ON THE PRESENCE OF PHENOMENA AKIN TO ADSORPTION IN BIOLOGY, AS A SOURCE OF FALLACY IN STATISTICAL INQUIRY.

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(With 2 Diagrams.)

Among the formulae which describe relationships found in physics, chemistry and biology is one commonly known as the "adsorption formula." It graduates for a considerable range of experiment the amount of adsorption which takes place at the bounding layers of solids, fluids and gases or immiscible fluids in presence of different concentrations of the substance adsorbed. It may be written in the form

$$x = aC^n$$

where x is the amount of the substance adsorbed; C the concentration of the adsorbed substance after equilibrium has been attained; n is an absolute constant less than unity; and a is a constant depending in value on the units of measurement. The form of this curve is such that x increases as C increases but more and more slowly with each further increase in the value of C. The formula, though it may arise on the theory of chemical equilibria, is really empirical as it does not hold with high values of the concentration.

The formula was, however, used at a date prior to its application to the problems of adsorption. So far as I can discover it was first introduced by Harcourt and Esson[1] who in the year 1867 used it to graduate the relationship of temperature to the rate of the chemical reaction between H_2O_2 and HI. In 1875 Dr Farr[1] showed that it closely represented the association of the death-rate and the density of population. In 1897, Pareto[1] recommended it as a graduation formula in the statistics of the distribution of wealth. It was only in 1909 that Freundlich[1] applied it to the theory of adsorption. With the addition of a constant to the abscissa, two subsidiary forms which accurately describe many relationships in biology arise, namely,

$$y = a\,(c + x)^n, \quad y = a\,(c - x)^n.$$

It is to be noted that when observations obeying these laws are plotted on double logarithmic paper they lie on a straight line while the value of n is given by the tangent of the angle between the line through the observations and the abscissa. It is thus easy to determine if the law holds and to evaluate the constant n.

[1] See References on p. 442.

The range of applications of the formula will now be considered. Three typical instances, chosen out of many others, are illustrated in Diagram I. The first is taken from the domain of economics, but in addition it has an importance in public health. It relates to the distribution of the numbers of houses of different rental in England. It was given and commented on by Mr Goschen (1887) in his Presidential Address to the Royal Statistical Society. Taking the initial rental as that of £10 per annum, the number of houses has been plotted on double logarithmic paper. The ordinates give the number of houses and the abscissae the rentals. The actual observations are denoted by circles; the straight line shown was fitted to these observations by the method of least squares. It is obvious that the formula very accurately describes the data.

As a second example, the distribution of incompetence in the improvident has been chosen. The figures are taken from a report of the Glasgow Parochial Board. The graph shows the comparison between the number of persons (the ordinates) and the number of times (the abscissae) each individual sought admission to the Poor House during a period of six months. Here again it is obvious that the observations are in close relation to the formula. The slope of the line which describes this distribution of incompetence is the same as that which describes the distribution of wealth as given by the rental of houses. This suggests that both depend on the same factors in human nature for which the wealth line describes the distribution of competence and the Poor House line that of its opposite.

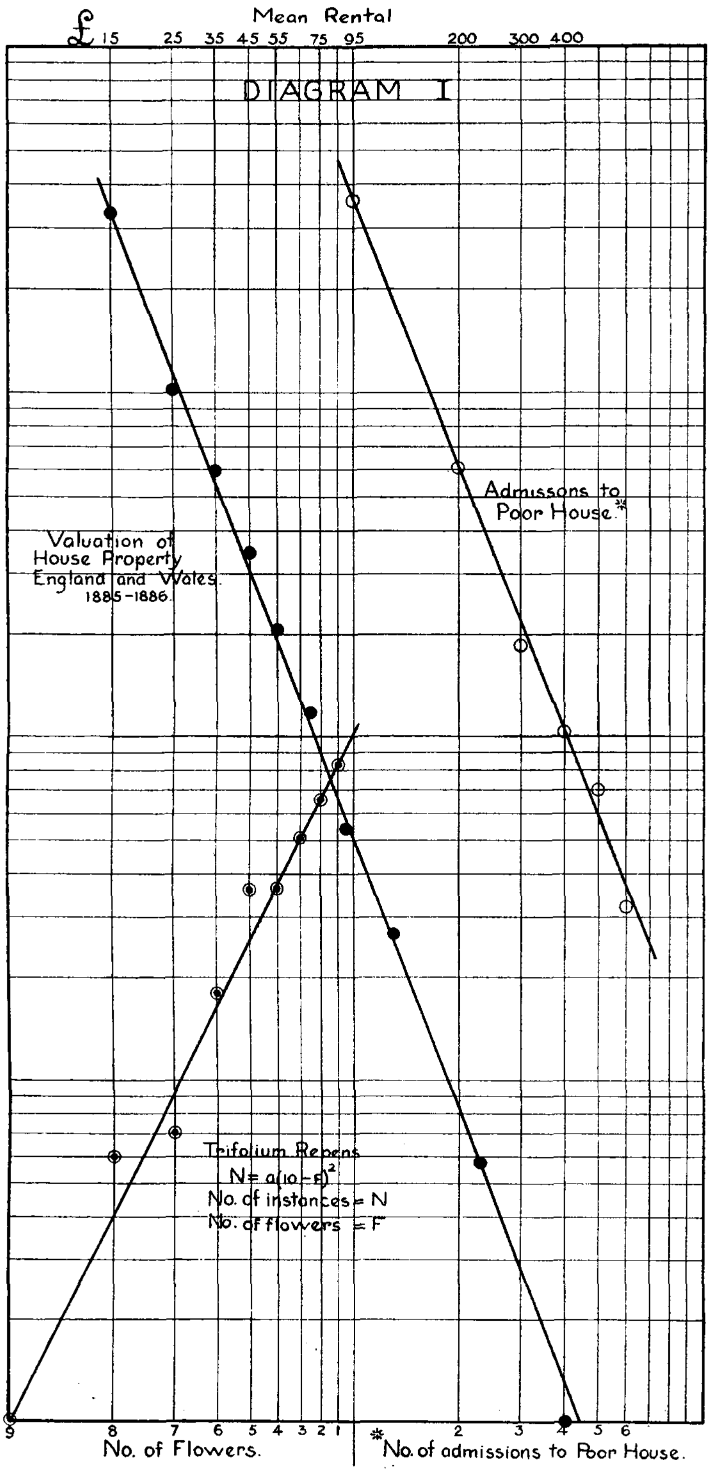
The third example is quoted by Prof. Pearson (1895) from De Vries. De Vries cultivated "a race of *Trifolium repens* in which the axis is frequently prolonged beyond the head of the flower and bears one to ten blossoms." If we look at this from the point of view that each additional blossom requires a greater expenditure of energy and that there is a maximum to the number of blossoms which may result, it is found that taking this maximum as 10 and plotting the frequency against the maximum less the number of flowers found, that the observations lie upon the straight line given by the formula

$$\log N = \log a + 2 \log (10 - F),$$

where N is the number of instances and F the total number of flowers.

As examples in the realm of biology three have been selected. Dr Ledingham (1912) has applied the formula to the phenomena associated with phagocytosis and finds that the binding of an opsonin and an organism and the phagocytosis of an organism by the leucocytes, both obey the same law, while Dr Martin and Miss Chick (1908) have shown that with certain types of disinfectant, the organism takes up the disinfectant in a manner consistent with the formula and that the destruction of the organism follows the same law.

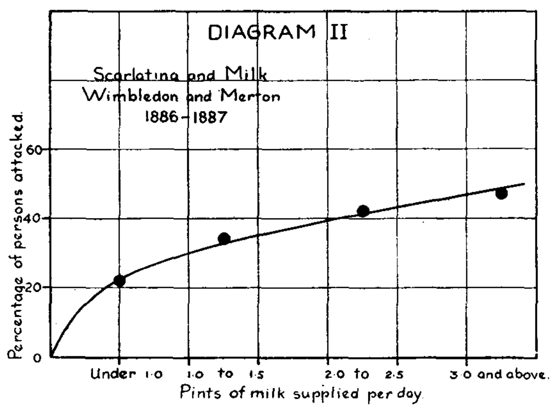
A third example taken from Carrel and Ebeling (1921) refers to the multiplication of fibroblasts *in vitro* when the growth is stimulated by adding embryonic juice to the medium of cultivation. The increase in the rate of



https://doi.org/10.1017/S0022172400034306 Published online by Cambridge University Press

growth is again closely described by the same type of formula suggesting that the action depends on adsorption.

As has been stated, the formula was first applied in the domain of public health by Dr Farr in 1875, when he used it to describe the relationship of the hygiene of a district to the concentration of the population. In making this application, Dr Farr took the crude death-rate as the measure of the hygiene of the district. The formula was later applied to the standardised death-rate and found not to give a good fit. I have shown elsewhere (Brownlee, 1920), however, that when the life table death-rate is used, the formula holds with a constant index, $n = 0{\cdot}1$, for each decade from 1861–1910. In the Boroughs of London also, the same relationship is found to obtain. The constant a under the influence of hygienic measures has continually decreased, but the index has not altered.



The formula also describes the data furnished by milk epidemics of scarlet fever (Diagram II). Of these, the most considerable in size was that of Wimbledon for the year 1886–87 (Power, 1886). It was carefully investigated and the observations published in detail. The amount of milk consumed per head in each family per day was recorded and compared with the number of the consumers who developed the disease. It will be observed that the rate of attack increases with the amount of milk consumed, at first rapidly, but the proportion infected to the amount of milk consumed falls with each increase in the amount of milk. The power of infection of the attacking organism varies in a manner consistent with the formula of adsorption. These observations thus confirm the work of Ledingham with reference to opsonins and phagocytosis.

The same phenomenon arises when the death-rates among patients suffering from diphtheria are classified according to the period after the beginning of the illness at which treatment is begun. This is illustrated by figures for Glasgow and London, figures which are in agreement with all others (Table I). It is found that with delay of treatment, the death-rate does not increase without limit. With treatment on the first day, a very low, or no mortality is found. Treatment on the second day is associated with a larger death-rate, but when treatment is delayed till the fifth or sixth day, a maximum mortality has been attained. Here again something like an adsorption balance between the human organism and the toxin is found. The late use of anti-toxin cannot ensure recovery, though in the event of recovery it may markedly hasten convalescence.

Table I.

Table showing the percentage mortality when cases of diphtheria are treated on different days of illness.

Day of treatment	Percentage mortality London, 1895	London, 1896	Glasgow, 1900
1st	4·6	5·3	—
2nd	14·8	14·8	9·0
3rd	26·2	21·9	15·5
4th	33·1	27·8	
5th and after	35·7	31·8	19·8

The phenomena considered have an important bearing on some present day problems in public health. Many inquiries with regard to the effect of environment on health and disease have been carried out and others are in contemplation. A considerable number of these have been conducted in a manner which, to my mind, seriously invalidates the conclusions, the inquiries not extending through a sufficiently wide range of environmental conditions. To illustrate this the facts regarding the density of population and the severity of scarlet fever and enteric fever in Glasgow in the five years 1898–1902 have been chosen. I have selected these figures because Glasgow was divided in 1872 into districts specifically on the ground of their sanitary conditions. The statistics thus afford much more homogeneous data than are obtainable elsewhere. These sanitary districts have been grouped according to the average number of persons per room. In the first column of the table (Table II) the populations of the grouped districts are given; in the second column the number of persons per room; in the succeeding columns the

Table II.

Table showing the number of cases and mortality of scarlet fever and enteric fever in the sanitary districts of Glasgow, grouped according to room density for the years 1898 *to* 1902.

Districts	Population 1901	Room density*	Scarlet fever Cases	Scarlet fever Deaths	Scarlet fever Mortality per cent.	Enteric fever Cases	Enteric fever Deaths	Enteric fever Mortality per cent.
Group I	34,868	·5 –1·0	864	20	2·3	106	10	9·4
,, II	83,255	1·0 –1·5	2148	76	3·5	389	50	12·8
,, III	201,098	1·5 –2·0	5439	226	4·1	1308	217	15·8
,, IV	87,885	2·0 –2·25	2184	110	5·0	711	116	16·3
,, V	237,161	2·25–2·5	5610	284	5·06	1743	296	16·9
,, VI	117,445	2·5 –2·75	2091	118	5·6	1003	164	16·3

* Room density means the average number of persons per room.

https://doi.org/10.1017/S0022172400034306 Published online by Cambridge University Press

number of cases of scarlet fever and enteric fever with their corresponding deaths and death-rates. It will be seen that something resembling the phenomena just discussed occurs here also. Life in unhealthy environment for a certain range of density of population acts so as to depress the vitality but there is a limit to the amount of injury which can take place. This may be different in different diseases. Taking first the statistics of enteric fever, it is seen that had an investigation as to the influence of insanitary conditions on mortality been carried out in those localities alone in which the number of inhabitants per room was over 1·5, the necessary conclusion would be that the environment of the person had no influence on the severity of the disease. The same also applies to scarlet fever but the upper limit of the death-rate is not reached till the concentration of the population is that of two persons per room. It is, however, only because of the very large number of cases of scarlet fever that it is possible to differentiate between the death-rate of Group III, and the death-rates of Groups IV, V and VI. These data regarding scarlet fever have been obtained automatically through the power of compulsory notification. It is not in the least likely that any health inquiry will be able to command the men and money to secure an equivalent number of observations.

I had intended to conclude this note by criticising some recent inquiries which have been carried out without the knowledge that, in addition to statistical fallacies, a biological fallacy might also be present. I have decided, however, to leave the matter as it stands as it is impossible to discuss whether this fallacy affects the conclusions of these statistical papers without working over the original statistical data on which they are based. I content myself therefore, with charting this shoal so that it may be given sea room in future investigations.

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(*MS. received for publication* 15. I. 1925.—Ed.)