From a comparison of these values, two important conclusions may be drawn. Firstly, it should seem that the reserve made by the Office is greatly in excess, in the instances here given, of the values of the policies as given by the Carlisle 3 per cent. table; and it will be found, I believe, throughout the whole of life, that the reserve made by the valuation of the Office is in excess of that required by the Carlisle 3 per cent. valuation; from which it results that if a valuation of the Society were made by the Carlisle 3 per cent. table, there would be a much larger cash bonus divided than is now allowed in reduction of premium. Secondly, it will be noticed that the reserve made for recent policies is for several years greater than the amount of the premiums received, so that in fact every new policy issued causes loss on the subsequent valuations—reduces the divisible surplus—and makes the abatement of the premium less than it would otherwise be.

This last observation opens up a wide and tempting field of investigation, but one which cannot be considered suitable for these pages. I therefore abstain from proceeding any further in that direction.

It will, of course, be understood that the values in the preceding table are not to be taken as the actual amounts reserved by the valuation of the Company. I believe that valuation is not conducted by the Carlisle table; and without being in possession of the table of mortality by which the valuations are conducted, it is impossible to assign the actual values of the policies. If the table in use is one which gives throughout a greater expectation of life than the Carlisle, then the values of the policies will be less than those given above; but it cannot be supposed that any table of mortality whatever would give such results as to vitiate the conclusions I have ventured to draw from a comparison of the values in the above table.

In conclusion, I should wish to add, that in writing these remarks I have not been in any way actuated by a desire to recommend the system pursued by the two Offices I have alluded to. I do not feel at liberty to express in these pages any opinion as to the merits of that system; and in all that I have said I have been careful to abstain from any expression of opinion, and to confine myself strictly to the discussion of questions of fact.

I am, Sir,

Your obedient servant,

T. B. SPRAGUE.

Equity and Law Life Assurance Society,
18, *Lincoln's Inn Fields,*
August, 1864.

ON MR. HODGE'S REMARKS UPON THREE-LIFE SURVIVORSHIPS.

To the Editor of the Assurance Magazine.

SIR,—I must beg the favour of a small space in your columns for a word or two in reference to Mr. Hodge's comments, at the last meeting of the Institute, upon Mr. Gray's account of my "Solutions of survivorship problems."

Mr. Hodge informed us that it was at one time his practice to calculate his three-life cases by Milne's formulæ, but that he afterwards found he could attain his results with sufficient accuracy by means of Simpson's well-known rule of substituting for two joint lives an equivalent single life. That is, I suppose, in calculating the values of the survivorship reversions, $\underset{1}{\mathfrak{A}}\mathbb{B}\mathbb{C}$, $\underset{1}{\mathfrak{B}}\cdot\underset{1}{\mathfrak{C}}\mathbb{A}$, &c., Mr. Hodge *now* (like everybody else, I presume) first finds the single life *D*, equivalent to the joint lives *BC*, and then determines the values of $\underset{1}{\mathfrak{A}}\mathbb{D}$, $\underset{1}{\mathfrak{D}}\mathbb{A}$, &c.

But what are we to understand from Mr. Hodge's account of his *former* practice? Does he mean that he calculated *accurately* each of the seven three-life annuities involved in the following formula, for instance, which is one of those given by Milne, and by far the least laborious of the series:—

$$\underset{1}{\mathfrak{A}}\mathbb{B}\mathbb{C}=\frac{1}{3}\left\{v-(1-v)ABC\right\}+\frac{1}{6\,{}_1a_1}\left(2A_1BC+\frac{A_1B_1C}{{}_1b_1}+\frac{A_1BC_1}{{}_1c_1}\right)$$
$$-\frac{1}{6\,{}_1b_1}\left(AB_1C+\frac{2AB_1C_1}{{}_1c_1}\right)-\frac{ABC_1}{6\,{}_1c_1},$$

or did he calculate these annuities *approximately* by Simpson's rule, and therewith determine the value of the reversion? If the former, it is to be hoped that these cases were not of frequent occurrence with him; and if the second supposition is the correct one, I think the process can scarcely afford a satisfactory test of the accuracy of other methods of solving the problem.

Again, Mr. Hodge did not explain whether his statement was confined to the *simple* survivorship problems, like the above, and others derived from them; or whether he included *also* those marked with an asterisk in in the synoptical table on page 195 of Milne's work. If he referred to the former *only*, it is scarcely necessary to point out that those problems were *not* the subject of discussion on the occasion referred to. If, on the other hand, his remarks applied to the *latter* problems *also* (which *did* form the subject of Mr. Gray's observations), Mr. Hodge is no doubt aware that Milne's solutions of these cases are but rude approximations; and, consequently, that the fact of any shorter methods giving results in near accordance with Milne's, is no proof that such results can be relied upon. Under either supposition, therefore, it is difficult to see what bearing Mr. H.'s remarks had upon the question before the meeting.

Before concluding, I may refer to one more point in Mr. Hodge's observations. In my former paper on the same subject (see *Assurance Magazine*, vol. x., p. 243), it will be seen that in the process of transforming

$$\underset{\text{iii}}{\mathfrak{A}}\underset{\text{ii}}{\mathbb{B}}\mathbb{C}=\underset{\text{ii}}{\mathfrak{A}}\mathbb{B}-\underset{1}{\mathfrak{B}}\mathbb{A}\mathbb{C}+{}_{\rceil\underset{1}{BC}}A(1-v)$$

into $$\underset{\text{iii}}{\mathfrak{A}}\underset{\text{ii}}{\mathbb{B}}\mathbb{C}=\underset{1}{\mathfrak{A}}\cdot\underset{1}{\mathfrak{C}}\mathbb{B}-\underset{1}{\mathfrak{A}}\mathbb{B}-{}_{\rceil\underset{1}{BC}}A(1-v).$$

I make use, incidentally, of the identity $(A-ABC)(1-v)=\mathfrak{A}\mathfrak{B}\mathfrak{C}-\mathfrak{A}$, making, however, no observation whatever upon it. Mr. Gray, who, as we all know, is rather curious in such matters, referred to it, in passing, as *noticeable*—his remark evidently having reference, not to any supposed difficulty in obtaining the equation (for it is so simple that I use it as a self-evident proposition), but solely to the symmetry exhibited in it.

I remain, Sir,

Your very obedient servant,

London, 1*st December*, 1864. W. M. MAKEHAM.

P.S.—As stated above, Milne's formula for $\underset{1}{\mathfrak{A}}\mathbb{B}\mathbb{C}$ is much less laborious than others of the series. The following is from his 23rd problem:—

$$\underset{\text{II.III}}{\mathfrak{A}}\mathbb{B}\overset{\circ}{\underset{1}{\mathbb{C}}} = \begin{cases} \frac{1}{2}\mathfrak{A} - \frac{1}{6}\mathfrak{A}\mathfrak{B}\mathfrak{C} - \frac{1}{6\,{}_1a_1}\left(A_1BC - \frac{3(AB)_1 - 2A_1B_1C}{{}_1b_1} + \frac{3(AC)_1 - A_1BC_1}{{}_1c_1}\right) - \\ \frac{1}{6\,{}_1b_1}\left(3AB_1 - 2AB_1C - \frac{AB_1C_1}{{}_1c_1}\right) + \frac{3AC_1 - ABC_1}{6\,{}_1c_1} + (\tfrac{1}{2} - \underset{1}{b}c)\rceil_{\gamma}\mathfrak{A} \\ - \frac{1}{2_1(ab)_1}\rceil_{\gamma}(AB)_1 + \frac{1}{2\,{}_1b_1}\rceil_{\gamma}AB_1 \end{cases}$$

This is the solution for the case where *C* is the oldest life, for Milne's formulæ for this problem vary according to the seniority of the lives involved; and it will be borne in mind that it is at best but a rough approximation. For the above I propose to substitute the formula

$$\underset{\text{II.III}}{\mathfrak{A}}\mathbb{B}\underset{1}{\mathbb{C}} = \underset{1}{\mathfrak{C}}\mathbb{A}\mathbb{B} - \rceil_{BC}A(1-v),$$

which is rigidly accurate—independent of seniority—and the two terms of which, as I shall hereafter show, admit of an easy and expeditious mode of calculation.

W. M. M.