# Migration of Celestial Bodies in the Solar System 

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#### Abstract

Migration of planetesimals and embryos of forming planets was investigated on the basis of computer runs of the evolution of disks of gravitating bodies orbiting the Sun. Our results obtained earlier with the use of the spheres' method are close to the results obtained recently by other authors by numerical integration. Due to the interaction with migrating planetesimals, the embryos of Uranus and Neptune, which acquired most of their masses near the orbit of Saturn, could migrate to the present distances from the Sun moving all time in nearly circular orbits. Each of the terrestrial planets incorporated planetesimals from all feeding zones of these planets.


## 1. An Algorithm for Computer Simulations

Our investigations of the process of planet formation were mainly based on the results of computer simulation of evolving gasless disks initially consisting of hundreds (up to 1000) gravitating bodies, moving about the Sun and coagulating under collisions. Some initial disks included also almost-formed planets. The densities of the bodies and planets were assumed to be similar to those of the contemporary planets. Below we summarize our results presented earlier and compare them with the recent results obtained by Chambers and Wetherill (1998) and Thommes et al. (1999). A lot of references to the papers devoted to the formation of the solar system can be found in the publications cited below.

In our runs, the gravitational influence was taken into account by the sphere's method, i.e., the relative motion of two bodies, encounting at the distance equal to the radius $r_{s}$ of the considered sphere, was investigated as a two-body problem, and while outside the sphere the particles were assumed moving around the Sun in unperturbed Keplerian orbits. Usually we used the sphere of action (the Tisserand sphere). Initially we chose pairs of encounting objects with the use of the "probability" algorithm. In this algorithm, the pairs of bodies encounting to $r_{s}$ were chosen in the proportion to the probability $p_{i j}$ of their encounter. Later on the efficient "deterministic" algorithm was developed (Ipatov 1992, 1993b), for which for a pair of encounting bodies the moment $\tau_{i j}$ of the first isolated encounter to $r_{s}$ is minimum. For encounting bodies $\tau_{i j}$ has a minimal but not an average value. Therefore, the results of computer runs showed that if the number of bodies in a disk is not small then for this algorithm the velocity of disk evolution is by an order of magnitude greater than that for the probability algorithm. Below we present only times of evolution obtained by using the deterministic algorithm. Our algorithms for calculations of $p_{i j}$ and $\tau_{i j}$ (Ipatov 1988, 2000) differed from the algorithms by E.J. Öpik and J.R. Arnold.

Analytical investigations of the dependence of disk evolution on the number of bodies in the disk were also made (Ipatov 1988, 2000). Some characteristics of the disk evolution depend on the initial number of bodies in the disk, and the real evolution differes from the computer simulation results, but basing on these results we can make some estimates of the planet accumulation.

## 2. Accumulation of the Terrestrial Planets

Our investigations of the evolution of disks consisted initially of identical bodies and corresponded to the terrestrial feeding zone (Ipatov 1981, 1987a, 1993a) are close to the results obtained by Wetherill (1985, 1988). For example, we also obtained that the number of embryos of terrestrial planets with masses greater than $0.1 m_{\oplus}$, where $m_{\oplus}$ is the Earth's mass, could be greater than the number of actual planets. At some stages of evolution, an average eccentricity of planetesimals exceeded 0.2-0.3 (initially it was small).

In our runs, initial bodies were divided into four groups depending on the values of their semimajor axes ( $0.4-0.6,0.6-0.8,0.8-1.0,1.0-1.2 \mathrm{AU})$. Computer simulation results showed that, if the mass of the formed planet was larger than $0.4 m_{\oplus}$, then the proportion of the bodies from different groups entered into the planet was almost the same as that for the initial disk. Such a strong mixing was obtained by Wetherill (1988) only for Venus, but not for Earth.

Eccentricities of the planets formed at the disk edges, as a rule, were larger than those of the planets formed in the disk center. These results show that Mercury and Mars could get their present eccentricities due to encounters with bodies from the feeding zone of the terrestrial planets. At some stages of evolution, the total mass of bodies with $a \geq 2 \mathrm{AU}$ exceeded 0.05 of the total mass of the disk. $9-14 \%$ of planetesimals were ejected into hyperbolic orbits. Most of the planetesimals that fell onto the Earth underwent collisional evolution. The time to form $80 \%$ of the mass of the Earth can be less than 10 Myr. The results obtained with the spheres' method are in an accordance with the results obtained by Chambers and Wetherill (1998) by numerical integration. Their times of evolution of disks are between the times obtained by deterministic and probability algorithms, but closer to those for the deterministic algorithm.

## 3. Formation of the Giant Planets

Evolution of various disks corresponded to the feeding zones of the giant planets was considered in (Ipatov 1987b, 1991, 1993a). The total mass of planetesimals from the feeding zone of Uranus and Neptune that were ejected into hyperbolic orbits was obtained to be by an order of magnitude larger than the mass of the bodies entered into planets. Jupiter (its envelope and nucleus) could incorporate more ices and rocks than any other planet. The total mass of the planetesimals that at some stages of evolution had $a>50 \mathrm{AU}$ exceeded $10 m_{\oplus}$.

Ipatov (1991, 1993a) considered the evolution of several disks, initially consisting of almost formed Jupiter and Saturn, of the embryos of Uranus and Neptune with masses equal to $10 m_{\oplus}$, and of about a thousand identical bodies with $a$ from 8 to 32 AU and a total mass ranged from $135 m_{\oplus}$ to $180 m_{\oplus}$. In one series of runs, initial values of $a$ of the above giant planets were taken equal to
$5.5,6.5,8,10 \mathrm{AU}$, respectively. Initial orbits of the planets were circular, and initial eccentricities of the bodies equaled to 0.02 . If the initial embryos had highly eccentric orbits, which crossed the orbit of Saturn, than it was obtained that the probability of the fact that an embryo remained in an elliptical orbit at the end of evolution was by an order of magnitude less than the probability of the ejection of this embryo into a hyperbolic orbit.

During evolution most of bodies migrated to Jupiter, which ejected them into hyperbolic orbits. The average eccentricity of bodies exceeded 0.3 during the most part of evolution. Due to gravitational interactions with migrating bodies, all four giant planets attained their present orbits. In the computer runs, the principal changes in the orbits of the embryos occurred over a time span less than 10 Myr , and their masses increased by less than $2 m_{\oplus}$. Later on Thommes et al. (1999) considered the similar evolution of the embryos of Uranus and Neptune using results of numerical integration and obtained similar results. In their runs the embryos of Uranus and Neptune got their present orbits in about 5 Myr , that is, a deterministic approach in the spheres' method gave similar times. For the actual disk of very large number of planetesimals, this time span can be ten fold larger. Probably, most of large asteroids and those large trans-Neptunian objects that were formed directly beyond Neptune's orbit could be formed directly from rarefied dust condensations but not by accretion of smaller planetesimals. A small part of planetesimals from the feeding zone of the giant planets could become "scattered trans-Neptunian objects".

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