

Chapter 6 introduces the crucial topic of differentials on a Riemann surface with the motivating question: 'does it make sense to differentiate a meromorphic function on a Riemann surface?'. Again the cases of the projective line and of the complex torus are treated in detail. The chapter builds towards Abel's theorem for complex tori and the Riemann–Roch formula. The latter is applied to give Chow's theorem for smooth plane curves and to prove the associativity of the addition law on smooth cubics.

More advanced material on singular curves is presented in the final chapter. First, the resolution of singularities is discussed. Then Newton polygons and the associated topic of Puiseux expansions are used to describe what a curve looks like near a singularity. Finally, the degree-genus formula for singular curves (Noether's formula) is proved using Puiseux expansions with the examples of cuspidal and nodal cubics being examined in detail. (These last two chapters would be very useful for anyone interested in the modern links between algebraic geometry and coding theory.) There are three appendices on algebra, complex analysis and topology.

There appear to be very few misprints; only the ones in 3.30 and in 5.23 are of any substance. The statement of 3.14 needs a little repair and there is a tiny formal difficulty (for example in 3.7) of resultants being viewed as having a non-zero degree when they might vanish. But these are minor cavils, set against the all-round excellence of this book.

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BRECHTKEN-MANDERSCHIED, U., *Introduction to the calculus of variations*, translated by P. G. Engstrom (Chapman and Hall, London 1991), pp. viii + 200, cloth 0 412 36690 8, £27.95, paper 0 412 36700 9, £13.95.

This book gives a conventional first course on the calculus of variations. A brief introductory chapter, which mentions the standard examples, is followed by one on Euler's equation and the Erdmann corner conditions. Then come four chapters on necessary conditions, including those of Weierstrass, Legendre and Jacobi, and sufficient conditions. In the second half of the book the earlier ideas are extended to problems involving variable boundaries or parametric representations, problems depending on several functions or multiple integrals, and problems constrained by side conditions. The final chapter gives the direct approximation method due to Ritz.

From this it will be seen that the material is entirely appropriate. Most of the worked examples are from mathematics, with an emphasis on geometrical properties, a few are from physics, and the Ramsey growth model from economics represents applications outside the physical sciences. There are some exercises for the reader but not always very many.

It is most welcome to have such a careful account of the calculus of variations available in paperback format at a very reasonable price. If you want a thorough yet concise treatment you can hardly do better. But I did sometimes feel that the very painstaking style tended to obscure what was going on. For the good student this would be an excellent text, but the weaker student might master the subject more easily from a less conscientious treatment.

The text is beautifully printed, with large clear type making it a pleasure to read. There are a few apparent infelicities of translation, for example the description on page 6 of the area enclosed by a plane curve as "surface area" just two pages after a problem about genuine surface area.

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