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SUBGLACIAL WATER FLOW UNDER ICE STREAMS AND WEST ANTARCTIC ICE-SHEET STABILITY

by

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ABSTRACT

An analysis is made of the steady-state width of an ice sheet whose base is below sea-level, whose basal temperatures are such that appreciable melting occurs at the base, and which is fringed by fast-moving ice streams that drain most of the outward ice flux. The fast ice velocities of the ice streams are considered to be a consequence of substantial subglacial water flow underneath the ice streams. The source of this water is the water melted from the base of the ice sheet which is diverted to flow beneath the ice streams. If the depth of the sea at the edge of the ice sheet is not a function of the width of the ice sheet, then an ice sheet with a steady-state width is in a situation of unstable equilibrium. Only if the sea-level depth at the edge of the ice sheet increases as a function of ice-sheet width at a rate greater than the 2/3rd power of the width can a stable, steady-state ice sheet exist. This condition (taking into account elastic rebound) is not satisfied for the West Antarctic ice sheet along an ice-flow path from the ice divide above Byrd station out to the Ross Sea. An increase of the mean precipitation, such as might occur under a CO_2-induced climatic warming, would cause growth of both stable or unstable steady-state ice sheets.

INTRODUCTION

Two ways have been suggested by which the West Antarctic ice sheet may become unstable. It may surge. Wilson (1964) proposed that ice ages are initiated by surges in Antarctica. Hollin (1969) has strongly advocated Wilson's surge hypothesis of ice ages. Or it may be "geometrically" unstable. Weertman (1974), following the proposal of Mercer (1968) and of Hughes (1973) that the West Antarctic ice sheet is inherently unstable and is presently disintegrating through the retreat of its grounding line towards the ice divide, showed mathematically that it may be impossible for an ice sheet whose bed is below sea-level to be in a stable, steady-state situation of mass balance. By steady-state mass balance, we mean that the total accumulation is exactly balanced by outflow into floating ice shelves. Without steady-state mass balance the ice-sheet dimensions must change. Thus the ice sheet either grows until it reaches the continental slope or it shrinks until it is no more.

It is not known whether the Antarctic or the Greenland ice sheets, or the Pleistocene ice sheets, have ever surged. However, nothing is known to prove that they could not have surged. Until surges in glaciers are well understood, this uncertainty about ice-sheet behavior will remain with us.

There is one major qualitative difference between a valley glacier and an ice sheet. The ice flow at the periphery of an ice sheet can be channeled into very fast moving ice streams or outlet glaciers. The ice flow of the West Antarctic ice sheet at its periphery occurs almost entirely through fast ice streams. This kind of flow does not exist within valley glaciers. The velocities of fast ice streams and outlet glaciers are so great that they cannot be accounted for through the mechanism of internal ice deformation. The only other possibility is that these large velocities are produced by fast sliding at the base. The velocities of ice streams are of the same magnitude as the fast surge velocities in large surging glaciers. It has been suggested (Weertman 1962 and 1969) that fast ice streams and outlet glaciers be considered to be permanently surging glaciers. (An actual surging glacier moves at surge velocities for only a relatively short period of time, say for only 1 to 2 a out of a time period of 20 to 50 a. During a surge the glacier is elongated and is thinned. Ice streams and outlet glaciers move at fast velocities, of the order of 1 km a^{-1}, year in and year out, without producing any readily apparent changes in the ice-sheet dimensions.) Weertman suggested that the same mechanism produces the fast sliding velocities of surging glaciers and ice streams, namely, a water lubrication mechanism.

A natural question to ask is whether the presence of ice streams at the border of the West Antarctic ice sheet alters the likelihood that this ice sheet can surge or changes the "geometrical" stability condition. It is the purpose of this paper to answer, at least partially, this question. Of course, this is not the first time that the importance of ice streams to the stability of the West Antarctic ice sheet has been examined. In particular, Hughes (1975 and 1977) has considered how the disintegration of this ice sheet might take place through retreat of the grounding line of its ice streams. In our paper, the new feature is that water-flow conditions needed for fast sliding velocities are brought explicitly into the analysis.

ICE-STREAM VELOCITY FROM MASS-BALANCE CONSIDERATIONS

An ordinary glacier cannot move at surge velocities and retain a steady-state profile. The glacier consequently only moves at surge velocities for limited periods of time. On the other hand, ice streams can move at surge-like velocities all the time and their ice sheet could retain a steady-state profile. To understand this, consider the following sample mass-balance calculation. Let a be the average accumulation rate, L the half-width of an ice sheet (considered, for simplicity, to be two-dimensional), H the thickness of an ice stream, W the width of an ice stream, W_s the spacing of ice streams, and V the average velocity of an ice stream. From mass balance under steady-state conditions

$$V = aLW_s/WH \ . \tag{1}$$

For $a = 0.15$ m a^{-1}, $W_s/W = 3$, $L = 800$ km, and $H = 0.5$ km (values in the range found in West Antarctica), the velocity $V = 480$ m^{-1}. This velocity is of the order of those found for the ice streams. Thus existence of surge velocities in ice streams is not necessarily an indication that the ice sheet is actually surging. These high velocities are merely a reflection that ice flow which is channeled into ice streams, of necessity, must occur at relatively high velocities.

WATER FLUX AVAILABLE FOR LUBRICATING FAST SLIDING MOTION

If ice is funneled into ice streams then the water that is produced under an ice sheet also will be funneled to the bed of these ice streams. The effect of the slope of the upper ice surface generally dominates the effect of the slope of the lower ice surface in determining the effective pressure gradient that drives the water flow (Weertman 1972). At ice streams, both because the bed is likely to be eroded into the bedrock and because the upper ice surface of an ice stream is lower than the surrounding ice sheet, the average pressure gradient is such that water flowing at an ice-sheet bed will be driven by this gradient into the bed of an ice stream. If λ is the average thickness of water melted per unit area and per unit time from the bottom of an ice sheet, then, on a mass balance argument, the volume Q of water flowing past a unit width of an ice stream is given by

$$Q = \lambda LW_s/W \ . \tag{2}$$

(The term λ is a thickness in which spatial variation of bottom melting and freezing is averaged out. It is possible, and indeed likely, that λ will be a function of the size of an ice sheet rather than a constant. This complication is ignored in this paper.)

THICKNESS OF A WATER FILM

Were the water flux Q of Equation (2) to flow in a sheet of thickness d, this thickness would have a value (Weertman 1962 and 1969) given by

$$d = (12\eta Q/P_g)^{1/3} \ , \tag{3}$$

where η is the viscosity of water ($\eta = 5.7 \times 10^{-11}$Pa a $= 1.8 \times 10^{-3}$ Pa s) and P_g is the effective pressure gradient that drives the water flow. (Equation (3) is valid for laminar flow of water. If the flow is turbulent the water thickness is somewhat greater than given by this equation.)

Table I lists values of d calculated with Equations (2) and (3) using the values of L and W_s/W given above. The effective pressure gradient P_g is set equal to $\rho g \alpha$ where α is the upper ice slope, ρ is ice density ($\rho = 0.9$ Mg m^{-3}), and g is the gravitational acceleration ($g = 9.8$ m s^{-2}). A typical value of α for an ice stream seems to be of the order of $\alpha = 0.002$ rad. Three values of λ are used in Table I. A value of $\lambda = 5$ mm a^{-1}, corresponding to the amount of ice that the average geothermal flux can melt off an ice mass if all the geothermal heat is used for melting ice, a value of 10 mm a^{-1}, and a value of 15 mm a^{-1}. The latter two values correspond to either high geothermal heat fluxes or a combination of geothermal heat and heat of sliding. (A sliding motion of 50 m a^{-1} under a shear stress of 0.5 bar (50 kPa) produces enough heat to melt 8 mm a^{-1} of ice from the bottom of a glacier. The West Antarctic ice sheet, of course, is partly in a volcanic region where higher than normal geothermal heat fluxes can be expected.)

TABLE I. THICKNESS OF WATER LAYER*

λ (in mm)	d (in mm)
5	8
10	10
15	11

* Calculated using $W_s/W = 3$, $L = 800$ km, $\alpha = 0.002$.

SLIDING VELOCITY OF AN ICE STREAM

Weertman's sliding theory (1969) predicts that, when a water film of average thickness d is present at the bed of a glacier or ice sheet, the sliding velocity S is given to a first approximation as

$$S = S_0(1 + 10d/d_c^*), \tag{4}$$

where S_0 is the sliding velocity when the water thickness is negligibly small and d_c^* is the dimension of a critical obstacle size. The value of both d_c^* and S_0 depends upon the value of the shear stress at the bed and bed roughness. A reasonable value of S_0 might be $S_0 = 20$ to 50 m a^{-1} and of d_c^* of the order of 2 to 5 mm. If d is of the order of or larger than d_c^*, then Equation (4) reduces to

$$S = S_0(10d/d_c^*). \tag{5}$$

A similar increase in sliding velocity that is produced by a water film is expected (Weertman 1979) in the Nye-Kamb theory of glacier sliding (Nye 1969 and 1970, Kamb 1970, Raymond 1980).

From Table I, it is seen that, for ice streams, d is larger than d_c^*. The sliding velocity S is thus of the order of 10 to 50 times larger than S_0 and consequently is of the order of the observed fast velocities of ice streams. The water lubrication mechanism appears, therefore, capable of accounting for the fast motion of ice streams.

The sliding velocity S can be expressed as

$$S = (10S_0/d_c^*)(12\eta\lambda\, LW_s/WP_g)^{1/3} \tag{6}$$

after combining Equations (2),(3), and (5). At fast sliding velocities $S \approx V$, because internal ice deformation of an ice stream makes a relatively small contribution to V.

In Equations (1) and (6) it has been assumed implicitly that the length of a fast moving ice stream is short compared with the half-width L of an ice sheet. This condition holds in West Antarctica. Hence, to a first approximation, V and S have a constant value down an ice stream. Actually, the velocity of ice streams seems to increase with distance between them (Hughes 1975 and 1977). The water that is produced by melting ice from the bottom of an ice stream is ignored in Equation (6). But the amount melted may be realtively large. For example, for a sliding velocity of 250 m a^{-1} under a basal shear stress of 50 kPa, the additional water flux Q at the end of a 100 km ice stream is 30% of that produced under the ice sheet for the case of λ = 5 mm, L = 800 km, and W_s/W = 3. The increased amount of water will cause an increase of the sliding velocity down an ice stream.

STEADY-STATE WIDTH OF AN ICE SHEET WITH ICE STREAMS

Under steady-state conditions the velocity V given by Equation (1) must equal approximately the sliding velocity S given by Equation (6). Combining these two equations results in the equation

$$L = (10S_0H/ad_c^*)^{3/2}(12\eta\lambda/P_g)^{1/2}(W/W_s). \qquad (7)$$

Equation (7) predicts a half-width L = 890 km, approximately the current value of the west Antarctic ice sheet, when S_0 = 50 m a^{-1}, d_c^* = 5 mm, λ = 5 mm a^{-1}, H = 500 m, W_s/W = 3, a = 0.15 m a^{-1}, and α = 0.002.

The value of H is approximately equal to $H_0(\rho_s/\rho)$, where H_0 is the depth of the sea at the border of the ice sheet where it loses contact with the bed. Here ρ_s is the density of sea-water. In the approximation that the basal shear stress τ of an ice stream is roughly a constant, the terms S_0, which is, according to sliding theory (Raymond 1980), roughly proportional to τ^2, and P_g, which is roughly equal to τ/H, are both approximately constants. Thus, for a fixed value of H_0 and the ratio W_s/W, the value of the half-width L of an ice sheet is determined. This steady-state width of an ice sheet is, however, not one of stable equilibrium if H_0 is independent of the value of L. According to Equation (1), V is proportional to L, and, according to Equation (6), S is proportional to $L^{1/3}$. Hence, if the half-width L were larger than the value given by Equation (7), the amount of ice that has to be transported through the ice streams under steady-state conditions requires higher velocities than are predicted by Equation (6). Thus, as time went by, ice would be added to the ice sheet and the ice sheet would grow both in elevation and horizontal extent. No equilibrium half-width position would be approached. By the same argument, if the half-width were smaller than that given by Equation (6), the ice streams would transport into the ice shelves, or calve off directly into the sea, more ice than could be supplied by the total accumulation on the ice sheet. In this circumstance, the ice sheet eventually would disappear. Only if H_0 had a functional dependence with L that increased faster than a proportionality of $H_0 \propto L^{2/3}$ could a stable, steady-state half-width be found. Of course, if the ratio W_s/W were to vary suitably with L a stable half-width could also be found. However, it seems unlikely that, once fast ice streams are formed, their spacing would thereafter change.

PROBABILITY OF WATER FILM BEING THE DOMINANT FLOW PATH

Because of past criticisms, the use of a water film of non-negligible thickness to account for fast sliding velocities requires some justification. It has been proposed that a water film of appreciable thickness cannot exist because water flow may take place primarily in channels (Röthlisberger 1972, Nye 1973). Consider Figure 1. It shows a schematic cross-section of the bed in which water flow occurs in parallel directions through channels of width w_c and thickness d_c and through a film of thickness d_f and width w_f. The water flux Q is now given by

$$Q \approx (P_g/12\eta)(w_cd_c^3 + w_fd_f^3)/(w_c + w_f), \qquad (8a)$$

which, when $w_c \approx d_c$ and $w_c \ll w_f$, reduces to

$$Q \approx (P_g/12\eta)(d_c^{\,4}w_f^{-1} + d_f^3). \qquad (8b)$$

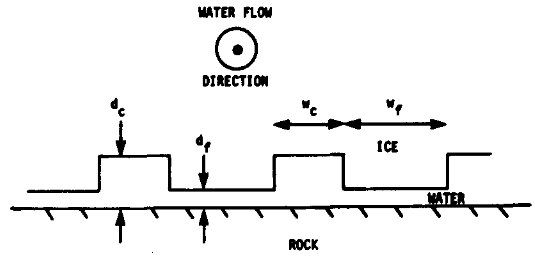

Fig.1. Water flow in parallel in channels and in films.

When $d_c^4 \gg w_fd_f^3$ most of the water is transported through the channels. Hence there is no need for appreciable transport of water in a water film.

If channels are to be the dominant water flow paths, it is essential that they should be able to capture the water from the water film. However, it has been shown (Weertman 1972) that, if an appreciable shear stress exists at the bed, this may not be possible. The pressure gradient in the vicinity of a channel is such that it drives water of the film away from the channel rather than into it. This is true for channels of arbitrary orientation that are incised upward into the ice (Röthlisberger, or R, channels) and for channels lying parallel to the ice-flow direction that are incised downwards into the bedrock (Nye channels). Only in the case of a tributary Nye channel (Fig.2) is there the possibility that the channel can capture water from a water

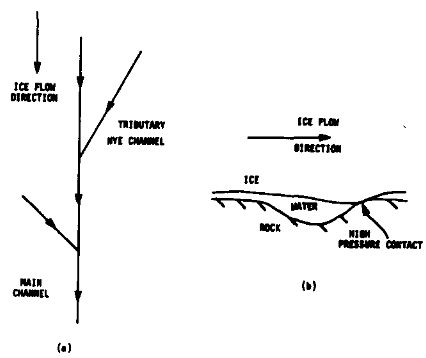

Fig.2. (a) Tributary Nye channel.
(b) Cross-section of tributary Nye channel.

film. The requirement is for a high pressure ice-rock contact to exist on the ice down-flow side of the channel that could form a barrier to water flowing in the film and force this water into the channel. But tributary Nye channels can be destroyed by the erosion produced under the sliding motion. Erosion by flowing water, of course, can deepen a channel cut into bedrock. The amount of water flowing at the end of a tributary channel, i.e. the end furthest from a main channel, is virtually zero. Hence the far ends of the Nye channels will be eroded away faster than they can be deepened. Once this occurs, even less water flows through the channel, and erosion, consequently, will shorten its length still further. So it is very doubtful that Nye channels can exist for long under ice streams. (For temperate glaciers, in which water can reach Nye channels in the bed through moulins, the situation is more complicated and this argument does not always apply. The upper surface of a West Antarctic ice stream is well below the freezing temperature of water and, therefore, surface melt water does not reach the bed. Our present, somewhat confused, understanding of the water-flow processes under temperate glaciers is summarized by Raymond (1980). Because the surface melt water production is an order of magnitude larger than the bottom melt production, because it is seasonal, and because this surface water arrives at the glacier bottom at discrete points, the water flow is strongly time-dependent and probably is time-dependent in the manner of flow, whether in channels or films.)

There is a second, additional argument against water from a film entering tunnels. The argument that the pressure gradient in the vicinity of a tunnel is such that it drives away water in the film considers the water pressure in the film to be the overburden pressure plus the change in pressure in the stress field of the tunnel itself. On the other hand, when there is sliding motion, the pressure in the water film, in general, is smaller than the hydrostatic pressure within the ice (see discussion in Robin and Weertman 1973). However, this reduced pressure does not change the direction of the pressure gradient. Moreover, it can require that the water pressure within the tunnel be a factor of the order of only one-half the overburden pressure if the water pressure in the tunnel is to be smaller than that in the film. But an R tunnel would creep shut virtually instantaneously (and a Nye channel fill itself with ice) under a pressure differential of this magnitude under a thick ice stream.

Lliboutry (1968 and 1979), in his theory of sliding, discounts the importance of water films in the determination of sliding velocity and argues that it is the water pressure at the bed of a glacier that is important for determining whether fast sliding velocities should occur or not. (Increased water pressure causes increased bed separation and faster sliding velocities.) He has, however, never made a detailed calculation of just how the water is transported down a glacier bed. His picture of water at the bed, we believe, can be schematicized as shown in Figure 3. Thick pockets of water exist behind

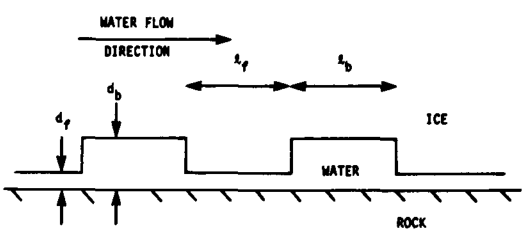

Fig.3. Water flow in series through pockets and films.

bumps (not shown) at the bed. These pockets are indicated in Figure 3 as a region of thickness d_b and width ℓ_b. Between these regions, the water flows in a thin film of thickness d_f and width ℓ_b. The direction of water flow is from the thick pockets into the thin films and back into the thick pockets. Thus the problem is of flow in series whereas, in Figure 1, it was flow in parallel. The average pressure gradient P_g driving the flow is given by

$$P_g = (\ell_b P_{gb} + \ell_f P_{gf})/(\ell_b + \ell_f), \qquad (9)$$

where P_{gb} is the pressure gradient in the pockets and P_{gf} is that in the film. Now $Q = P_{gb} d_b^3/12\eta = P_{gf} d_f^3/12\eta$. Thus, when d_b is larger than d given by Equation (3), the following limits can be set on d_f

$$(12\eta Q/P_g)^{1/3}[\ell_f/(\ell_b + \ell_f)]^{1/3} < d_f < (12\eta Q/P_g)^{1/3} \quad . \qquad (10)$$

Since it is difficult to conceive that ℓ_f is ever very small compared with ℓ_b, Equation (10) predicts that d_f is essentially given by Equation (3). Under ice streams the thickness of the water film between the pockets cannot be of negligible thickness, as seems to be required in Lliboutry's analyses. Hence a Lliboutry-type picture of the bed must reduce to that which we have considered of a water film of appreciable thickness when there is a large quantity of water flowing at the bed that is not diverted into channels.

DISCUSSION

We conclude that, if water is produced under the West Antarctic ice sheet in appreciable amounts, it would be diverted to flow under the ice streams existing at the periphery of the ice sheet. If the amount of water melted from the bottom is only of the order of 5mm a^{-1}, a water layer of thickness sufficient to cause the ice streams to slide with fast velocities would exist under these ice streams. It does not seem likely that the water flow under the ice streams would be further diverted into tunnels. (It is known, of course, from one drill hole and from radar reflection data that, at a number of basal positions, the West Antarctic ice sheet is indeed at the melting temperature.)

The preliminary calculations of our paper indicate that the grounding line of ice streams cannot be at a stable, steady-state equilibrium position unless the sea floor deepens with distance from the ice divide at a rate that is larger than distance to the 2/3rd power. This condition (taking into account elastic rebound) is not satisfied for the West Antarctic ice sheet along an ice flow path from the divide above Byrd station out to the Ross Sea. Of course, if the average bottom melt rate or the accumulation rate are rapidly varying functions of the size of the ice sheet this 2/3rd power dependence stability would be modified.

Because the West Antarctic ice streams already move with surge velocities and these velocities are of the order of magnitude that is required to remove the continuously present total accumulation, it is difficult to see how or why this ice sheet could surge in its present configuration.

What would be the effect of an increased accumulation rate such as might occur under a CO_2 induced climatic warming? When the sea water depth increases at a rate smaller than a 2/3rd power of ice sheet width relationship the (unstable) equilibrium width of the ice sheet decreases with increased precipitation. Hence a steady-state ice sheet would undergo uncontrolled growth. If the sea-water depth increased at a rate greater than the 2/3rd power of width relationship the (stable) equilibrium width of the ice sheet would increase with increased precipitation, and such an ice sheet would grow until it reached a new stable equilibrium width. In either case, the final state of the ice sheet would require at least several thousand years to be reached because of the relatively slow rate of growth of a non-equilibrium ice sheet.

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