# SOME REMARKS ON THE PAPER BY B. G. MARSDEN "AN ATTEMPT TO RECONCILE THE DYNAMICAL AND RADAR DETERMINATIONS OF THE ASTRONOMICAL UNIT"

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While Marsden's solution C leaves residuals with the relatively small $[vv]$ of 13.73, it should be realized that this representation of the observations of Eros does not satisfy the fundamental principle of the least squares method, in so far as the associated value of $[vv]$ is not a minimum with respect to small arbitrary deviations from solution C. As a matter of fact, there is an infinite number of " solutions " with $[vv]$ between the 13.73 of Marsden's solution C and the 8.66 of his solution A, each of these being associated with a certain arbitrarily prescribed value of the mass of Mars and with a related mass of Earth + Moon. Of this infinite series of solutions, only solution A is a least squares solution in the true sense, with a minimum value of $[vv]$. This can be seen and verified as follows.

Let $X^i_A$ (for $i = 1, 2, \ldots, 16$) represent the individual results from solution A for the 16 unknowns which affect the observations equations of Eros, and $X^i_C$ the corresponding results of solution C. Further, let $v^j_A$ (for $j = 1, 2, \ldots, 74$) denote the various residuals of solution A, and $v^j_C$ those of solution C. Since the 74 differences $v^j_C - v^j_A$ are linear functions of the 16 differences $X^i_C - X^i_A$ between the two solutions A and C, the infinite series of solutions N, with

$$X^i_N = X^i_A + N\,(X^i_C - X^i_A) \qquad (0 < N < 1), \tag{1}$$

will leave residuals obtainable by linear interpolation according to

$$v^j_N = v^j_A + N\,(v^j_C - v^j_A). \tag{2}$$

For $N = \frac{1}{2}$, for example, the elements of Eros and of the Earth and all the other masses and corrections represented by $X^i_N$ will have values

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exactly halfway between those of Marsden's solutions A and C, and the residuals will also be equal to the arithmetical means of those of A and C.

Of principal interest are the [vv] values associated with different values of N. Equation (2) has been used to compute the residuals associated with $N = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$, and also those for $N = -\frac{1}{4}, -\frac{1}{2}$. The related square sums [vv] are listed in table I, together with the associated mass values for Mars and Earth + Moon. In figure 1 the [vv] have been plotted

TABLE I.

*[vv] of Eros for interpolated solutions.*

| N. | [vv]. | $\frac{1}{m_{♂}}$. | $\frac{1}{m_{\oplus+☾}}$. | Remarks. |
|---|---|---|---|---|
| −0.50.......... | 9.60 | 3 123 600 | 328 418 | |
| −0.25.......... | 8.82 | 3 106 200 | 328 499 | |
| 0.00.......... | 8.66 | 3 088 900 | 328 579 | Solution A |
| +0.25.......... | 9.10 | 3 071 800 | 328 659 | |
| +0.50.......... | 10.03 | 3 054 900 | 328 740 | |
| +0.75.......... | 11.58 | 3 038 200 | 328 820 | |
| +1.00.......... | 13.73 | 3 021 700 | 328 900.5 | Solution C |

against N, and the curve connecting the computed points clearly exhibits the significance of solution A as the only true least squares solution, as far as the representation of the Eros motion is concerned. Marsden's

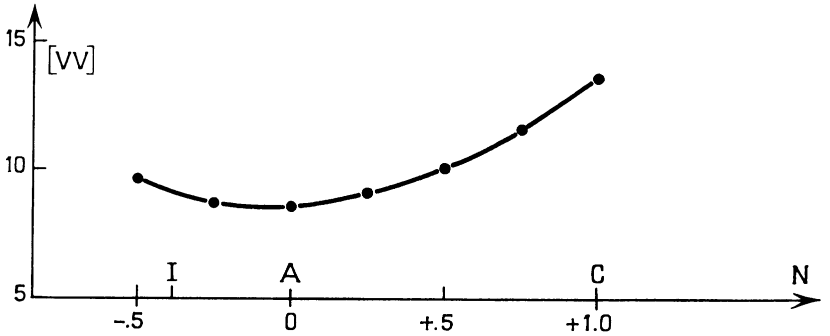


Fig. 1. — [vv] as a function of N.

solution A is the equivalent of my own solution I, the N-value of which $\left(\text{as determined by } \frac{1}{m_{\oplus+☾}} = 328\,452\right)$ is represented by the I in figure 1. It may be noted that between I and A, or on the left of A, the slope of the curve is less steep than on the right of A.

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While the shallowness of the minimum of the [$vv$] curve at A explains the sensitivity of $m_{\oplus+☾}$ and thus of the solar parallax against any interference with a „ free " solution from Eros alone, the slope at C is already so steep, and the related value of [$vv$] so far above the minimum at A, that it seems far from justified to consider the solution C as one doing justice to the Eros observations. It should be noted that each residual of Eros is based on a normal place representing numerous individual observations. The only argument in favor of solution C is its agreement with the Venus radar determinations of the astronomical unit. The further argument, that solution C gives a sufficiently close representation of the Eros motion, has no real weight, because an infinite number of such solutions with even better residuals exists on both sides of solution A, as illustrated in figure 1.