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ABSTRACT

The formulation to calculate the precise refraction by means of the primitive equations by an electronic computer was made to be used for the calculation of the atmospheric refraction in the astrometry and the satellite geodesy with upper air data obtained with the atmospheric soundings.

This formulae were applied to calculate the astronomical refraction in the results of latitude observation at the Mizusawa Latitude Observatory. It was concluded that the results show larger refraction as compared with values due to Radau's and Pulkovo Refraction Tables.

The refraction calculated with the upper air data should be interpolated at each time of observation to be applied to the results of the astronomical observation.

To make an effective application of the formulae, the astronomical refractions were calculated on each clear night and the expressions to represent the actual astronomical refraction from the ground surface to the upper air were derived from these results.

It was found that these expressions can be given in a form

$$\begin{aligned} \log(n-1) &= a + bh && \text{from ground to tropopause} \\ &a' + b'(h-h_0) && \text{from tropopause to the upper air,} \end{aligned}$$

where  $n$  is the refractivity.

The quantities  $a$ ,  $b$ ,  $a'$  and  $b'$  have almost same values during a night and these formulae can be used for the interpolation of the astronomical refraction at the time on the observation.

## 1. EXPERIMENTAL FORMULA

The astronomical refraction is one of the most important and difficult problem in the astrometry. We usually use the Refraction Table or experimental formulae to calculate the constitution of the atmosphere by means of the atmospheric factors measured at the observing site. Many attempts have been made to obtain the much more precise value of the astronomical refraction by means of the data of the actual atmospheric factors observed, for example, by the Radio Sonde. We attempted, in this report, to derive experimental formula to obtain the precise value of refraction for each astronomical observation.

According to the Fermat's principle, we derived the following primitive differential equations to compute the path of a ray (Takagi, 1974)

$$\frac{d\varphi}{dr} = \frac{N_{\varphi} \cos \varphi}{r \sqrt{n^2 r^2 \cos^2 \varphi - N_{\theta}^2 - N_{\varphi}^2 \cos^2 \varphi}},$$

$$\frac{d\theta}{dr} = \frac{N_{\theta}}{r \cos \varphi \sqrt{n^2 r^2 \cos^2 \varphi - N_{\theta}^2 - N_{\varphi}^2 \cos^2 \varphi}},$$

and

$$\frac{dN_{\varphi}}{dr} = \frac{\partial n}{\partial \varphi} \sqrt{1+r^2 \left(\frac{d\varphi}{dr}\right)^2 + r^2 \cos^2 \varphi \left(\frac{d\theta}{dr}\right)^2} - \frac{nr^2 \cos \varphi \cdot \sin \varphi \cdot \theta r^2}{\sqrt{1+r^2 \left(\frac{d\varphi}{dr}\right)^2 + r^2 \cos^2 \varphi \left(\frac{d\theta}{dr}\right)^2}},$$

$$\frac{dN_{\theta}}{dr} = \frac{\partial n}{\partial \theta} \sqrt{1+r^2 \left(\frac{d\varphi}{dr}\right)^2 + r^2 \cos^2 \varphi \left(\frac{d\theta}{dr}\right)^2}$$

where  $n$  is the refraction index and  $N_{\theta}$ ,  $N_{\varphi}$  and  $N_0$  are defined by

$$N_{\theta} = nr^2 \cos^2 \varphi \frac{d\varphi}{dr} / N_0, \quad N_{\varphi} = nr^2 \frac{d\varphi}{dr} / N_0,$$

$$N_0 = \sqrt{1+r^2 \left(\frac{d\varphi}{dr}\right)^2 + r^2 \cos^2 \varphi \left(\frac{d\theta}{dr}\right)^2}$$

having a polar coordinate system fixed to the Earth in which  $r$  is the radius of a point from the geocenter,  $\theta$  the east longitude and  $\varphi$  the latitude. We applied these formulae to the actual data obtained in the vicinity of the Mizusawa Observatory and reduced them to the value at Mizusawa by the interpolation. We calculated the value of refraction for various zenith distances in the previous paper (Takagi and Goto, 1975) and found the constant and the seasonal systematic differences between our value and these given by the Radau and the Pulkovo refraction tables. Moreover, our results shows that the refraction is not symmetric around the zenith. For instance, the difference between the refractions for

north star and south star with zenith distance 20° amounts to 0!13 and shows slight seasonal variation with about 0!01 amplitude.

2. SOME ASPECTS OF REFRACTION INDEX

Difficulties in deriving the actual refraction are in the fact that we have hardly any data of the Radio Sonde at the time and at the site of the astronomical observation. There are many complex factors to determine the conditions of the atmosphere and these factors vary from time to time and from place to place. It is very difficult problem for the meteorologist to know detailed values of the meteorological factors from the data of Radio Sonde observation (Teleki 1974). We tried in this paper to devise a process to make interpolation of the atmospheric factors. We can see from our previous results (Takagi and Goto, 1975) that the refraction index can be expressed by a linear equations in three regions.

- a. from the surface to the top of the boundary layer (about 1.5 km height) (Region I)
- b. from the top of the boundary layer to the tropopause, (Region II)
- c. from the tropopause to the top of the atmosphere. (Region III)

By assuming a formula

$$\log(n-1) = a + bh$$

where n ; refraction index  
 h ; height (km)

we calculated the values a and b for the regions II and III for stations around the Mizusawa Observatory in a period from 1 January to 17 January, 1971. The values of a's will vary from time to time and from place to place, whereas the values of b's will not vary in a region with a small area. We will give the average values of a's and b's in this period for each station in Table 1 together with the standard deviation.

Table 1.

Station	Region II			Region III				
	Long. (E)	Lat. (N)	a	b	S.D.	a	b	S.D.
Sapporo	141.3	43.1	0.4814	-0.0503	0.0011	-0.0159	-0.0670	0.0008
Akita	140.1	39.7	0.4750	-0.0494	0.0015	-0.0688	-0.0672	0.0011
Sendai	140.9	38.3	0.4707	-0.0492	0.0015	-0.0029	-0.0668	0.0013
Wajima	136.9	37.4	0.4718	-0.0489	0.0012	-0.0024	-0.0673	0.0011

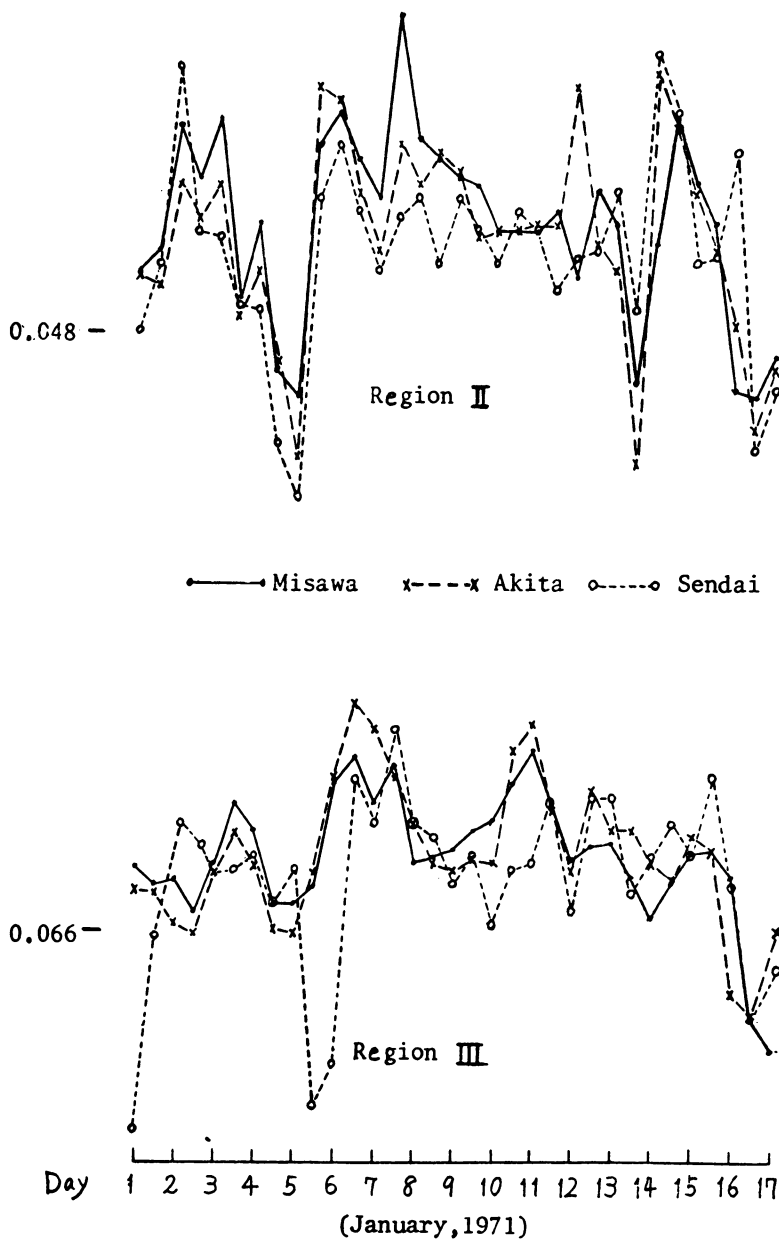


Fig. 1

The figure 1 shows the time variation of  $b$ 's in the regions II and III

at the same station. This figure shows us that the time variations in the three stations are very close and have almost the same tendency in the two regions, which reveals us the possibility to get the values of  $b$ 's by the regional and time interpolation from the results obtained by the Radio Sonde observation. This problem should be solved with a numerical analysis which will need further investigation.

### 3. SUMMARY

In consideration of the results of our investigations we would like to suggest the following procedures to determine the precise refraction at any station.

1) the atmospheric factors in the boundary layer should be observed at the observing site with a new device simultaneously with the astronomical or earth satellite observation.

2) the atmospheric refraction above the top of the boundary layer can be interpolated from the data obtained at the stations around the observing site by assuming the simple formula of the linear variation of the refraction index.

Our investigations are not sufficient to detect the fact whether we can use the average value of  $b$ 's for the both regions II and III obtained in many years or not. Our next investigations will be stressed to find a period when we can use the data to be applied for the interpolation at the time and at the site of the observation.

### REFERENCES

- Takagi, S.: 1974, Publ. Int. Latit. Obs. Mizusawa, 9, No. 2, 241.
- Takagi, S. and Goto, Y.: 1975, Publ. Int. Latit. Obs. Mizusawa, 10, No. 1, pp. 41-51.
- Teleki, G.: 1974, Publ. Obs. Astr. Beograd, No. 18, pp. 213-234.

### DISCUSSION

J. Milewski: remarked that the variations of refraction table values mentioned in the presented paper show systematic changes, which can be connected with the change of relative composition of atmosphere layers, particularly with the permanent increasing of CO<sub>2</sub> component in the atmosphere. He proposed an investigation of the variations of atmosphere composition with time and its influence of different refraction table values elaborated at various period of time.

J. A. Hughes: The change of the index of refraction due to the presence

of CO<sub>2</sub> may be explicitly accounted for by using the expressions given by Owens. This could be used for the proposed investigation.

G. Teleki: regret that the authors did not analyse the variations of refractive indices at the boundary layer, which are dominant in the calculation of refraction values, and that they did not compare their results with similar investigations already published by Sugawa and Kikuchi.