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BOOK REVIEWS

WOJTASZCZYK, P., *Banach spaces for analysts* (Cambridge Studies in Advanced Mathematics 25, Cambridge University Press, 1991), pp. xiv+382, 0 521 35618 0, £50.

As the title implies, this is intended as a survey of the interplay between Banach space theory and other branches of analysis, not a book on Banach spaces for their own sake. Given such an objective, the selection of material is perhaps bound to be somewhat personal, and it does not take long to detect that harmonic analysts figure prominently among the "analysts" these Banach spaces are "for". The topics studied are viewed largely through this lens. The result is an original book with a wealth of information on the particular Banach spaces of interest in harmonic analysis. Moreover, Wojtaszczyk loses no opportunity to demonstrate how the methods and special functions of harmonic analysis can be applied in the study of Banach spaces, including some spaces (like $L_p(\mu)$) that are by no means the special preserve of harmonic analysis. There were instances where the reviewer was left unconvinced that these methods were preferable (such as the estimation of the projection constant of l_2^n), but there are quite enough successes for the book to merit the subtitle "harmonic analysis for Banach space theorists".

This does not pretend to be a book for beginners. The introductory survey of topics assumed already known includes such things as the Krein–Milman theorem, the Riesz–Thorin theorem, Khintchine's inequality, Sidon sets, Hardy spaces and Sobolev spaces. It would be a pity if this formidable opening deterred some potential readers from starting, because in fact large sections of the book require only a fraction of this material. The author recognises in the introduction that "it will be unusual for a reader to read this book from beginning to end" and advises anyone interested in a particular topic to "start right there". Within reason, the book does indeed allow the user to do this.

After a brisk survey of general theory, nine selected topics are treated in depth. The headings are: (A) L_p-spaces, type and cotype, (B) Projection constants, (C) $L_1(\mu)$-spaces, (D) $C(K)$-spaces, (E) The disc algebra, (F) Absolutely summing and related operators, (G) Schatten–von Neumann classes, (H) Factorization theorems, (I) Absolutely summing operators on the disc algebra. However, the bare titles do not really convey the whole story. The author's emphasis leads to an extensive collection of deep results on his chosen list of special spaces, while leaving out a number of other theorems that one might expect to find under the above headings. The section on factorization concentrates on Nikishin's theorem and related results, but does not include the standard theorems on factorization through Hilbert space. There is no mention of Kwapień's characterization of Hilbert spaces as those having both type and cotype 2, or of Maurey's generalization. Though L_p-spaces figure prominently, no use is made of lattice properties such as p-concavity. The chosen applications of p-summing operators include the logarithmic estimate for power-bounded operators, Sidon sets and the Menchoff–Rademacher theorem, but the far more immediate applications to projection constants and the Dvoretsky–Rogers theorem (arguably the historical origin of the notion p-summing) only appear as exercises. The two sections on the disc algebra are particularly detailed, and culminate in an exposition of the recent work of Bourgain.

Each section concludes with extended remarks and exercises. The "remarks" contain a thorough historical survey together with further theorems and open questions; they will be of great value to many users. The exercises are substantial and include a good number of theorems not included in the text: some very compressed hints for them are given at the end.

The author's urge to get straight to serious applications can result in a tendency to economise on the really simple results and examples that would help the reader (this reader, anyway) to understand and get used to a new concept. For example, when introducing type and cotype, it would surely be helpful to point out that for Hilbert spaces and $p=2$, the two quantities being compared are exactly equal. Within two pages of meeting the definition of p-summing operators, one is confronted with one of the deepest results on them, Grothendieck's theorem (naturally with the harmonic analyst's proof using Paley operators!). The elementary fact that for operators between Hilbert spaces, the 2-summing norm coincides with the Hilbert–Schmidt norm only appears as a "remark" 43 pages later.

However, the main objective of the book is to give an in-depth treatment of the particular Banach spaces of interest in harmonic analysis, and in this there is no doubt that it is outstandingly successful. Here is an authoritative book by a scholarly mathematician who is active in the field. The selection of topics is too special for it to become established as a substitute for other books on Banach spaces, but it will be a very useful complement to them.

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