

EFFECTS OF LARGE-SCALE STRUCTURE UPON THE DETERMINATION OF H_0 FROM TIME DELAYS

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Abstract. We have analyzed the effects of both large-scale inhomogeneities in the mass distribution and cosmological gravitational waves upon the time delay between two images in a gravitational lens system. We have shown that their leading order effect, which could potentially bias the determination of the Hubble parameter, is indistinguishable from a change in the relative angle between the source and the lens axis. Since the absolute angular position of the source is not directly measurable, nor does it enter the relationship between the Hubble parameter and the lens observables, the determination of H_0 from gravitational lens time delays follows in the usual way, as if the metric perturbations were absent.

We have considered a thin gravitational lens in the weak field approximation, embedded in a spatially-flat FRW cosmology, with small-amplitude and long-wavelength scalar and tensor metric perturbations $h_{\mu\nu}$. Our approach relied upon Fermat's principle, which is also valid in the non-stationary space-time under consideration (Kovner 1990). We extremized the time of travel within a family of zig-zag null photon paths composed of two segments, each one a geodesic of the perturbed FRW background in the absence of the deflector.

Our main result is that a lens system with the source located at an absolute angular position β (bold characters denote two-components angular

vectors measured with respect to the lens axis) in the presence of scalar and tensor metric perturbations is, to leading order, indistinguishable from an identical lens system in the absence of perturbations, but with a different source alignment β_{eff} given by (Frieman et al. 1994, Surpi et al. 1995),

$$\beta_{\text{eff}} \equiv \beta + \beta_{\text{pert}} \quad \text{with} \quad \beta_{\text{pert}} = \frac{1}{r_d} \int_0^{r_d} dr \Delta(r) - \frac{1}{r_s} \int_0^{r_s} dr \Delta(r) \quad , \quad (1)$$

$$\Delta^i(r) = -\frac{1}{2} \int^r dr (h_{00,i} + h_{33,i} - 2h_{i3,3} - 2h_{i3,0}) \quad . \quad (2)$$

Here r is the comoving coordinate distance along the lens axis, coincident with the z -axis. $r = 0$, r_d and r_s correspond to the observer, deflector and source respectively. The index i takes the values $i = 1, 2$. The result is valid both for scalar as well as tensor perturbations, the former described in the longitudinal gauge and the latter in the transverse-traceless gauge.

The change in the effective alignment induced by a single mode of the metric perturbations can be as large as the perturbation amplitude $|h_{\mu\nu}|$. Compatible with current limits, fixed by the cosmic microwave background anisotropy, it could be of order $10^{-5} \approx 1''$. Thus, the time delay directly induced by metric perturbations could be as large as the intrinsic delay due to the lens geometry. The alignment angle is not, however, directly observable. In particular, β_{eff} cancels out from the expression that allows determination of H_0 from the time delay and other lens observables (Refsdal 1964). We thus conclude that the leading order effect of scalar and tensor metric perturbations does not compromise the program to determine H_0 from gravitational lens time delays.

Our analysis is limited to the effects of small-amplitude (i.e. linear) perturbations. Non-linear structure on small scales (e.g. galaxies) could also significantly affect the distance measure, which we approximated by its FRW expression. We have also limited ourselves to long-wavelength metric perturbations, and have discarded terms $\xi/\lambda \lesssim 10^{-3}$ times smaller than the leading order terms, where ξ is the maximum transverse separation between the image paths (of order 10 kpc) and λ is the wavelength of the perturbation (larger than 10 Mpc). The neglected terms would compromise the lens reconstruction only at levels below tenths of arc seconds in the images' angular separation (Seljak 1994).

References

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