CHAOS IN THREE-BODY DYNAMICS: INTERMITTENCY, STRANGE ATTRACTOR, KOLMOGOROV-SINAI ENTROPY

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The temporal structure of chaos in three-body dynamics is analyzed; the emphasis is made on a similarity and difference between three-body chaos and basic patterns of chaotic behaviour known in nonlinear physics.

1. With the use of homology mapping (Agekian and Anosova 1967), we study a set of computer models of thee-body systems in a stationary spherically symmetric potential well (Valtonen *et al.* (1994); the well confines the bodies, and because of this the system can generate fairly long time series. Typical time series reveal sequences of seemingly periodic motion and short bursts of strong chaos that appear in an irregular manner (Heinämäki *et al.* 1998). The quasi-ordered states are associated with hierarchical homology, and the quasi-period of the low-amplitude oscillations is very near the period of the temporary close binary in the system. The high-amplitude irregular states are mostly due to active three-body interplay when each of the bodies interacts with the two others with almost equal intensity. In the evolutionary history of most systems, these two extreme kinds of states alternate in an apparently random way producing together a non-stationary pattern of unpredictable behaviour.

The time behaviour of this type is similar in its appearance – and actually in its physical nature too – to the phenomenon of intermittency observed in the ocean flows. A time series close in shape to what we observe in three-body dynamics may also be found in laboratory hydrodynamical experiments with Rayleigh-Bènard flow. The chaotic behaviour with this shape of time series is classified as type-III intermittency.

2. We use the correlation integral method (Lehto *et al.* 1993) and find that the dimension D of the time series generated by three-body systems is between 2 and 2.1 (Heinämäki *et al* 1998). It means that the number of the major physical parameters of the systems is 3; one can expect that they are the eccentricities of the binary orbit and that of the third body and also the ratio of semi-major axes of these two orbits, in hierarchical states of the systems. Two first of these parameters are related to the compact 2-dimensional homology hypersurface of the whole phase space. The third parameter corresponds to the infinite configuration coordinate in the whole non-compact phase space of the system.

For the system motions, the area of the homology hypersurface contracts with time onto the set of hierarchical states of lower dimensionality $D_H = D - 1 \leq 1.1$. The phenomenon of contraction (which is similar to the contraction of the whole phase space in dissipative systems) is seen from the fact that a typical system spends about 2/3 of its life-time in hierarchical states with temperary close binaries; these

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states attract almost all the trajectories, no matter where they start. The trajectories approach these states in highly irregular, chaotic way, which is similar to what is observed in the cases of strange attractors of dissipative systems.

However, some of the trajectories may not be trapped in the set of hierarchical states forever; they can leave it, and this proceeds also in a chaotic way. Such a behaviuor with transient trapping is different from what is observed in standard attractors and indicates one of the special features of three-body chaotic attractor. Because almost any three-body system ends its chaotic evolution with the decay, the system comes to the attracting set ultimately.

Note that the dimension of the time series in three-body dynamics proves to be near to one for the Lorenz dissipative system, introduced first in meteorology. A low dimension (close to 2) of the time series is an important sign of intermuttent chaos in nonlinear physics where this is considered as an evidence for low-dimension strange attractors that may be behind the time series.

One of the possible representations of the three-body strange attractor is given in a return map (Heinämäki *et al.* 1998). The object is cone or a piramid-like structure which can almost be reduced to a two-dimensional surface, except for the sparce loops that appear to avoid self-crossing of the trajectories.

3. An effective way to study the onset of chaos in dynamical systems involves a notion of the phase drop; it can be reformulated for three-body dynamics with the use of homology mapping. To quantify the process of deformation and spreading of the 'homology' drop in a coarse-grained description, we calculate the average exponential growth rate of the area occupied by the drop: it proves to be $h = 0.7 - 1.3$ in most of the observed cases of our computer simulations (Heinämäki *et al* 1999). The growth rate h has a close analogue in the Kolmogorov-Sinai entropy, defined in a similar manner for the whole phase space. With this values of h, one can conclude that the state of developed chaos occurs in about one crossing time, in three-body dynamics.

The 'fine' structure of the homology drop reflects the behaviour of individual trajectories. The average growth rate of divergence of the trajectories σ in the homology mapping which is a close analogue of the Lyapunov first exponent defined in the phase space. We finds that $\sigma = 0.4 - 1.5$. On the order of magnitude, h and σ are close, and the both prove to be close to the inverse time of correlation decay, τ_c, estimated for tree-body chaos by Ivanov *et al.* (1995). The relation $h \approx \sigma \approx 1/\tau_c$ is characteristic for the 'standard' patterns of chaotic behaviour in nonlinear physics.

References

Agekian, T.A. and Anosova, J.P.: 1967, *Sov. Astron.*, **44**, 1261.
Heinämäki P., Lehto H., Valtonen M.J., Chernin A.D.: 1998, *MNRAS*, **298**, 790.
Heinämäki P., Lehto H., Valtonen M.J.: Chernin A.D, 1999, *MNRAS*, in press
Ivanov A.V., Filistov E.A., Chernin A.D: 1995, *Astron. Rep.*, **39**, 368.
Lehto H.J., Czerny B., McHardy M.: 1993, *MNRAS*, **216**, 125.
Valtonen M.J., Mikkola S., Heinämäki, Valtonen H.:, 1994, *ApJS*, **95**, 69

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