem was thus the first of Hilbert's problems to be solved. Although several improvements and clarifications of Dehn's proof have appeared, this prompt solution seems to have led mathematicians to regard the third problem as rather an uninteresting one. Indeed, the problem seems not to have been discussed at all at the American Mathematical Society's 1974 Symposium on Hilbert's problems.

The book under review seems to be the third book entirely devoted to this problem. The previous ones were both written by V. G. Boltianskii (the first was published in Russian in 1956 and in English in 1963, the second and larger book in Russian in 1972 and the English translation was published by J. Wiley in 1978). Someone who wants a quick, clear and elementary account of the problem and its solution should read one of Boltianskii's books; indeed the books are probably accessible to a bright sixteen year old. Someone who wants to find a research problem in geometry might profitably read Sah's book.

The solution of the problem can be explained as follows. If P is a polyhedron in $\mathbb{R}^3$ whose edges have lengths $l_1, l_2, \ldots, l_n$ and whose corresponding dihedral angles are $\theta_1, \theta_2, \ldots, \theta_n$, define its Dehn invariant $d(P)$ to be $\sum l_i \otimes \theta_i$ whose values are in $\mathbb{R}\otimes(\mathbb{R}/\pi\mathbb{Z})$. Two polyhedra P_1, P_2 are equidecomposable if each can be cut into pieces $P_{i1}, \ldots, P_{ir}$ with P_{1j} congruent to P_{2j}. It is not hard to check that if P_1, P_2 are equidecomposable then their Dehn invariants are equal. To give an example for Hilbert's problem it is enough therefore to find two polyhedra P_1, P_2 with equal volume but $d(P_1) \neq d(P_2)$. Two such are the cube and the regular tetrahedron of unit volume. They have Dehn invariants $12\otimes(\pi/2)=0$ and $12.\sqrt[3]{3}\otimes\cos^{-1}(1/3)\neq 0$ respectively. In 1965, J.-P. Sydler proved that two polyhedra in $\mathbb{R}^3$ are equidecomposable if and only if they have the same volume and the same Dehn invariant. This would seem to be the final word on Hilbert's third problem in its original form.

Sah's book discusses many variants of the original problem and ties them in with other parts of mathematics. He starts by giving an axiomatic and abstract framework in which to discuss the problem and its variants. These variants are many: the real numbers are replaced by as general a field as possible, the n-dimensional version is treated, the affine, euclidean, hyperbolic and spherical cases and even versions for more general manifolds are handled. As well as the usual congruence in space, equivalence of polyhedra under various restricted groups of transformations (such as translations) are considered. All these possibilities give a large number of problems and many of them have not been solved yet. I found some of these variants more interesting than I

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