showed that the rational field possesses it. Lang shows how results of this kind are tied up with Siegel's theorem. He also gives a treatment of the case when k is the rational field from first principles (essentially a streamlined version of the original).

" In the large " the book is first class. Lang has the gift of dissecting out the concepts which play the leading role in a theory and exhibiting them with full clarity. For example, the concept of the " height " of a point on a variety (which has been distilled from Fermat's " infinite descent ") is analysed and its formal (" functorial ") properties displayed. The strategies of the various proofs and of the book as a whole are brought out clearly and there are most enlightening discussions of the relations of the results to one another and to others in the literature (I was particularly struck by the discussion of the various generalisations of Roth's theorems to include p-adic valuations).

" In the small " the effect is less satisfactory. To quite a large extent this is hardly the author's fault, but is caused by the chaotic state of algebraic geometry. Every few years there is a new formulation of the elements of algebraic geometry, and although the professional geometer can doubtless transpose a theorem or a theory without effort from the style of " the Italian School " to that of Weil's " Foundations " or to the language of schemas, this all increases the difficulties of the mere number theorist, who should also be vitally interested in this book. Lang writes for geometers and, besides assuming the facility of transposition mentioned above, is apt to justify his steps by a vague reference to some whole theory (e.g. " by the theory of Chow co-ordinates " without even a reference to where it may be found in one formulation or another). Difficulties of comprehension are increased because one is discouraged from a too curious analysis of mysterious arguments by the lurking suspicion that what is in the book is not quite what was meant: there are some strategically placed misprints, and the muddle near the top of page 136 can hardly be blamed on the printer.

But one would infinitely prefer to have the book, even with these blemishes, than no book at all, particularly in these days when so many new ideas either circulate for years in restricted circles before (if ever) they appear in print, or appear only in arid unappetising form. Professor Lang has greatly added to our already deep debt of gratitude by producing this fascinating account of a topical and important field.

J. W. S. CASSELS

WHITEHEAD, A. N. AND RUSSELL, B., *Principia Mathematica* to *56 (Students' Edition) (Cambridge University Press, 1962), xlvi+410 pp., 17s. 6d.

Here we have a book which is widely known about—but little known. This students' edition makes the book much more readily accessible; it is to be hoped that it will be more widely read.

This book is a paperback edition of a part of the first volume of *Principia Mathematica*; the whole of Part I and Section A of Part II are included, with Appendices A and C. This restriction on the size of the book appears to be a very reasonable compromise between weight, expense and usefulness.

R. M. DICKER

GÖDEL, K., *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*, translated by B. MELTZER with an Introduction by R. B. BRAITHWAITE (Oliver and Boyd, 1962), viii+72 pp., 12s. 6d.

This little book is one half a translation of Gödel's famous paper and the other half an introduction to this paper. The translation itself is very good; the symbolism

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of the original text is retained and the original page numbers are inserted in the margin, so that reference to the original is easy.

The Introduction by Professor Braithwaite is almost as long as Gödel's paper; it is accurately and precisely written, primarily with the needs of a philosophical logician in mind. The Introduction begins by explaining the state of our knowledge of metamathematics at the time when Gödel's paper was written, and this is followed by accounts of arithmetisation, recursiveness, the " unprovability " theorem, and consistency. The final section deals with the syntactical character of Gödel's theorems.

There are accounts of Gödel's theorems in a number of books on mathematical logic. Thus this book will be valuable to mathematicians as a readily accessible source of the original paper. The Introduction by Professor Braithwaite is an account of Gödel's paper from a philosopher's point of view. The reviewer's reaction to this is perhaps what one would expect of a mathematician. The Introduction (especially the first, second, and final sections) is clear and informative, but in places one feels that a more mathematical approach is preferable.

R. M. DICKER

BURKILL, J. C., *A First Course in Mathematical Analysis* (Cambridge University Press, 1962), vi+186 pp., 22s. 6d.

This course is based on the idea of a limit and is intended for students who already have a working knowledge of the calculus. The analytical treatment of the calculus includes chapters on the Riemann integral and on differentiation of functions of several variables, but not on multiple integrals; the chapters on sequences and series exclude uniform convergence, upper and lower limits, and the general principle of convergence. There are examples after each section, and notes on these are given at the end of the book.

The author makes skilful use of informal explanations, and in general the ideas are presented extremely clearly and at a well chosen level. Thus the discussion of real numbers in the first chapter leads in a natural way to a statement of Dedekind's theorem as an axiom, and this is used to prove the existence of the supremum and infimum of a bounded set. More than average ability is necessary to follow the proofs of theorems, however, as steps have sometimes been omitted which might well have been included in an elementary textbook. Occasionally there is a serious gap; for example, the author does not prove that a convergent sequence is bounded. (This result is *assumed* on page 30; after the discussion of the sequence $1/(n-10)$ on page 24, the reader may doubt whether it is *true*).

These reservations are small, however, and it is a pleasure to be able to welcome a book on analysis written by an author who has a sense of style and who avoids the excessive use of symbolism which can make the subject unnecessarily difficult for the student.

P. HEYWOOD

DITKIN, V. A. AND PRUDNIKOV, A. P., *Operational Calculus in Two Variables and its Applications*, translated by D. M. G. WISHART (Pergamon Press, 1962), x+167 pp., 50s.

This book contains an account of the operational calculus in two variables based on the two-dimensional Laplace Transform,

$$F(p, q;\ a, b) = \int_0^a \int_0^b e^{-px-qy} f(x, y)\,dx\,dy.$$

The authors divide the text up into two distinct parts—the first part containing the

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