ON THE COMPUTATION OF ACCURATE EARTH ROTATION BY THE CLASSICAL ASTRONOMICAL METHOD

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Improved series of Universal Time and latitude measurements back to 1962 have been provided to the BIH by several observatories recently. New techniques are currently or will soon provide Earth rotation data that are independent from the classical astronomical observations. In the meantime, the BIH has acquired experience on possible methods for achieving better accuracy. These reasons make it worthwhile to apply our present practical knowledge to the past data. The method which will be used for computing Earth rotation data from the updated BIH files is presented.

INTRODUCTION

The worldwide net of about eighty stations which provide Earth rotation data by classical astronomical methods may be considered as a unique instrument, the function of which is to monitor the orientation of the Earth in an inertial reference frame. As any measuring instrument, it may be described by its accuracy and precision. Only accuracy will be considered in the present paper. An instrument is accurate when it is able to provide measures without any systematic trend, or, in other words, when its measurements remain fixed to the same reference frame.

In the operation of the global astronomical instrument for measuring Earth rotation, the Earth is represented by a sphere of station zeniths (Danjon, 1962), and the inertial reference frame is provided by stars that are observed through dedicated telescopes. Accuracy can be achieved by a comprehensive analysis and an adequate correction of all spurious effects which may appear. The questions which are to be answered for this purpose are the following:

1 - What causes deformations to the sphere of zeniths, and how should these effects be minimized;

2 - What causes motions and deformations to the celestial reference frame, and how should their effects be minimized;

3 - If one wants to devise a method to get rid of systematic errors what is the error power spectrum associated with 1 and 2.

The method presented below is a possible answer to these questions. It

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will be applied to the improved data provided by the observatories. This new reduction will benefit from the comparison with the completely independent Doppler method for polar motion. It is hoped that it will provide a better basis for future comparisons with lunar laser ranging and long base interferometry determinations of the Earth's rotation.

1. THE CAUSES OF INACCURACY

The perturbations with effects larger than $0\overset{''}{.}001$ are listed in Tables 2 and 3. Some comments follow.

1.1. Practical realization of the terrestrial reference frame

Global motions. Some terms of the Earth tides have local effects; some cause a real change in the Earth's rotational speed. Continental drift gives motions that are common to subsets of stations; absolute velocities (Solomon and Sleep, 1974) have to be considered.

Local motions. Real local tectonic motions may take place. The instruments and their operation (Niimi, et al., 1976) and atmospheric conditions (Hughes, 1974) may give rise to large-scale errors and to flicker noise (Barnes, 1969).

Table 1 shows the rms differences between series of observations obtained with similar instruments and identical programs. The averaging time is 0.05 years; when the instruments are not located in the same observatory the BIH results were used as an intermediate reference. The figures in Table 1 show also the large differences which may exist in the precision of the results from identical instruments.

Table 1. Rms differences between identical instruments with the same program.

Instruments	Dates	UT	Latitude
2 astrolabes	1971.05 - 1971.40	$0\overset{s}{.}0128$	$0\overset{''}{.}097$
2 astrolabes	1971.75 - 1973.80	0.0201	0.138
2 PZTs	1974.00 - 1975.95	0.0051	0.076
2 PZTs	1975.00 - 1976.95	0.0056	0.043
2 transit inst.	1972.00 - 1974.85	0.0206	--

1.2. Practical realization of the celestial reference frame

Fundamental constants. The erroneous conventional precession constant has no effect (Fricke, 1977); the use of an erroneous value for nutation in longitude gives a common effect on observations (Feissel and Guinot, 1976). The aberration constant is now sufficiently well known, but its change in 1968.0 has effects on the time series.

The FK 4 is intended to provide an inertial reference frame. This is realized to a certain precision. Yet, some instruments are devised in such a way that the stars observed (with some exceptions) cannot be taken from the FK 4, and the programs cannot remain unaltered for many years. We have, then, to consider separately the fundamental system and the local systems used by these instruments.

Fundamental catalog. It has been studied by Fricke (1972), Lederle (1978) and others. Independent information on positions and proper motions is given by catalogs of FK 4 stars which were obtained from instruments used for the determination of the Earth's rotation (Pavlov, et al., 1971; Billaud, 1972; Afanas'eva and Gorshkov, 1974; Billaud, et al., 1978).

Local stellar systems. They usually are taken from the General Catalog. Studies of PZT catalogs (Yasuda and Hara, 1964; McCarthy, 1973; Takagi, et al., 1976; Greenwich, 1976) and those for zenith telescopes (at Blagovestchensk, Borowiec, Engelhardt, etc.) show the initial errors in positions and proper motions of such programs as compared to the fundamental catalog. Changes of programs, of positions, or of proper motions of stars are made in order to improve the local system. A side effect of these changes is an alteration of the local reference system.

2. CORRECTION OF INACCURACIES

We use the following notation:

t - date; T - local sidereal time of observation;
$[\mathrm{UT0}(i)-\mathrm{UTC}](t)$ - UT measurement of station i at date t;
$[\phi(i)](t)$ - latitude measurement of station i at date t;
L_i, F_i - reference longitude and latitude of station i;
$[\mathrm{UT1}-\mathrm{UTC}](t)$, $x(t)$, $y(t)$ - UT and polar coordinates at date t;
$\xi(t)$, $\eta(t)$ - coordinates of the pole of the catalog.

The classical equations (1) and (2) for deriving the Earth's rotation are relevant only if the data are accurate.

$$[\mathrm{UT0}(i)-\mathrm{UTC}]\,(t) = (-x(t)\sin L_i + y(t)\cos L_i)\tan F_i + [\mathrm{UT1}-\mathrm{UTC}](t). \tag{1}$$

$$[\phi(i)](t) - F_i = x(t)\cos L_i + y(t)\sin L_i. \tag{2}$$

Accuracy will be achieved by adding to (1) and (2) correcting terms for all causes of inaccuracy. The chosen method of correction will depend on whether the perturbation involved is constant or variable, modeled or not, local or common to all stations.

Table 2. Perturbations with a constant effect ($\geq 0\overset{''}{.}001$).

Form	Cause and amplitude (UT0(i); ϕ(i))	
periodic	Common	Local
period < 0.1y	Earth tides ($0\overset{s}{.}002$; 0) diurnal nutation (0; $0\overset{''}{.}001$-$0\overset{''}{.}006$)	Earth tides* ($0\overset{s}{.}0005 \tan F_i$; $0\overset{''}{.}01 \genfrac{}{}{0pt}{}{\sin}{\cos} 2F_i$) diurnal nutation (($0\overset{s}{.}0001$-$0\overset{s}{.}0004$) $\tan F_i$; 0)
period $= 0.5$y	Earth tide ($0\overset{s}{.}005$; 0)	Earth tide* (0; $0\overset{''}{.}001 \sin 2F_i$) change of aberration constant ($0\overset{s}{.}0001 \sec F_i$; $0\overset{''}{.}001 \sin F_i$)
period $= 0.95$y	diurnal nutation (0; $0\overset{''}{.}001$)	diurnal nutation ($0\overset{s}{.}0001 \tan F_i$; 0)
period $= 1.0$y	Earth tide ($0\overset{s}{.}002$; 0) diurnal nutation (0; $0\overset{''}{.}011$) FK 4 position errors function of α($0\overset{s}{.}005$; $0\overset{''}{.}05$)	Earth tide* ($0\overset{s}{.}001 \tan F_i$; $0\overset{''}{.}02 \cos 2F_i$) diurnal nutation ($0\overset{s}{.}001 \tan F_i$; 0) local catalog position errors ($0\overset{s}{.}01$; $0\overset{''}{.}1$) change of aberration constant (0; $0\overset{''}{.}005 \cos F_i$) change of program ($0\overset{s}{.}01$; $0\overset{''}{.}1$) change of star coordinates ($0\overset{s}{.}005$; $0\overset{''}{.}05$)
period $= 9.3$y	Earth tide ($0\overset{s}{.}001$; 0)	
period $= 18.6$y	Earth tide ($0\overset{s}{.}15$; 0)	Earth tide* (0; $0\overset{''}{.}001 \sin 2F_i$)
biases	FK 4 position errors function of δ ($0\overset{s}{.}005$; $0\overset{''}{.}05$)	local catalog bias Earth tide* (0; $0\overset{''}{.}01 \sin 2F_i$)
steps		change of aberration constant ($0\overset{s}{.}002 \sec F_i$; 0)
white noise	FK 4 mean error of positions	local catalog mean error of positions
flicker noise	FK 4 mean error of proper motions	local catalog mean error of proper motions

*and deflection of the vertical

Table 3. Perturbations with a variable effect ($\geq 0''.001$/y).

Form	Cause and amplitude (UT0(i); ϕ(i))	
	Common	Local
variation of annual term	nutation (principal term) (0; $0''.02$, period 18.6y FK 4: μ errors function of α ($0^{s}.0001$/y; $0''.001$/y)	nutation (principal term) ($0^{s}.002$ tan F_i, period 18.6y; 0) local catalog: μ errors ($0^{s}.001$/y; $0''.01$/y) change of star proper motions ($0^{s}.001$/y; $0''.01$/y) refraction, instrument ($0^{s}.002$; $0''.03$)
drifts	FK 4: μ errors function of δ ($0^{s}.0001$/y; $0''.001$/y)	continental drift ($0^{s}.0001$/y; $0''.001$/y) instrument, local tectonic motions (extremely variable)
steps		change of program instrument + equipment local tectonic motions
flicker noise		climate observer, plate measurement

The corrections are of three different kinds: 1) conventional expressions for modeled perturbations; 2) addition of auxiliary unknowns

$$w(t)\tan F_i = (\xi(t)\cos T + \eta(t)\sin T)\tan F_i \text{ in (1)},$$

$$z(t) = -\xi(t)\sin T + \eta(t)\cos T \text{ in (2)};$$

3) empirical corrections regularly updated by a prediction method according to the type of noise in the data (Feissel, 1976)

$$C_i(t) = a_i + b_i \sin 2\pi t + c_i \cos 2\pi t + d_i \sin 4\pi t + e_i \cos 4\pi t. \quad (3)$$

The complete treatment which will be applied is summarized in Table 4.

Table 4. Correction of the perturbation.

Perturbation	Correction
nutation, Earth tides, deflection of the vertical, change of astronomical constants	conventional expressions
pole of catalog (residual motion) proper motions (common error)	auxiliary unknowns $w(t)$, $z(t)$
polar reference $\neq$ CIO star position errors function of δ	initial calibration of L_i, F_i (or a_i)
refraction star position errors function of α	calibration of b_i, c_i, d_i, e_i versus global solution
continental drift, proper motion errors in FK 4, change of program or local catalog	updating of a_i
climatic variations, proper motion errors in local catalog, change of local program or local catalog	updating of b_i, c_i, d_i, e_i
instrumental deformations	optimal prediction of a_i
refraction, observer, thermal effects on instruments	optimal prediction of b_i, c_i, d_i, e_i

REFERENCES

Afanas'eva, P. M., and Gorshkov, V. L.: 1974, Astron. Zh. 51, pp. 652-7.
Barnes, J. A.: 1967, Nat. Bureau of Standards (USA) Report 9284.
Billaud, G.: 1972, Astron. Astrophys. 19, pp. 181-188.
Billaud, G., Guallino, G., and Vigouroux, G.: 1978, Astron. Astrophys. 63, pp. 87-95.
Danjon, A.: 1962, Bull. Astron. 23, pp. 187-230.
Feissel, M.: 1976, 29th Journées Luxembourgeoises de Géodyn. (unpub.).
Feissel, M. and Guinot, B.: 1976, Mitt. Lohrmann Obs. Dresden 33, p. 949
Fricke, W.: 1972, Ann. Review Astron. Astrophys. 10, pp. 101-128.
Fricke, W.: 1977, Astron. Astrophys. 54, pp. 363-366.
Greenwich Time Report: 1976, January-March.
Hughes, J. A.: 1974, Publ. Obs. Astron. Beograd No. 18, pp. 63-81.
Lederle, T.: 1978, Bull. Centre de Données Stellaires 14, pp. 62-68.
McCarthy, D. D.: 1973, Astron. J. 78, pp. 642-649.
Niimi, Y., Oguma, I. and Matsunami, N.: 1976, Publ. Astron. Soc. Japan 28, p. 693.
Pavlov, N. N., Afanas'eva, P. M. and Staritsyn, G. B.: 1971, Trudy Glavnoj Astron. Observ. V. Pulkovo 78, pp. 4-27.
Solomon, S. C. and Sleep, N. H.: 1974, J. Geophys. Res. 79, p. 2557.
Takagi, S., Murakami, G., Kitago, H., Sakai, S., Iwadate, K.: 1976, Publ. Int. Latitude Obs. Mizusawa 10, pp. 179-191.
Yasuda, H. and Hara, H.: 1964, Ann. Tokyo. Astron. Obs. 8, pp. 162-168.