Proceedings of the Edinburgh Mathematical Society (1990) 33, 497–498 ©

BOOK REVIEWS

PISIER, G., *The volume of convex bodies and Banach space theory* (Cambridge Tracts in Mathematics 94, Cambridge University Press, 1989), pp. 250, 0 521 364655, £30.

This excellent book contains a full account of recent deep results on the geometry of finite dimensional normed spaces that are related to the volumes of convex bodies. Until a decade ago there were only a few beautiful classical results in this area of mathematics; since then several major theorems have been proved. The author gives a detailed account of some of the classical and recent theorems relating volumes of convex bodies to geometrical properties. Methods and ideas from geometry, probability theory, approximation theory, and the local theory of Banach spaces all play a role. The following technical concepts are introduced, carefully defined and used: maximal volume ellipsoids in convex sets in $\mathbb{R}^n$, Gaussian processes in finite dimensional spaces, entropy and approximation numbers of operators, and the type and cotype of normed spaces. Some of the most important results in the later chapters are proved in more than one way, to illustrate different methods and show how the results interlink.

However, these notes are more than a monograph on recent deep research. In the earlier chapters several related classical results are proved in detail with simpler modern proofs where these are known. The Brunn–Minkowski Theorem [H. Brunn (1887), H. Minkowski (1910)] is proved using K. Ball's approach to the problem (1986), and the isoperimetric and Urysohn's inequalities are deduced from it; modern forms of F. John's Theorem (1948) on the uniqueness of the maximal volume ellipsoid in a finite dimensional convex set are proved; and Dvoretzky's Theorem (1961) on almost ellipsoidal sections of convex bodies is proved using Gaussian processes. Inequalities on the volume ratio are proved, from which it is deduced that there are two closed linear subspaces X and Y of the usual L^2-space $L^2[0,1]$ whose direct sum is $L^2[0,1]$ and on X and Y the L^1 and L^2 norms are equivalent. This much is covered in the first one hundred pages as background and motivation for the main results; it is this part of the book which will be of most interest to the general reader. Experts in Banach space theory will be more concerned with the subsequent sections, where recent theorems on Milman's ellipsoids, volume number and weak type 2 and cotype 2 are covered. There is little point in discussing the technicalities of these sections here as most of those in Banach space theory will soon have the book anyway.

The book is as well written as the author's previous lecture notes, *Factorization of linear operators and the geometry of Banach spaces* (CBMS Regional Conference Series No. 60, AMS 1986), but is more technical because of the nature of the material. The calculations are done in sufficient detail for easy reading without too many steps, and the background theory is developed, or sketched. The most important theorems, classical and modern, are each in a separate chapter with their corollaries and closely related theory. Each chapter ends with brief notes and remarks giving an account of the development of the results and detailed references. There is a bibliography and index. This book is essential for those who wish to follow recent progress in Banach space theory, and it will probably be as influential as the author's previous notes.

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