COMET BRIGHTNESS PARAMETERS: DEFINITION, DETERMINATION, AND CORRELATIONS

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Introduction

Visual estimates of comet total magnitude have been made for well over one hundred years. In this paper no attempt has been made to review all previous work on comet magnitudes. Instead we prefer to concentrate on developing a conceptual framework upon which previous work can be evaluated. We have tried to unify the approach as much as possible by filling in gaps that occur between previously published accounts. The present work represents an extension and revision of an earlier attempt at understanding comet magnitudes (Meisel, 1970).

Part I. Comet Brightness Formulae

Comet total brightness (luminosity) is usually defined by a power-law formula

$$B_{☄} = B_0 r^{-n} \Delta^{-2}$$

which can be directly converted to an expression using stellar magnitudes.

$$m = m_0 + 2.5n \log r + 5 \log \Delta \quad (1)$$

where m = total apparent comet magnitude, r = the comet heliocentric distance, Δ = the comet geocentric distance, n = the parameter "index of variation" (n = 2 for pure reflection), and m_0 is the unit or "absolute" magnitude of the comet. Least-squares solutions of the power-law formula occur throughout astronomical literature in numbers far too numerous to mention explicitly here.

Levin (1943) proposed an alternative formula originally based on the desorption of gases,

$$m = A + B\sqrt{r} + 5 \log \Delta \quad (2)$$

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While desorption processes are no longer considered relevant to the comet problem, this formula can be used for interpolation purposes and Bobrovnikoff (1951) and Meisel (1970) have shown the conditions under which expression (2) converges to (1). Oort and Schmidt (1951) used the Levin formula in an attempt to distinguish photometrically between "old" and "new" comets. Because solutions using (2) appear from time-to-time in the literature, we have developed a formalism to convert the parameters of such solutions to m_0 and n sets.

First, note that (1) can be written (ignoring the geocentric variation) as

$$m = m_0 + 2.5 \times 0.43 \times n \times \ln r$$

which is the integral $m = m_0 + \int_1^r \frac{\partial m}{\partial r} dr$. Then (2) can be written as

$$m = A + B\sqrt{r}$$

which is the integral $m = A + \int_q^r \frac{\partial m}{\partial r} dr$. Comparing the integral expressions

$$m_0 = A + \int_q^1 \frac{\partial m}{\partial r} dr \text{ and } m = A + \int_q^1 \frac{\partial m}{\partial r} dr + \int_1^r \frac{\partial m}{\partial r} dr$$

These imply that $m_0 = A + B$ and

$$B = \int_q^1 \frac{\partial m}{\partial r} \quad \text{with} \quad \int_1^r \frac{\partial m}{\partial r} dr = \sqrt{r} \int_q^1 \frac{\partial m}{\partial r} dr$$

by formal definition. The above expression predicts that B (and by implication n and m_0) is q dependent in accord with the empirical findings of Oort and Schmidt (1951). By differentiation we obtain

$$n = \frac{r}{2.5 \times 0.43} \frac{d}{dr} \left[\sqrt{r} \int_q^1 \frac{\partial m}{\partial r} dr \right]$$

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Taking average values for n and B gives the direct variable transformation

$$n = 0.43\ B \left\langle \sqrt{r} \right\rangle$$

while $m_o = A + B$ as the formal transformation equations.

In the case of an elliptical orbit the computation of $\left\langle \sqrt{r} \right\rangle$ can be a problem. In the case of a parabola, the expressions are complicated, but straightforward. We have adopted the parabolic assumption for our parameter conversion (because of its simplicity) even in the case of most periodic comets. The error involved is largest for orbits of small eccentricity at aphelia. Even for Comet Encke, the worst case in our list, the maximum possible error in $\left\langle \sqrt{r} \right\rangle$ is only 25%. In the appendix, the average $\sqrt{r}$ expressions are given for a parabolic orbit. Exact conversion of least-squares $m = A + B\sqrt{r}$ and $m = m_o + 2.5n \log r$ solutions is also given, but the additional complexity of the exact conversion does not appear to be necessary at least for the several test cases that have been investigated.

While the physical reasons for originally adopting the Levin formula are invalid in the light of modern research, expression (2) can be useful for avoiding the mathematical singularity encountered with least-squares solutions using the power-law formula when $r \to 1$ A.U. Because the association of Levin's name with expression (2) sometimes connotes a physical interpretation in terms of adsorption, we suggest that the notion of a $\sqrt{r}$ variation be dropped and near r = 1 A.U. a generalized <u>series formula</u> be adopted to avoid the singularity: $m = C + Dr^{n_o}$ where C and D are found by least-squares assuming a value of n_o. The final C, D values are found by trial values of n_o until minimum solution residuals are obtained. The usual (m_o,n) set can then be compared with the corresponding transformed parameter set defined as $n_1 = 0.4\ n_oD$ and $m_1 = C + 2.5(n_1/n_o)$ to see if solution diver-

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gence due to the small log r values and fluctuations in the magnitude data (either real or observational) are present. The possibility of this type of solution divergence is obviously greatest for objects which have $q \to 1$ and the power-law parameters derived when $r \to 1$ and/or $q \to 1$ should always be suspect. Divergences due to small ranges in log r values may also be present and these are not as easily identified in a consistent manner. However, solutions based on observations made over a short time period should always be considered less certain.

Early work by Bobrovnikoff (1941a, 1941b) showed the necessity of investigating the possibility of instrumental systematic effects thoroughly before applying expression (1). Later Öpik (1963) proposed a modification of the usual power-law formula (1) which attempted to correct explicitly for instrumental effects.

$$m = m_o - 2.5(s-2) \log(D/67.8) + 2.5s \log \Delta + 2.5n \log r \quad (3)$$

where D is the telescope diameter (in millimeters) and s is the index of variation of brightness within the comet coma such that the comet surface brightness has a radial dependence $B(\rho) = B_o\rho^{-s}$ where ρ is the projected distance from the comet central condensation. When s = 2, the Öpik formula (3) reduces to (1). In a previous investigation (Meisel, 1970) with two comets, an attempt was made to justify empirically (3), but this failed presumably because $s \to 2$ for both objects. Delsemme (1973a) and O'Dell and Osterbrock (1962) have cited many reasons for adopting an exponential decay model to describe a comet coma. Haser (1957) investigated this model in some detail with the result that no single value of s can describe the entire coma. In view of the lack of theoretical as well as empirical justification for the Öpik formula, continued use of expression (3) only compounds the difficulty of interpreting the derived comet photometric parameters.

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We believe that expression (1) still represents the best approximation to visual comet brightness behavior. Bobrovnikoff's method of comparison and reduction appears to give consistent results even when reflecting telescopes are used although the mean aperture correction for reflectors has been shown by Morris (1973a) to be less than Bobrovnikoff's value for refractors. Meisel's (1970) earlier work suggested aperture corrections result from clipping of the object spatial frequencies by telescope apertures, but only for certain radial coma brightness distributions will analytical expressions be obtained. In an equivalent analysis for the fixed field stop case, Delsemme (1973a) has derived expressions which take into account the aperture effect without the need for correction of individual observations.

We have investigated the possibility that Delsemme's theory might be applicable to photometric solutions derived from visual magnitude estimates where the size of the effective field stop is not predetermined. In the case of solutions based on estimates made with a single instrument such as those given by Beyer, the connection with the fixed field stop theory is straightforward and empirical systematic corrections can be applied to m_0 and n with confidence as will be done later for the Beyer data. In the cases where a variety of instruments and apertures have been used, averaging in Fourier transform space must be carried out before the Delsemme results can be applied. For two well-studied cases (Meisel, 1970) the preliminary results of a direct conversion using Delsemme's theory and an inversion of the statistical distribution of apertures shows no significant advantage of a direct correction of (m_0,n) values based on visual magnitude estimates either statistically or computationally. At the present time we see no reason to abandon the simpler procedure of instrumental correction using linear aperture correlations prior to least-squares solution in favor of a

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more direct approach. Only if a significant improvement in accuracy of the individual visual magnitude estimates could be made would the complicated Fourier inversion procedure be worthwhile. One final point should be noted about comet brightness formulae. Over the past several decades there have been numerous attempts at interpreting the (m_0,n) parameters in terms of unique physical processes. However, if evaporative processes predominate in comet gas production as argued by Delsemme (1973c) and Huebner (1965) such attempts are largely futile since the number of possible mechanisms is much greater than the number of distinct parameters which can be determined empirically from visual magnitude estimates. [Recall the difficulty of deriving Öpik's "s" parameters directly from observation. (Meisel, 1970)] Traditional interpretations of n have centered around two mechanisms--fluorescence ($n \rightarrow 4$) and dust reflection ($n \rightarrow 2$). But it is quite clear that many other influences may be involved and until some means of establishing the possible heliocentric variations of these other mechanisms is available no physical interpretation of n (or even m_0) should be attempted. All that can be concluded at this point is that the n coefficient somehow characterizes an unknown combination of the following physical processes.

(a) Gas Evaporation Rate
(b) Dust Production Rate
(c) Dust Destruction Rate
(d) Parent Molecular Dissociation Rates
(e) Daughter Molecular Dissociation Rates
(f) Fluorescence $\propto r^{-4}$
(g) Dust Reflection $\propto r^{-2}$
(h) Gas and Dust Velocity Fields

How m_0,n values relate to these various processes is a topic for future investigations.

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Part II. Treatment of Instrumental Effects

There are three main methods of comet-star comparison in the literature. We summarize these here:

(a) "In-Out Method" - Sidgwick (1955)

[Memorize to compare focused comet and out-of-focus star]

(b) "Bobrovnikoff Method" - Bobrovnikoff (1941a, 1941b)

[Compare out-of-focus star with (same size) out-of-focus comet image and apply empirical aperture corrections]

(c) "Beyer Method" - Beyer (1952)

[Observe relative extinction of grossly out-of-focus star and comet images]

The Sidgwick method requires considerable skill unless binoculars with individual focus mounts are available. This method can have systematic effects if aperture corrections are ignored. The Bobrovnikoff method is the easiest to do consistently for relatively inexperienced observers. Always requires "aperture" corrections when comparisons between different instruments are made. The Beyer method is quite sensitive to sky background illumination. As shown later this method leads to systematic effects unless aperture corrections are applied.

Bobrovnikoff (1941a, 1941b) first introduced the notion of systematic "aperture" corrections in a purely empirical way. Meisel (1970) has demonstrated that the Bobrovnikoff and Sidgwick extrafocal comparison methods produce a flux mismatch in the focal plane. Furthermore it was demonstrated that this mismatch is really an effect of focal ratio. However, since focal length and aperture are frequently correlated, Bobrovnikoff's empirical use of aperture as the correlation parameter can be justified. Using numerous

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visual observations, Morris (1973a) has demonstrated a definite difference of mean aperture correction between reflecting and refracting telescopes for the Bobrovnikoff method. It is straightforward (but tedious) to show that the Bobrovnikoff method of equal image-size comparison gives the smallest possible aperture correction for a given optical configuration. Only when the star and comet are put out-of-focus with the same apparent size are the instrument entrance and exit pupils in a maximum flux transmitting configuration. The subject of aperture corrections has been very controversial and Bobrovnikoff's work criticized. But his investigation along with that by Morris (1973a) has shown the persistence of the correlation. It is easy to forget that in optical imagery one is dealing with diffraction patterns which involve Fourier transforms of apertures and not the apertures themselves. It is this lack of understanding of the image formation process that has made acceptance of instrumental corrections in visual comet photometry very slow. We therefore digress to present the following theoretical development outlining the nature of the problem.

First we define a function ψ which gives the comet/star flux ratio in the instrument focal plane.

$$\psi = \frac{\int_0^\infty r' I_f^{\text{☄}}(r')_g \, dr'}{\int_0^\infty r' I_f^{\star}(r')_g \, dr'}$$

where r' is the radial coordinate in a plane perpendicular to the optical axis, the f subscript indicates the intensities are those in the focal plane. The subscript g indicates that these are geometrical optics projections of the objects.

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If the intensities all have circular symmetry we may use Fourier-Bessel (F-B) transforms defined as

$$B\Big(I(r')\Big) = G(\rho') = 2\pi \int_0^\infty r' I(r') J_0(2\pi r'\rho')dr'$$

and

$$I(r') = B\Big(G(\rho')\Big) = 2\pi \int_0^\infty \rho' G(\rho) J_0(2\pi r'\rho')d\rho'$$

where ρ' is the spatial frequency. Thus ψ becomes

$$\psi = \frac{\int_0^\infty r' \left[\int_0^\infty \rho' G^{\#}(\rho)_g J_0(2\pi r'\rho')d\rho'\right] dr'}{\int_0^\infty r' \left[\int_0^\infty \rho' G^{*}(\rho)_g J_0(2\pi r'\rho')d\rho'\right] dr'}$$

The relationships between the geometrical $G(\rho)_g$ and the instrumental $G(\rho)_i$ in the focal plane are given by

$$G^{\#}(\rho)_i = H_0(\rho) G^{\#}(\rho)_g$$

and

$$G^{*}(\rho)_i = H_0(\rho) G^{*}(\rho)_g$$

Here $H_0(\rho)$ is the so-called optical transfer function (OTF) of the instrument. Since a star is a point source, we assume by definition that $G^{*}(\rho)_g = P$ and

$$H_0(\rho) = G^{*}(\rho)_i/P$$

where P is a scalar. Then ψ becomes

$$\psi_f = \frac{\int_0^\infty r' \left[\int_0^\infty \rho' G^{\#}(\rho)_g J_0(2\pi r'\rho')d\rho'\right] dr'}{P \int_0^\infty r' \left[\int_0^\infty \rho' J_0(2\pi r'\rho')d\rho'\right] dr'}$$

and with $G^{\#}(\rho)_g = \Big(G^{\#}(\rho)_i/G^{*}(\rho)_i\Big) P$, we finally obtain

$$\psi_f = \int_0^\infty r' \left[\int_0^\infty \rho' \Big(G^{\#}(\rho')_i/G^{*}(\rho')_i\Big)_f J_0(2\pi r'\rho')d\rho'\right] dr'$$

In the cases of real optical systems there is always band limiting in ρ'

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such that

$$\psi_f = \int_0^{r'_L} \left[\int_0^{\rho'_0} \rho' \left(G^{\text{☄}}(\rho')_i / G^*(\rho')_i \right)_f J_0(2\pi r'\rho')d\rho' \right] dr'$$

where $\rho'_0 = R/\lambda f$ with R = aperture, λ = wavelength and f = focal length.

Each method has its own criteria for determining a match between star and comet.

(a) Sidgwick Method

$$\int_0^{r'_0} r' I^*_{\Delta f_1}(r')dr' = \int_0^{r'_0} r' I^{\text{☄}}_f(r')dr'$$

(b) Bobrovnikoff Method

$$\int_0^{r_1} r' I^*_{\Delta f_2}(r')dr' = \int_0^{r_1} r' I^{\text{☄}}_{\Delta f_2}(r')dr'$$

(c) Beyer Method

$$\int_0^{R_L} r' I_{sky}(r')dr' = \int_0^{R_L} r' I^*_{\Delta f_3}(r')dr' = \int_0^{R_L} r' I^{\text{☄}}_{\Delta f_3}(r')dr'$$

Note that Δf_1, Δf_2 and Δf_3 are such that

$$0 \le \Delta f_1 \le \Delta f_2 \le \Delta f_3$$

In the focal plane, we have the spectral ratio

$$\Phi_f(\rho') = \left(G^{\text{☄}}(\rho')_i / G^*(\rho')_i \right)_f$$

If the star and comet are both thrown out of focus, then the spectral ratio is different.

$$\Phi_{\Delta f}(\rho') = \left(G^{\text{☄}}(\rho')_i / G^*(\rho')_i \right)_{\Delta f}$$

The defocusing process is one which attempts to make the comet image <u>identical</u> in appearance to that of the star. Thus in <u>both</u> the Beyer and Bobrovnikoff method, the aim is to make

$$\Phi(\rho')_{\Delta f} = \left(G^{\text{☄}}(\rho')_i / G^*(\rho')_i \right)_{\Delta f} \Rightarrow \text{constant for all } \rho'$$

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If the match is to be <u>perfect</u> then

$$\Phi(\rho')_{\Delta f} = 1 \quad \text{for all } \rho' \quad 0 \text{ to } \infty.$$

This condition then requires the "defocus" function $\sigma(\Delta f,f)$ to be such that

$$G^{\mathscr{G}}(\rho')\Delta f = \sigma(\Delta f,f)G^{\mathscr{G}}(\rho')f$$

and

$$G^{*}(\rho')\Delta f = \sigma^{-1}(\Delta f,f)G^{*}(\rho')f$$

These imply that

$$\left(\frac{G^{\mathscr{G}}(\rho')\Delta f}{G^{*}(\rho')\Delta f}\right) = \sigma^{2}(\Delta f,f)\left(\frac{G^{\mathscr{G}}(\rho')f}{G^{*}(\rho')f}\right)$$

It can be shown [Goodman (1968)] that a misfocused system requires as a first approximation

$$\sigma^{2} = \exp\left[-2\pi j \left(\frac{\Delta f}{f}\right) \lambda(\rho')^{2}\right]$$

Because this involves $\Delta f(\rho')^2$ it can be seen that as Δf increases there will be a corresponding decrease in the system bandwidth.

Thus while there is an advantageous degree of spectral smearing in extrafocal methods of comparison (i.e., comet and star can be smoothed to look identical), the further one goes out-of-focus, the more the effective system bandwidth is cut. It is this decrease of effective system bandwidth which is responsible for the net "aperture" effect in extrafocal comparisons. Since the Bobrovnikoff method requires the <u>least</u> amount of Δf for a given focal ratio, it will always have the smallest instrumental correction.

It also should be noted that objects which have different brightness profiles will have different Δf distances before the star and comet match can be made. However, because the Bobrovnikoff method has minimum Δf, it will also display minimum sensitivity to the effective s parameter (exponent of the change of coma brightness with distance from the nucleus).

Explicit proofs for the above are too involved to give here, but the

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lesson of the above discussion is clear. IN ANY EXTRAFOCAL METHOD OF COMPARISON, SYSTEMATIC EFFECTS ARE MINIMIZED IF IMAGES ARE THROWN OUT-OF-FOCUS BY THE LEAST AMOUNT NECESSARY TO MAKE THE EXTENDED OBJECT LOOK SIMILAR TO THE STAR. In all the methods of extrafocal comparison there is an approximation of

$$\left.\frac{G^{\mathscr{G}}(\rho')\Delta f}{G^{*}(\rho')\Delta f}\right]_{\rho_0'} \quad \text{by} \quad \frac{\left.G^{\mathscr{G}}(\rho')f\right]_{\rho_0'}}{\left.G^{*}(\rho')\Delta f\right]_{\rho_i'}}$$

where

$$\rho_i' = R/\lambda(f + \Delta f_i)$$

The instrumental corrections thus depend inversely on two ratios

(a) f/R = 2 x focal ratio

(b) $\Delta f/R$ = "defocus" ratio

For a given optical system $\Delta f \simeq kf$ and hence

$$\rho_i' \simeq R/\lambda f(1 + k)$$

In Bobrovnikoff's scheme the lowest possible eyepiece magnification is recommended. If the magnification is below a certain critical amount, the focal ratio will be determined by the pupil of the eye not the aperture of the telescope. At the critical magnification there is a perfect flux match often referred to as the "richest-field" condition. A suitable descriptive parameter Z is obtained by normalizing to this condition (assuming the pupil of the dark-adapted eye is 7.6 mm)

$$Z = (\rho_i'/\rho_k') \simeq \frac{(\text{f-ratio})\ \text{minimum}\ (1 + k\ \text{min})}{(\text{f-ratio})\ \text{actual}\ \ (1 + k)}$$

upon substitution

$$Z = 0.13\ (1 + k\ \text{min})\ D\ (\text{min})/(1 + k)M$$

where M = instrument magnification. To a sufficient approximation, $(1 + k'/M) = 1+k$ and as $M \to 0$ we have $(1 + k) \to M^{-1}$ with the result that $Z \propto D$.

Thus for instruments used visually the minimal defocusing process effectively renders the appropriate parameter to be the aperture <u>alone</u>. THIS IS IN

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ACCORD WITH FINDINGS OF BOBROVNIKOFF AND DEMONSTRATES WHY IT IS VALID TO USE APERTURE CORRECTIONS WHEN DISCUSSING VISUAL MAGNITUDE ESTIMATES.

Since the above derivation does not depend explicitly on the method of extrafocal comparison used, we conclude that no method will be free from aperture effects.

It is important to remember that the aperture correlation only applies to a fixed exit pupil situation near the focal plane. For photographic extrafocal magnitudes the appropriate correlation parameter is focal ratio, not aperture.

Obtaining analytical expressions for aperture corrections is difficult. In all realistic cases of interest, numerical convolutions must be carried out. However, the mathematical procedures are simplified if we take $B = B_0 \rho^{-s}$ (as first proposed by Öpik) as a zero order approximation. Under that assumption (even if it is a bit unrealistic), it can be shown that the aperture effects of the three principal methods of obtaining comet magnitude are simply related.

(a) Slopes of Linear Aperture Correlation

$$\left(\frac{\Delta m}{\Delta D}\right)_{\text{In-Out}} \simeq 2s+1 \left(\frac{\Delta m}{\Delta D}\right)_{\text{Bobrovnikoff}}$$

$$\left(\frac{\Delta m}{\Delta D}\right)_{\text{Beyer}} \simeq 2s \left(\frac{\Delta m}{\Delta D}\right)_{\text{Bobrovnikoff}}$$

where s = index of the radial brightness relation.

(b) Intercepts of Linear Aperture Correction

(D_0) In-Out $\simeq (2s+1) \times (67.8$ mm)

(D_0) Beyer $\simeq (2s) \times (67.8$mm)

These relationships, however, are of little use in practice because:

(a) Random errors contribute to wide scatter.

(b) The Bobrovnikoff aperture-effect parameters are sensitive to instrument type as well as aperture (Morris, 1973a).

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(c) It is not clear how to determine the effective s value to be used in these expressions since the validity of the Öpik formula has been questioned.

It is therefore simpler to derive empirical (mean) aperture corrections for each object when possible. Otherwise mean corrections for all available comets should be applied. If $s \simeq 2$ the above relations reduce to

$$\left(\frac{\Delta m}{\Delta D}\right)_{\text{In-Out}} \simeq 5x \left(\frac{\Delta m}{\Delta D}\right)_{\text{Bobrovnikoff}}$$

with $(D_0)_{\text{In-Out}} \simeq 340$ mm and

$$\left(\frac{\Delta m}{\Delta D}\right)_{\text{Beyer}} \simeq 4x \left(\frac{\Delta m}{\Delta D}\right)_{\text{Bobrovnikoff}}$$

with $(D_0)_{\text{Beyer}} \simeq 270$ mm.

It is therefore expected that on the average, Beyer method will lead to fainter estimates with "large" telescopes and brighter estimates with "small" telescopes compared with those made on the Bobrovnikoff system at the same heliocentric and geocentric distances. Therefore magnitude reductions using only those m's obtained with the Beyer method should give n values which are systematically higher than those obtained using the Bobrovnikoff system. With the proper aperture corrections, individual Beyer estimates could probably be reduced to the Bobrovnikoff system but such an approach for past observations is time-comsuming because of the need for re-doing the least-squares or graphical solutions. As will be shown later, however, the Beyer (m_o,n) values show systematic differences which enable mean corrections to be made without explicit derivation of aperture correlations. Such a systematic effect follows directly from the above discussion since Beyer used essentially the same instruments for all the estimates upon which his solutions are based.

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Part III. Lists of Photometric Parameters

We have been able to locate 150 separate sets of comet brightness parameters that appear to be on (or convertible to) a common photometric system. Prior to the year 1963, we have drawn from the lists of Bobrovnikoff (1941a, 1941b), Beyer (1970, 1972) and Schmidt (1951). After 1963, both published and unpublished values by Morris, Meisel, Bortle, Minton, and Beyer have been used. Each comet appearance has been listed separately regardless of whether the sightings represent a reappearance of the same object or not. All values given as Levin (A,B) sets have been converted using the parabolic conversion equations listed in the appendix. The original data have been separated into three categories -- (a) solutions where pre-perihelion observations dominate (Table I); (b) solutions where perihelion falls in the middle of the observational period (Table III); and (c) solutions where post-perihelion observations predominate (Table II). In our lists we give the comet designation, its perihelion distance, the appropriate m_0 and n values, the number of observations upon which the solution is based, the span of the observation period in months, the mean sunspot number over that observation period, and notes giving the source of the solution, the Oort-Schmidt orbit classification, and possible solution divergence. Finally a solution weight defined as the product of the time span in months and the number of points is given as a rough guide to the likelihood that the (m_0,n) are characteristic of the comet behavior.

Although it is difficult to distinguish between "normal" and "abnormal" brightness behavior, cases where it is obvious that observational bias or intrinsic brightness flares (or fading) have rendered the solution completely unreliable are omitted. In spite of this prior screening there

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however may be certain solutions where spurious values have gone unrecognized.

Instrumental corrections are known to have been applied before trying a least-squares or graphical solution for all but the Beyer values. Since the previous Fourier transform discussion of aperture effect suggests that the Beyer method leads to systematic effects, we have tested these for the available material by comparing means and standard deviations for the two groups.

Parameters	Beyer Values	Non-Beyer Values
$\langle n \rangle$	5.2 ± 2.5	3.7 ± 2.1
$\langle m_0 \rangle$	6.9 ± 2.5	6.3 ± 1.9
$\langle q \rangle$	1.1 ± 0.7 A.U.	1.0 ± 0.7 A.U.
Object Nos	67	83

The $\langle n \rangle$ difference is significant at a 99.9% level and the $\langle m_0 \rangle$ difference is significant at an 85% level. WE THEREFORE ADJUSTED THE ORIGINAL BEYER DATA BY $\Delta m_0 = -0.6$ and $\Delta n = -1.5$ for use in the statistical discussions. However, both the original and adjusted values have been listed.

The limitations of the mean correction to individual Beyer data is illustrated by the entry for Comet 1968I in Table II where both the Bortle-Morris solution and the Beyer solution are given. The n value differences for this one comet in common are in agreement with the mean adjustment relation but the m_0 values do not agree very well. In several other cases where direct comparisons can be made (but where the Beyer solutions have not been included in our lists because of relatively low precision), the systematic tendency of Beyer's n values to be too high persists, but the m_0 negative correction does not. Thus while we are confident that the

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n correction is generally valid, the m_o correction term needs to be investigated further as indicated by the somewhat lower confidence level (85%) found for the $\langle m_o \rangle$ differences.

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LIST OF VISUAL BRIGHTNESS PARAMETERS

Table I. Pre-Perihelion Dominated Solutions
(Original Beyer Data in Parentheses)

Comet	q(A.U.)	m_0	n	N	Δt (mos.)	$\hat{R}$	Wt	Notes
1858 VI	.58	3.39	3.49	25	1	64.3	25	B, O
1874 III	.68	6.24	4.78	48	1	66.8	48	B, O
1882 I	.06	7.6	2.9	13	1	59.4	13	S, N
1884I	.78	5.21	3.13	103	5	72.9	515	B, O
1886 II	.48	6.66	2.05	76	3	31.1	228	B, N
1886 IX	.66	4.79	2.63	27	2	14.2	54	B, N
1902 III	.40	6.77	2.63	89	2	7.4	178	B, N
1903 IV	.33	6.49	2.38	128	2	24.8	256	B, N(P)
1908 III	.95	4.00	5.00	109	3	51.7	327	B, N, d
1911 VI	.79	6.31	3.55	81	1	4.5	81	B, O
1915 II	1.00	5.65	1.66	58	4	43.3	232	B, N, d
1919 III	.48	10.44	5.76	64	1	62.3	64	B, O
1919 V	1.12	10.8	6.6	6	3	60.5	18	S, N, d
1925 III	1.63	5.9	2.0	3	3	57.3	9	S, O
1930 II	.67	8.34	4.27	53	1	55.3	53	B, N
1932 IX	1.62	7.5 (8.1)	3.0 (5.5)	9	1	11.9	9	Be, O
1937 VI	.33	9.4 (9.96)	4.5 (5.95)	10	4	110.8	40	Be, (Encke) O
1941 I	.37	5.2 (5.81)	0.5 (1.99)	32	2	58.6	64	Be, O
1947 XI	.34	9.3 (9.90)	4.8 (6.32)	20	1	145.6	20	Be, (Encke) O
1951 III	.34	9.2 (9.83)	1.2 (2.73)	15	2	70.6	30	Be, O
1952 VI	1.20	8.3 (8.87)	4.2 (5.68)	28	1	29.5	28	Be, N, d
1954 VII	.77	4.1 (4.66)	2.8 (4.33)	76	6	6.6	456	Be, O
1954 X	.97	5.3 (5.86)	2.1 (3.65)	76	6	4.7	456	Be, N(P)
1956 IV	1.18	4.1 (4.68)	4.9 (6.38)	46	5	118.2	230	Be, O
1959 VIII	.94	9.6 (10.18)	7.9 (9.43)	28	2	143.7	56	Be, O
1960 II	.50	7.2 (7.78)	2.3 (3.80)	37	4	122.4	148	Be, N
1960 III	1.20	7.2 (7.83)	9.1 (10.56)	24	4	122.4	96	Be, O, d(hn)
1961 I	.34	9.6 (10.19)	2.0 (3.52)	14	1	53.5	14	Be, O
1962 III	.03	5.6 (6.24)	0.8 (2.30)	11	1	45.4	11	Be, N(P)
1962 V	1.12	10.7 (11.31)	8.8 (10.32)	35	3	43.5	105	Be, O, d(hn)

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Table I. (cont.) Pre-Perihelion Dominated Solutions (Original Beyer Data in Parentheses)

Comet	q(A.U.)	m_0	n	N	Δt (mos.)	$\hat{R}$	Wt	Notes
1962 VIII	2.13	1.5 (2.14)	2.0 (3.49)	51	11	42.0	561	Be, O
1963 I	.63	5.7 (6.29)	3.2 (4.70)	20	2	42.7	40	Be, O
1965 VIII	.01	6.10	3.28	59	1	18.8	59	Milon, Solberg Minton O
1967 II	.42	9.44	2.53	18	2	88.3	36	Bortle, N(P)
1969 VII	.77	7.70	3.75	11	1	86.3	11	Bortle, O
1970 II	.54	5.41	4.31	20	2	115.6	40	Bortle, O
1970 ℓ	.34	9.75	4.23	10	1	83.8	10	Bortle, Encke, O
1973 f	.14	5.37	2.52	63	3	34.0	189	Morris & Bortle, N

B = Bobrovnikoff, Be = Beyer, S = Schmidt, N = New comet, O = old comet, N(P) = parabolic comet, d = solution divergence possible, d(hn) = solution divergence, high n, d(ln) = solution divergence, low n, hn = high n group, ln = low n group.

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LIST OF VISUAL BRIGHTNESS PARAMETERS

Table II. Post-Perihelion Dominated Solutions

Comet	q(A.U.)	m_0	n	N	Δt (mos.)	$\hat{R}$	Wt	Notes
1853 III	.31	5.7	3.3	2	5	26.1	10	S, N
1858 VI	58	4.3	4.5	21	1	67.6	21	B, O
1861 I	.92	6.5	11.7	4	5	79.0	20	S, O, d(hn)
1861 II	.82	5.08	0.47	66	2	77.0	132	B, O, ln
1862 III	.96	5.35	8.63	80	2	58.0	160	B, O, d(hn)
1881 III	.77	5.65	2.40	106	4	56.7	424	B, O
1882 II	.01	0.8	3.2	3	5	56.8	15	S, O
1890 II	1.91	5.47	2.55	30	8	11.1	240	B, N
1892 I	1.03	3.3	1.9	2	10	73.8	20	S, N, d(ln)
1893 II	.68	6.42	2.24	35	1	85.7	35	B, N
1893 III	.67	6.2	2.8	4	4	85.6	16	S, O
1894 II	.98	5.8	7.4	3	2	79.3	6	S, O, d(hn)
1896 III	.57	9.0	5.1	11	2	43.8	22	S, N
1898 I	1.10	4.62	5.93	53	3	26.5	159	B, O, d(hn)
1898 VIII	2.28	4.3	4.9	3	7	17.4	21	S, O
1899 I	.33	6.49	3.77	56	4	13.5	224	B, N
1900 II	1.02	8.62	6.55	59	2	7.5	118	B, N, d(hn)
1904 I	2.71	3.36	3.45	146	8	46.2	1168	B, N
1905 IV	3.34	5.3	2.0	3	1	62.6	3	S, N
1906 VII	1.22	7.58	6.89	37	1	60.9	37	B, N, d(hn)
1907 I	2.05	6.9	2.2	2	11	55.2	22	S, N
1907 IV	.51	4.32	3.58	98	7	54.6	686	B, O
1910 I	.13	4.7	3.8	3	3	29.6	9	S, N(P)
1911 II	.69	7.90	4.14	58	1	5.3	58	B, O
1912 II	72	6.28	3.21	113	3	3.0	339	B, N
1913 IV	1.25	5.71	9.56	58	2	2.4	116	B, N, d(hn)
1915 II	1.00	6.19	2.99	20	3	55.2	60	B, N, d
1917 I	19	5.1	1.8	2	0.5	95.2	1	S, O
1922 II	2.26	7.6	0.2	2	10	6.3	20	S, N, ln
1925 I	1.10	5.88	3.28	39	2	44.0	78	B, N, d
1925 VII	1.57	5.8	1.6	4	7	63.2	28	S, N
1927 IV	3.68	4.3	2.2	23	48	83.3	1104	S, N
1930 III	.48	8.67	4.67	99	2	45.6	198	B, O
1930 IV	2.08	8.2	0.4	2	3	34.0	6	S, N, ln
1931 III	1.04	4.3	5.2	3	9	15.7	27	S, O
1932 I	1.26	9.3	2.8	3	1	12.1	3	S, O
1932 V	1.04	7.36	11.40	130	2	11.8	260	B, O, d(hn)
1932 VI	2.31	5.08	2.46	28	4	7.7	112	B, N
1932 X	1.31	8.92	2.09	47	2	8.1	94	B, O
1933 I	1.00	9.3	1.9	11	1	7.7	11	Be, N(P), d(ln)
		(9.92)	(3.39)					
1937 II	.62	10.21	3.74	32	2	119.1	64	B, O
1941 II	.94	10.3	0.64	14	1	54.7	14	Be, O*
		(10.86)	(2.14)					
1947 I	2.41	2.5	4.2	22	13	141.6	286	Be, N
		(3.12)	(5.71)					

*Badly placed

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Table II. Post-Perihelion Dominated Solutions
(cont.)

Comet	q(A.U.)	m_0	n	N	Δt (mos.)	$\hat{R}$	Wt	Notes
1948 I	.75	5.7 (6.31)	1.5 (2.97)	52	3	138.2	156	Be, N
1948 IV	.21	6.9 (7.54)	3.9 (5.39)	36	2	137.7	72	Be, N(P)
1948 V	2.11	3.8 (4.37)	2.9 (4.44)	132	12	139.2	1584	Be, N
1948 X	1.27	5.4 (6.01)	5.0 (6.48)	8	4	136.8	32	Be, O
1948 XI	.14	4.8 (5.36)	2.2 (3.66)	29	5	137.5	145	Be, N
1949 IV	2.06	5.0 (5.59)	4.0 (5.53)	79	9	118.3	711	Be,N
1950 I	2.55	4.3 (4.94)	3.9 (5.36)	33	4	102.7	132	Be, N*
1950 VII	1.39	8.3 (8.87)	7.1 (8.59)	11	1	69.5	11	Be, O*, hn
1951 II	.72	8.8 (9.40)	2.0 (3.47)	30	10	80.3	300	Be, N(P)
1951 IV	1.12	9.6 (10.23)	9.5 (11.05)	20	1	70.0	20	Be, O, d(hn)
1952 I	74	8.3 (8.88)	2.8 (4.33)	6	1	40.8	6	Be, N
1954 III	.56	12.2 (12.78)	4.6 (6.06)	11	1	4.2	11	Be, O
1954 VIII	.68	3.3 (3.94)	6.9 (8.42)	10	2	7.6	20	Be, N*, hn
1955 III	.54	6.3 (6.85)	3.7 (5.21)	13	1	35.1	13	Be, O
1955 IV	1.43	4.2 (4.79)	5.7 (7.24)	25	4	60.0	100	Be, O
1955 V	89	6.2 (6.85)	3.2 (4.67)	35	3	59.8	105	Be, N(P)
1956 IV	1.18	4.4 (5.02)	2.4 (3.93)	15	3	150.6	45	Be, O
1957 V	.36	3.0 (3.63)	0.7 (2.21)	23	2	198.4	46	Be, N
1958 III	1.32	6.2 (6.75)	5.8 (7.25)	27	2	191.7	54	Be,N
1959 I	1.63	6.5 (7.10)	3.4 (4.86)	46	6	174.8	276	Be, N
1959 VIII	.94	9.6 (10.18)	7.9 (9.43)	28	2	143.7	56	Be, O, hn

*Badly Placed

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Table II. Post-Perihelion Dominated Solutions
(cont.)

Comet	q(A.U.)	m_o	n	N	Δt (mos.)	$\hat{R}$	Wt	Notes
1961 II	1.06	5.9	7.5	21	4	48.5	84	Be, N(P), d(hn)
		(6.53)	(8.95)					
1961 V	.04	8.6	4.5	19	2	52.3	38	Be, N
		(9.19)	(6.02)					
1962 III	.03	4.5	2.1	15	1	44.0	15	Be, N(P)
		(5.12)	(3.55)					
1962 IV	.65	9.9	3.7	15	2	41.8	30	Be, N(P)
		(10.47)	(5.21)					
1963 I	.63	5.7	3.2	32	2	33.9	64	Be, O
		(5.36)	(3.56)					
1963 VIII	2.21	8.6	0.5	55	3	15.7	165	Be, O
		(9.23)	(2.00)					
1964 IX	1.26	5.7	4.5	63	4	8.1	252	Be, N(P), d
		(6.29)	(6.03)					
1965 VIII	01	6.43	3.63	55	3	24.0	165	Morris, O
1966 IV	.88	6.7	1.9	10	1	58.0	10	Be, N(P)
		(7.26)	(3.44)					
1968 I	1.70	3.87	4.85	243	9	107.7	2187	Morris & Bortle, N
1968 I	1.70	2.52	4.35	86	9	107.7	774	Be, N
		(3.12)	(5.85)					
1968 VII	1.77	7.5	1.2	17	2	107.1	34	Be, N(P)
		(8.08)	(2.74)					
1967 VII	.18	7.27	2.8	67	1	85.8	67	Meisel, N(P)
1969 IX	.47	5.8	1.6	29	5	117.4	145	Be, O
		(6.39)	(3.06)					
1970 II	.54	3.42	3.54	31	6	86.0	186	Bortle, O
1970 X	.41	7.9	3.0	8	2	88.0	16	Be, N(P)
		(8.49)	(4.48)					
1971a	1.23	6.67	4.12	66	5	62.6	330	Morris, N, d
1972d	.99	9.57	4.22	33	5	70.4	165	Morris, O, d
1973f	.14	6.47	2.51	120	3	26.6	360	Morris & Bortle, N

B = Bobrovnikoff, Be = Beyer, S = Schmidt, N = New comet, O = old comet, N(P) = parabolic comet, d = solution divergence possible, d(hn) = solution divergence, high n, d(ln) = solution divergence, low n, hn = high n group, ln = low n group.

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LIST OF VISUAL BRIGHTNESS PARAMETERS

Table III. Combined Pre- and Post-Perihelion Periods

Comet	q(A.U.)	m_0	n	N	Δt (mos.)	$\hat{R}$	Wt	Notes
1873 V	.38	7.2	4.9	3	3	54.3	9	S, N
1886 I	.64	8.1	5.4	14	8	30.7	112	S, N
1889 I	1.81	4.9	1.8	5	8	6.0	40	S, N
1889 II	2.26	8.2	0.3	2	5	6.3	10	S, N, ln
1898 VII	1.70	6.5	2.0	3	6	24.0	18	S, N
1907 IV	.51	4.32	3.58	98	7	54.6	686	B, O
1910 II	.59	5.70	3.71	254	8	27.5	2032	B, O (P/Halley)
1910 IV	1.95	7.1	1.6	2	9	12.4	18	S, N
1911 V	.49	5.60	3.43	466	5	4.3	2330	B, O
1914 V	1.10	1.78	3.50	260	3	16.3	780	B, N
1917 III	1.69	8.29	1.97	67	6	100.9	402	B, N
1921 II	1.01	6.94	5.53	96	2	28.1	192	B, N, d
1932 VIII	1.87	8.7	3.0	8	1	11.9	8	Be, O
		(9.3)	(5.5)					
1935 I	.81	9.81	2.88	51	3	22.4	153	B, O
1936 II	1.10	6.75	4.62	346	3	77.8	1038	B, O, d
1937 IV	1.73	6.18	3.25	121	7	115.8	847	B, N
1937 V	.86	6.20	0.72	349	2	111.7	698	B, N, ln
1941 VIII	.88	6.3	2.1	50	6	48.1	300	Be, N(P)
		(6.91)	(3.62)					
1943 I	1.35	4.6	1.4	494	6	19.4	2964	Be, O
		(5.22)	(2.93)					
1946 VI	1.14	4.4	2.3	19	9	111.7	171	Be, N, d
		(4.98)	(3.81)					
1951 I	2.57	6.6	0.0	35	11	74.6	385	Be, N
		(7.15)	(1.54)					
1952 III	1.19	6.8	5.1	57	5	37.7	285	Be, O, d
		(7.39)	(6.57)					
1953 I	1.67	-0.2	12.2	26	5	25.3	130	Be, N, hn
		(0.43)	(13.73)					
1953 III	1.02	7.0	9.0	7	1	17.2	7	Be, O, d(hn)
		(7.61)	(10.47)					
1955 VI	3.87	2.4	2.8	60	1	58.8	60	Be, N(P)*
		(3.05)	(4.33)					
1957 III	.32	4.6	2.8	60	10	181.8	600	Be, N
		(5.15)	(4.35)					
1966 V	2.39	1.3	5.2	40	3	47.2	120	Be, N
		(1.86)	(6.67)					
1968 IV	.68	10.7	4.1	10	1	121.5	10	Be, N(P)
		(11.28)	(5.56)					

*Badly placed

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Table III. Combined Pre- and Post-Perihelion Periods (cont.)

Comet	q(A.U.)	m_0	n	N	Δt (mos.)	$\hat{R}$	Wt	Notes
1968 VI	1.16	5.2 (5.78)	2.5 (3.98)	74	5	106.7	370	Be, N(P), d
1970 XV	1.11	5.0 (5.56)	1.1 (2.60)	61	4	97.3	244	Be, N(P), d

B = Bobrovnikoff, Be = Beyer, S = Schmidt, N = New comet, O = old comet, N(P) = parabolic comet, d = solution divergence possible, d(hn) = solution divergence, high n, d(ln) = solution divergence, low n, hn = high n group, ln = low n group.

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Part IV. Statistical Analysis of Comet Brightness Parameters

(a) Photometric Groups

Various authors have attempted to use visual brightness parameters to classify comet behavior. Oort and Schmidt (1951) and Oort (1951) established that there was a significant statistical difference in the photometric parameters for "old" and "new" comets as well as a noticeable perihelion distance correlation. Unfortunately their analysis was based on the erroneous Levin model. We have re-examined the available power-law parameters not only for a q correlation and the "old" and "new" comet distinction, but also for a possible statistical difference between pre-perihelion and post-perihelion parameters. In addition to these groupings we have also examined two others of possible significance--one group of unusually high n values and one group of unusually low n values. The mean and standard deviations characteristics of these groupings are summarized in Table IV. NOTE THAT THESE STATISTICS INCLUDE THE CORRECTED BEYER VALUES. If the uncorrected Beyer values are used instead, somewhat different means are obtained.

Comparison of the data in Table IV shows that grouping according to perihelion distance is even more significant than the "old" and "new" distinction. While the pre-perihelion/post-perihelion behavior of single comets may be very different, groupings according to period of visibility produces only slight changes in the parameter means.

The high n and low n groups appear to be quite distinctive when means are compared but it remains to be demonstrated that these groups are not simply the result of least-squares solution divergence.

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Table IV. Grouped Data Means and Standard Deviations of Comet Brightness Parameters

GROUP	MEANS AND STANDARD DEVIATIONS	GROUP	MEANS AND STANDARD DEVIATIONS
All (m_o,n) sets (N = 150)	$\langle n\rangle$ = 3.6 ± 2.3 $\langle m_o\rangle$ = 6.3 ± 2.2 $\langle q\rangle$ = 1.0 ± 0.7 A.U.	Pre-perihelion Dominant (N = 38)	$\langle n\rangle$ = 3.6 ± 2.0 $\langle m_o\rangle$ = 6.9 ± 2.2 $\langle q\rangle$ = 0.7 ± 0.5 A.U.
New Comets (N = 85)	$\langle n\rangle$ = 3.2 ± 2.0 $\langle m_o\rangle$ = 6.1 ± 2.0 $\langle q\rangle$ = 1.2 ± 0.8 A.U.	Post-perihelion Dominant (N = 82)	$\langle n\rangle$ = 3.9 ± 2.3 $\langle m_o\rangle$ = 6.3 ± 2.1 $\langle q\rangle$ = 1.1 ± 0.3 A.U.
Old Comets (N = 65)	$\langle n\rangle$ = 4.2 ± 2.5 $\langle m_o\rangle$ = 6.6 ± 2.5 $\langle q\rangle$ = 0.9 ± 0.5 A.U.	Both pre-and post-perihelion periods (N = 30)	$\langle n\rangle$ = 3.4 ± 2.4 $\langle m_o\rangle$ = 5.9 ± 2.1 $\langle q\rangle$ = 1.3 ± 0.8 A.U.
q > 1.25 A.U (N = 40)	$\langle n\rangle$ = 3.1 ± 2.2 $\langle m_o\rangle$ = 5.6 ± 2.3 $\langle q\rangle$ = 2.0 ± 0.6 A.U.	High n group (N = 13)	$\langle n\rangle$ = 8.7 ± 2.4 $\langle m_o\rangle$ = 6.6 ± 2 7 $\langle q\rangle$ = 1.1 ± 0.2 A.U.
q < 1.25 A.U. (N = 110)	$\langle n\rangle$ = 4.4 ± 2.7 $\langle m_o\rangle$ = 6.6 ± 2 1 $\langle q\rangle$ = 0.7 ± 0.4 A.U.	Low n group (N = 11)	$\langle n\rangle$ = 0.5 ± 0.2 $\langle m_o\rangle$ = 6.8 ± 1.9 $\langle q\rangle$ = 1.3 ± 0.9 A.U.

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(b) Residual Systematic Effects

The most striking differences due to grouping occurs for the perihelion distance q as might have been expected on the basis of the Oort-Schmidt (1951) results. However, as pointed out in Part I of this paper, an empirical dependence on q for n and m_0 is expected on purely pedigogical grounds. The dependence of the solutions on q for our data is significant at the 99.5% level. For the combined list (N = 150), the relationship is $m_0 = 7.0 \pm 0.3 - (0.7 \pm 0.2) \times q(A.U.)$ with $r' = -0.23 \pm 0.08$ for the correlation coefficient. The negative correlation represents an observational selection effect--observers with small telescopes tend to see intrinsically fainter objects only when the perihelia are small. The n values do not show a significant q correlation.

From time-to-time, there are various suggestions for a solar modulation of comet brightness. Over the long time scale represented by the data available to us, there are only sunspot numbers available as indicators of solar activity. We have therefore calculated mean sunspot numbers for each period of comet observation. Since solar rotation would present the same average level of activity to the comet as it does to the earth.

We have searched for a statistically significant solar modulation of the (m_0,n) with little success. A direct linear correlation with $\hat{R}$ for either m_0 or n has at most only a 20-30% chance of being non-zero. In addition, a logarithmic correlation for n (i.e. $\log n = \log n_0 + C_1\hat{R}$ for a regression equation) is absent. We conclude that the perihelion distance correlation along with remaining random errors and solution divergence problems obscure any real solar effect that might be present. We should point out that a higher degree of solar correlation is obtained if the raw Beyer values are

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used in the analysis. However, this can be entirely attributed to the fact that the average level of solar activity of the Beyer data ($\langle \hat{R} \rangle$ = 78 ± 53) is significantly higher than for the non-Beyer data ($\langle \hat{R} \rangle$ = 48 ± 32) and this couples with the Beyer systematic observational effect to produce an apparent solar correlation of n values.

The distribution of n values with the assigned solution weight agrees well with expectation of Poisson statistics except for sixteen solutions. Eleven of these (1861 I, 1862 III, 1913 IV, 1932 V, 1936 II, 1937 V, 1951 IV, 1959 VIII, 1960 III, 1961 II, and 1962 V) can readily be ascribed to the previously mentioned solution divergence problem. The remaining five (1927 IV, 1943 I, 1951 I, 1953 I, and 1968 I) cannot be assigned to this category and therefore must represent verified intrinsic cometary variations from the mean of the n value. These objects deserve a more detailed discussion than we can give here.

Summary and Recommendations

We have reviewed the power-law definition of comet brightness and discussed possible systematic influences that can affect the derivation of m_0 and n values from visual magnitude estimates. We have provided a rationale for the Bobrovnikoff aperture correction method and argue for its continued use. We have demonstrated that the Beyer extrafocal method leads to large systematic effects which if uncorrected by an instrumental (aperture or focal ratio) relationship, results in n values significantly higher than those derived according to the Bobrovnikoff guidelines.

We present a series of (m_0,n) parameter sets which have been reduced to essentially the same photometric system (Bobrovnikoff). In order that future

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observations are reduced to this same system we offer the following recommendations.

For observers

(a) Make extrafocal comparisons using the smallest possible aperture and magnification.

(b) Be sure to note instrument size, instrument type, focal ratio, and magnification.

(c) Use stars with spectral type G or earlier for comparison.

(d) Throw the images out-of-focus only by an amount needed to make the star and comet look identical. The more an image is out-of-focus, the greater is the required instrument correction. Beyer's and Sidgwick's methods are to be avoided if possible, since they can lead to serious problems even when used by skillful observers.

For users of visual magnitudes

(a) Do not attempt solutions for photometric parameters using observations for which aperture corrections have not or cannot be obtained as the values will be systematically affected.

(b) If possible, aperture corrections should be derived for each comet individually and for each type of instrument separately and for each method of extrafocal comparison (if necessary) separately. As a last resort, the mean aperture relationships derived by Bobrovnikoff (1941a) and Morris (1973a) can be used.

(c) Solutions obtained for comets with $q \to 1$ or $r \to 1$ should always be treated carefully. If necessary, a series expansion formula around $r = 1$ A.U. is to be preferred to the usual logarithmic formula.

(d) Most n values outside 3.6 ± 2.0 should always be suspect as poss-

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ible cases of solution divergence.

(e) The probability of solution divergence is roughly proportional to $1/\sqrt{N}$ where N is the number of observations available.

(f) Since $\langle n \rangle = 3.6$, the use of $n = 4$ and $n = 6$ in making comet brightness predictions may be erroneous, particularly when attempting to use photographic observations (whose instrumental effects are related to focal ratio) combined with visual estimates (whose instrumental effects are functions of aperture).

(g) Solutions which are based on the Beyer system will on the average have an "apparent" n value that is 1.5 units too high compared to the Bobrovnikoff system. Insofar as the Beyer method represents the extreme of all magnitude estimates which have no aperture corrections, we could use $3.6 < n < 5.1$ for prediction of visual magnitudes and the standard $n = 4$ is a reasonable compromise.

(h) When combining individual observations, there are many sources of random error that produce poorly defined m_0,n values even when least-squares techniques are applied. Few of these are reported in the literature along with the raw observations, so considerable care must be exercised by the investigator to make sure that the following are recognized in each analysis:

1. Sky Background Effects - Moonlight (Meisel, 1970) and Twilight
2. Air mass effects (Meisel, 1970)
3. Observer inexperience - can be very large $\pm 0\overset{m}{.}5$
4. Comparison Star-Comet Color Mismatch - can be very large $\pm 0\overset{m}{.}3$ to $\pm 0\overset{m}{.}5$
5. Inconvenient location of comparison objects
6. Poor comparison star magnitudes
7. Drastic change in comet physical form and/or activity
8. Variations in observing site quality
9. Use of unreliable or unsuitable instrumentation.

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Appendix. Conversion of Levin Parameters to an Equivalent Power-Law Set

Since the Levin (A,B) parameters appear from time-to-time in the literature, it is sometimes desired to convert to the equivalent (m_0,n) set without re-analyzing the original observations.

Two approaches are possible:

(a) If the (A,B) values were derived by least-squares, we can obtain an equivalent set of (m_0,n) values that would have been obtained by least-squares from the same observations. This conversion, though exact, is tedious. We list the necessary averages here for completeness.

$$n = \frac{B}{2.5 \times 0.43} \; \frac{\langle \ln r_i \rangle}{\langle \sqrt{r_i} \rangle} \; \frac{\langle r_i \rangle (1-B) - A\langle \sqrt{r_i} \rangle - \frac{1}{k}\langle \sqrt{r_i} \rangle^2}{\langle (\ln r)^2 \rangle - A\langle \ln r_i \rangle - B\langle \sqrt{r}\,\ln r_i \rangle - \frac{1}{k}\langle \ln r_i \rangle}$$

Explicitly taking time averages and converting

$$\langle \ln r_i \rangle = \frac{\int \ln r_i\,dt}{\int dt} = \frac{\int r^2 \ln r\,d\nu}{\int r^2 d\nu}$$

where ν is the true anomaly.

$$\langle (\ln r)^2 \rangle = \frac{\int r^2 (\ln r)^2 d\nu}{\int r^2 d\nu}$$

$$\langle r_i \rangle = \frac{\int r^3 d\nu}{\int r^2 d\nu}$$

$$\langle \sqrt{r_i} \rangle = \frac{\int r^{3/2} d\nu}{\int r^2 d\nu}$$

$$\langle \sqrt{r}\,\ln r \rangle = \frac{\int r^{3/2} \ln r\,d\nu}{\int r^2 d\nu}$$

(b) If it can be assumed that the least-squares or graphical methods are convergent to the correct A,B values, a more direct conversion can be performed.

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$$m_o = A + B$$

and $$n = \frac{1}{2.172} B \langle \sqrt{r} \rangle$$

where $$\langle \sqrt{r} \rangle = \frac{\int \sqrt{r}\, d\nu}{\int d\nu}$$

The assumption of a parabolic orbit was used in this study to compute the $\langle \sqrt{r} \rangle$ required above. The other means required in (a) above could also be computed under the parabolic assumption but we do not give these explicitly here because they are quite complicated. The evaluation of $\langle \sqrt{r} \rangle$ in the parabolic approximation is complicated but straightforward. We quote here the results for the time averaged $\sqrt{r}$. The average $\sqrt{r}$ obtained by integration over the true anomaly is considerably simpler. Since observations are not generally evenly distributed over the true anomaly, however, the longer expression is usually preferred.

$$\langle \sqrt{r} \rangle = \sqrt{q} \left[\frac{(B_2C_2 - B_1C_1) + \frac{3}{8} \ln \left\{\frac{A_2(1+B_2)}{A_1(1+B_1)}\right\}}{D_2 - D_1 + \frac{1}{3}(E_2-E_1)} \right]$$

with $q < r_1 < r_2$,

$$U_i = r_i/q$$

$$A_i = \sqrt{U_i}$$

$$B_i = \sqrt{1-U_i^{-1}}$$

$$C_i = (U_i^2/4 + \frac{3}{8} U_i)$$

$$D_i = \sqrt{\frac{r_i}{q} - 1}$$

$$E_i = \sqrt{\left(\frac{r_i}{q} - 1\right)^3}$$

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where q is the perihelion distance, r_1 is the comet heliocentric distance of the first observation and r_2 is the heliocentric distance of the last observation. Equivalent expressions can be derived covering orbit segments which span the perihelion point by letting $\langle\sqrt{r}\rangle = \frac{1}{2}(\langle\sqrt{r}\rangle_{r_1 \to q} + \langle\sqrt{r}\rangle_{q \to r_2})$.

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