

Session I

Primordial Nucleosynthesis and the First Stars in the Universe



Gary Steigman during his talk.



Volker Bromm during his talk.

Primordial Nucleosynthesis After WMAP

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Abstract. During its early evolution, the hot, dense Universe provided a laboratory for probing fundamental physics at high energies. By studying the relics from those early epochs, such as the light elements synthesized during primordial nucleosynthesis when the Universe was only a few minutes old, and the relic, cosmic microwave photons, last scattered when the protons, alphas, and electrons (re)combined some 400 thousand years later, the evolution of the Universe may be used to test the standard models of cosmology and particle physics and to set constraints on proposals of physics beyond these standard models.

Keywords. early universe, nucleosynthesis, abundances, cosmic microwave background.

1. Introduction

Primordial, or Big Bang, Nucleosynthesis (BBN) provides a key probe of the physics and early evolution of the Universe, as do observations of the cosmic microwave background (CMB) radiation. These probes offer windows on the early Universe at two widely separated epochs: BBN when the Universe was only ~ 20 minutes old and the CMB some 400 thousand years later. BBN and the CMB provide complementary tests of the consistency of the standard, hot big bang cosmology and offer observational tests of its quantitative predictions. For a recent review, see Steigman (2007); for a comparison between the predictions of BBN and the CMB, see Simha & Steigman (2008).

The standard models of cosmology, and of standard, big bang nucleosynthesis (SBBN), employ the general theory of relativity (GR) to describe an expanding Universe, filled with radiation (including three flavors of light neutrinos) and matter (including non-baryonic dark matter). For our purposes here, the presence or not of dark energy is irrelevant since dark energy (or a cosmological constant) plays no role in the physics of the early Universe which concerns us here. For BBN, the relic abundances of D, ^3He , ^4He , and ^7Li are predicted as a function of two cosmological parameters: the baryon density parameter η_{B} and the expansion rate parameter $S \equiv H'/H$, where H is the standard model value of the Hubble parameter and $S \neq 1$ allows for a large class of non-standard models of particle physics and/or cosmology. For SBBN it is assumed that the Hubble parameter assumes its standard model value ($S = 1$), so that the relic abundances depend on only one cosmological parameter, η_{B} .

Here, to assess the current status of the standard models of particle physics and cosmology, we'll ask if the BBN-predicted and observationally inferred relic abundances of the light nuclides agree and, if the BBN-determined values of η_{B} and H are consistent with the values inferred from independent (non-BBN) cosmological observations, including those of the CMB and of the Large Scale Structure (LSS) observed in the Universe.

2. Defining The Cosmological Parameters

Baryon Density Parameter

In the relatively late, early Universe, at the time of BBN (and later), the only baryons present are the nucleons, the neutrons and protons. Hence, “baryons” and “nucleons” will be used interchangeably. As the Universe expands, all densities decrease so, to define a parameter which provides a measure of the baryon/nucleon abundance, it is convenient (and conventional) to compare the baryon number density to the number density of CMB photons: $\eta_B \equiv n_B/n_\gamma$. Since η_B is very small, it is convenient to introduce $\eta_{10} \equiv 10^{10}\eta_B$. In terms of the baryon density parameter, the baryon *mass* density parameter $\Omega_B \equiv \rho_B/\rho_{\text{crit}}$ may be written as $\Omega_B h^2 = \eta_{10}/274$, where the present value of the Hubble parameter is $H_0 \equiv 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Steigman (2006b)).

The annihilation of relic electron-positron pairs in the early Universe, when $T \lesssim m_e c^2$, produces “extra” photons which are thermalized and become part of the CMB observed today. Since BBN occurs after e^\pm annihilation is complete and, since baryons are conserved, the numbers of baryons and CMB photons in every comoving volume in the Universe are (should be) unchanged from BBN to recombination to the present epoch ($N_B/N_\gamma = \eta_B = \text{constant}$). As a result, the value of η_B inferred from the comparison of the BBN predictions with the abundance observations should agree with the value inferred from the CMB (supplemented by observations of LSS).

Expansion Rate Parameter

For the standard cosmology the expansion rate (the Hubble parameter) during the early evolution of the Universe is determined solely by the mass/energy density of the Universe: $H^2 \propto G\rho$, where G is Newton’s gravitational constant and ρ is the energy density which, during the early evolution of the Universe, is dominated by “radiation” (e.g., massless or relativistic particles).

In the presence of non-standard physics and/or cosmology (e.g. modifications to GR or, to the standard model particle content leading to $\rho \rightarrow \rho' \neq \rho$),

$$S^2 = (H'/H)^2 = G'\rho'/G\rho \equiv 1 + 7\Delta N_\nu/43, \quad (2.1)$$

where

$$\Delta N_\nu \equiv (\rho' - \rho)/\rho_\nu. \quad (2.2)$$

ΔN_ν parameterizes any difference from the standard model, early Universe predicted energy density, normalized to the contribution from one additional light neutrino. For SBBN, $N_\nu = 3$ and $S = 1$ ($\Delta N_\nu = 0$). However, since any departures from the standard models may arise from new particle physics and/or new cosmology, ΔN_ν does not necessarily count additional flavors of neutrinos and, indeed, N_ν may be < 3 or > 3 (i.e., $S < 1$ or $S > 1$). N_ν (or ΔN_ν) is simply a convenient way to parameterize a non-standard, early Universe expansion rate. For example, if the particle content of the standard model of particle physics ($\rho' = \rho$) is adopted,

$$G'/G = 1 + 7\Delta N_\nu/43, \quad (2.3)$$

the BBN or CMB determined values of N_ν can constrain any deviations between the early Universe magnitude of Newton’s constant and the value measured terrestrially today.

3. Primordial Nucleosynthesis

Standard BBN

For standard BBN (SBBN) the relic abundances of the light nuclides produced during

primordial nucleosynthesis depend only on the baryon density parameter η_B . Over a limited range in $\eta_B \equiv 10^{10}\eta_{10} \approx 6 \pm 1$, the SBBN-predicted abundances (Kneller & Steigman (2004), Steigman (2007)) depend on the baryon density parameter as, $(D/H)_P \propto \eta_{10}^{-1.6}$, $({}^3\text{He}/H)_P \propto \eta_{10}^{-0.6}$, and $({}^7\text{Li}/H)_P \propto \eta_{10}^{2.0}$. A good fit to the SBBN-predicted ${}^4\text{He}$ mass fraction is $Y_P = 0.2485 \pm 0.0005 + 0.0016(\eta_{10} - 6)$. The SBBN-predicted relic abundances of D and ${}^7\text{Li}$ are most sensitive to the baryon density parameter, while those of ${}^3\text{He}$ and ${}^4\text{He}$ are less (for the latter, much less) sensitive to η_B .

Non-Standard BBN

For non-standard BBN ($S \neq 1$, $N_\nu \neq 3$), it is the primordial abundance of ${}^4\text{He}$ which provides the most sensitive probe. According to Kneller & Steigman (2004), for $0.85 \lesssim S \lesssim 1.15$ ($1.3 \lesssim N_\nu \lesssim 5.0$) and $5 \lesssim \eta_{10} \lesssim 7$, $(D/H)_P \propto \eta_D^{-1.6}$, where $\eta_D \equiv \eta_{10} - 6(S - 1)$ and $({}^7\text{Li}/H)_P \propto \eta_{\text{Li}}^{2.0}$, where $\eta_{\text{Li}} \equiv \eta_{10} - 3(S - 1)$. In contrast to the relatively weak dependences on S of the D and ${}^7\text{Li}$ abundances, a good fit to the primordial ${}^4\text{He}$ abundance is $Y_P = 0.2482 \pm 0.0006 + 0.0016(\eta_{\text{He}} - 6)$, where $\eta_{\text{He}} \equiv \eta_{10} + 100(S - 1)$. While deuterium (or ${}^7\text{Li}$) probes the baryon density parameter, ${}^4\text{He}$ is sensitive to N_ν .

Deuterium: The Baryometer Of Choice

Of the BBN-synthesized light nuclei, deuterium is the baryometer of choice. One key reason is that the post-BBN evolution of deuterium is simple: as gas is cycled through stars, deuterium is destroyed. As a result, the deuterium abundance measured anywhere in the Universe, at any time during its evolution, provides a **lower limit** to the primordial D abundance. In particular, if the D abundance is measured in high-redshift, low-metallicity systems where post-BBN stellar synthesis has been minimal, the observed abundances should approach the primordial value (“deuterium plateau”: as $Z \rightarrow 0$, $(D/H)_{\text{OBS}} \rightarrow (D/H)_P$). Furthermore, as already noted, the relic D abundance is sensitive to the the baryon density parameter. For SBBN, a $\sim 10\%$ determination of $(D/H)_P$ results in a $\sim 6\%$ constraint on η_{10} . That’s the good news. The bad news is that high precision, high spectral resolution observations of D at high-redshifts and low-metallicities (e.g., in high- z , low- Z QSO Absorption Line Systems (QSOALS; see, e.g., Pettini *et al.* (2008) and earlier reference therein) are difficult, requiring significant observing time on

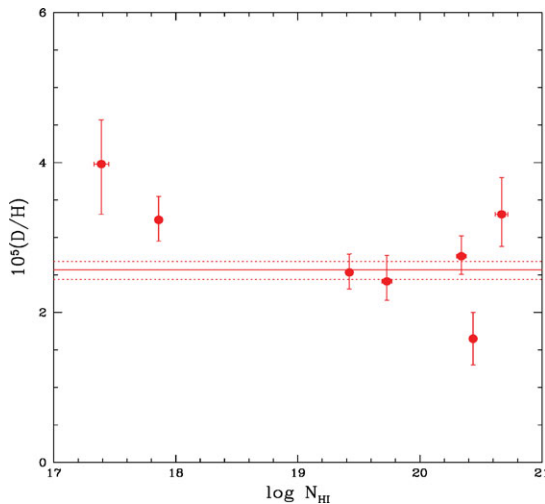


Figure 1. The deuterium abundances, $y_D \equiv 10^5(D/H)$, inferred from observations of high-redshift, low-metallicity QSOALS, as a function of the log of the corresponding H I column densities. The solid line is at the value of the weighted mean of the seven y_D values, $\langle y_D \rangle = 2.7$, and the dotted lines show the estimated uncertainty, $\sigma(y_D) = \pm 0.2$.

large telescopes, equipped with high resolution spectrographs. As a result, as shown in Figure 1, at present there are only seven, relatively reliable D abundance determinations.

The weighted mean of the seven D abundances is $y_{\text{DP}} \equiv 10^5(D/H)_P = 2.7$ (note that the weighted mean of $\log(y_{\text{DP}})$ is 0.45, which corresponds to $y_{\text{DP}} = 2.8$) but, as may be seen from the Figure 1, only three of the seven abundances lie within 1σ of the mean. Indeed, the fit to the weighted mean of these seven data points has a $\chi^2 = 18$ ($\chi^2/dof = 3$). Either the quoted errors in the inferred D abundances are too small or, one or more of the determinations are wrong, perhaps contaminated by unidentified (and, therefore, uncorrected) systematic errors. In the absence of further evidence identifying the reason(s) for such a large dispersion, the best that can be done at present is to adopt the mean D abundance but to inflate the error in the mean in an attempt to account for the unexpectedly large dispersion among the D abundances (Steigman (2007)).

$$y_{\text{DP}} \equiv 10^5(D/H)_P = 2.7 \pm 0.2. \quad (3.1)$$

Note that the Pettini *et al.* (2008) value of $\log(y_{\text{DP}}) = 0.45 \pm 0.03$ corresponds to $y_{\text{DP}} = 2.8 \pm 0.2$, consistent, within the errors, with the weighted mean of the individual y_{D} values. For quantitative comparisons, the value of y_{DP} from eq. (3.1) is adopted here.

For SBBN, this value of the primordial D abundance corresponds to $\eta_{10} = 6.0 \pm 0.3$ or, $\Omega_{\text{B}}h^2 = 0.022 \pm 0.001$ (Steigman (2007)), a 5% determination of the baryon density parameter. For comparison, if the Pettini *et al.* (2008) value were adopted, a slightly lower (but consistent) value, $\eta_{10} = 5.8 \pm 0.3$ or, $\Omega_{\text{B}}h^2 = 0.021 \pm 0.001$, would be found.

Non-BBN Determinations Of The Baryon Density Parameter: CMB And LSS

The baryon density parameter determined by SBBN and the deuterium observations reflects the value of this parameter when the Universe is some ~ 20 minutes old. According to standard model physics and cosmology, the value of this parameter should be unchanged some ~ 400 thousand years later, at recombination (and, at present, some ~ 14 Gyr later). The comparison between the BBN-determined baryon density parameter and that inferred from observations of the CMB (see, e.g., Spergel *et al.* (2007) and further references therein) and of LSS provides a test of the standard models of particle physics and cosmology (Steigman (2007)). According to Simha & Steigman (2008), the combination of the CMB plus LSS data results in $\eta_{10} = 6.1 \pm 0.2$ or, $\Omega_{\text{B}}h^2 = 0.022 \pm 0.001$. More recent results from the WMAP team (Dunkley *et al.* (2009), Komatsu *et al.* (2009)) and others (Sanchez *et al.* (2009)) suggest a slightly higher value of $\eta_{10} = 6.22 \pm 0.17$ or, $\Omega_{\text{B}}h^2 = 0.0227 \pm 0.0006$. Within the uncertainties, $\eta_{\text{B}}(\text{BBN}) \approx \eta_{\text{B}}(\text{CMB/LSS})$; the number of baryons (nucleons) in a comoving volume of the Universe is unchanged between BBN and recombination (as it should be for the standard model).

4. Testing SBBN

Having found agreement between $\eta_{\text{B}}(\text{BBN})$ and $\eta_{\text{B}}(\text{CMB/LSS})$, we still need to test the consistency of SBBN. That is, do the abundances of the other light nuclides (^3He , ^4He , ^7Li) predicted by SBBN, using the D-determined value of η_{B} , agree with their observationally inferred primordial abundances?

Helium-3:

In contrast to deuterium, the post-BBN evolution of ^3He is complex. When gas is cycled through stars, D is burned to ^3He , any prestellar ^3He (prestellar D + ^3He) is burned away in the hot interiors (of all but the least massive stars), but preserved in their cooler, outer layers and, new ^3He is synthesized via stellar nucleosynthesis in the interiors of lower mass stars (e.g., Iben (1967), Rood (1972), Rood, Steigman, & Tinsley (1999)). Competition between destruction, preservation, and synthesis, complicates the

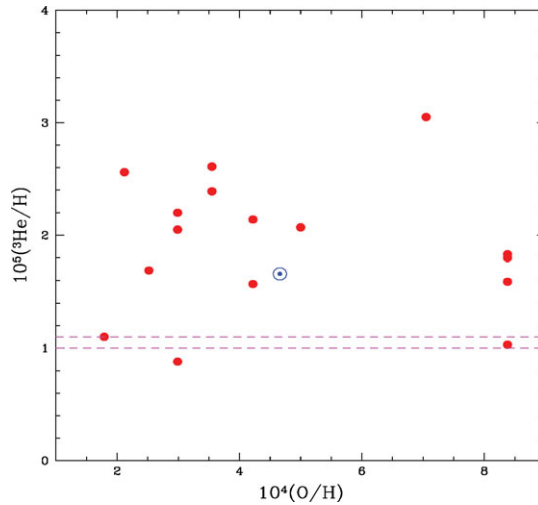


Figure 2. The ^3He abundances, $y_3 \equiv 10^5(^3\text{He}/\text{H})$, from observations of Galactic H II regions (Bania, Rood, & Balser (2002)), as a function of the H II region oxygen abundances. The solar symbol is the pre-solar nebula ^3He abundance (Geiss & Gloeckler (1998)). The dashed lines show the $\pm 1\sigma$ range around the SBBN-predicted primordial abundance, $y_{3P} = 1.05 \pm 0.05$.

process of using the observations to infer the primordial ^3He abundance. The data (see Bania, Rood, & Balser (2002)) don't help. Observations of ^3He are limited to the solar system Geiss & Gloeckler (1998) and to H II regions in the Galaxy (see Steigman (2006a) and Steigman (2007) for further discussion and references). If, in the course of Galactic chemical evolution there is a net increase in the abundance of ^3He from its primordial value, the ^3He abundance and metallicity should be correlated (and, the ^3He abundance in the interstellar medium (ISM) of the Galaxy at present should exceed the presolar nebula abundance). As may be seen in Figure 2, no clear trend with metallicity is revealed and, while many of the observed ^3He abundances do exceed the solar abundance, some don't. All that may be inferred from this data (in this author's opinion) is that the *lowest* ^3He abundances observed are consistent with the SBBN-predicted primordial abundance and, the remaining ^3He abundances suggest net production of ^3He in the course of Galactic chemical evolution. D, ^3He , and the CMB/LSS are in agreement with SBBN.

Lithium-7:

^7Li is a relatively fragile, weakly bound nuclide, easily destroyed at the high temperatures inside most stars. However, as with ^3He , some ^7Li may survive in the cooler, outer layers of stars and, stellar production and cosmic ray nucleosynthesis likely increase the post-BBN abundance of ^7Li . The data reveal that while lithium is depleted in many stars, the overall trend is a lithium abundance which increases with metallicity. As the metallicity approaches zero (primordial), the ^7Li abundances are expected to plateau (the "Spite plateau") at the primordial abundance. In Figure 3 the lithium abundances, $[\text{Li}] \equiv 12 + \log(^7\text{Li}/\text{H})$, are shown as a function of the iron abundances (on a log scale, normalized to the solar iron abundance) for the most metal-poor halo stars (and for the globular cluster NGC 6397). Where is the Spite plateau? The data in Figure 3 (see, also, the contributions to these proceedings by Sbordone *et al.* and Melendez *et al.*) fail to reveal clear evidence for a plateau as $[\text{Fe}/\text{H}] \rightarrow 0$. Even more disturbing is the fact that **none** of the lithium abundances inferred from these observations of the oldest, most metal-poor, most nearly primordial stars in the Galaxy, come even close to the SBBN-predicted abundance. The observed abundances are too low by factors of

$\sim 3 - 5$. This gap seems too wide to be closed by observational uncertainties. Either this conflict between the predictions of SBBN and the observations is pointing to “new” physics and/or cosmology or, our understanding of the structure and evolution of the oldest, most metal-poor stars in the Galaxy is seriously incomplete.

Helium-4:

After hydrogen, helium (${}^4\text{He}$) is the most abundance element in the Universe. In the post-BBN Universe, as gas cycles through stars, the helium abundance (the mass fraction of ${}^4\text{He}$, Y) increases from its primordial value Y_{P} . While the correction for stellar produced ${}^4\text{He}$ is uncertain, its contribution can be minimized by restricting attention to the most metal-poor sites, the low-metallicity, extragalactic H II regions (Blue Compact Galaxies). The current status of the search for Y_{P} is an object lesson in the difference between quantity and quality. With more than ~ 100 helium abundance determinations, statistical uncertainties are very small: $\sigma(Y_{\text{P}})_{\text{stat}} \lesssim 0.001$ (Izotov *et al.* (2007)). However, most analyses fail to deal adequately with the many identified (but often ignored) sources of systematic errors, whose values are estimated to be larger, $\sigma(Y_{\text{P}})_{\text{sys}} \gtrsim 0.006$ (see Steigman (2007) for a discussion of these and other related issues and, for further references). Following Steigman (2007) and Simha & Steigman (2008), here we adopt $Y_{\text{P}} = 0.240 \pm 0.006$ as an estimate of the primordial ${}^4\text{He}$ mass fraction. For SBBN, this corresponds to a very small value of the baryon density parameter, $\eta_{10}({}^4\text{He}) \lesssim 3$ (note that the simple fitting formula for Y_{P} in §3 is not valid for such a low helium abundance and the result here is from the full BBN code). Such a low estimate of the baryon density parameter is clearly in conflict with the value determined by SBBN and D, which is otherwise consistent with the CMB/LSS determined value. How serious is this conflict? That is, given the estimate of the error in the observationally determined value of Y_{P} , how bad is the disagreement? For $\eta_{10}(\text{D}) = 6.0$, the SBBN-predicted primordial helium abundance is $Y_{\text{P}} = 0.249$. The observationally-inferred abundance differs from the predicted abundance by only $\sim 1.5\sigma$, a not very serious disagreement. However, this tension

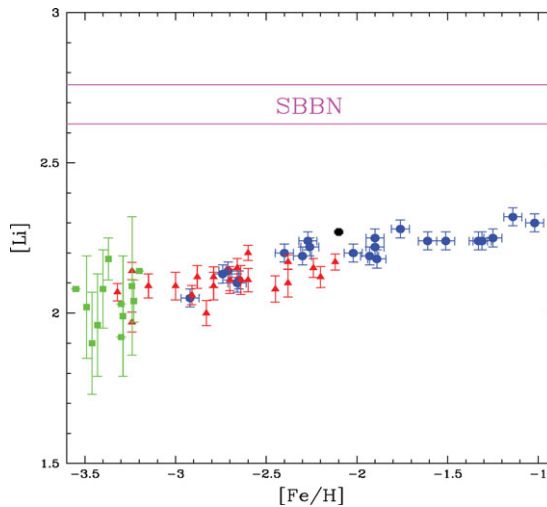


Figure 3. The log of the ${}^7\text{Li}$ abundances, $[\text{Li}] \equiv 12 + \log({}^7\text{Li}/\text{H})$, from observations of old, very metal-poor halo stars, as a function of the stellar iron abundances. Blue filled circles (Asplund *et al.* (2006)), red, filled triangles (Boesgaard *et al.* (2005)), green filled squares (Aoki *et al.* (2009)). The black filled circle (Lind *et al.* (2009)) is for the globular cluster NGC6397. The solid lines shows $\pm 1\sigma$ range around the SBBN-predicted primordial abundance $[\text{Li}]_{\text{SBBN}} = 2.70 \pm 0.06$.

between the predicted and observed helium abundances could be a hint of non-standard physics and/or cosmology.

5. Extension Of the Standard Models: $N_\nu \neq 3$ ($S \neq 1$)

If, indeed, the tension between the observationally-inferred and SBBN-predicted primordial helium abundances is taken seriously, it is of interest to explore non-standard models of particle physics and/or cosmology, with $S \neq 1$ ($N_\nu \neq 3$). For BBN and the D and ^4He abundances adopted here, $\eta_{10} = 5.6 \pm 0.3$ and $N_\nu = 2.4 \pm 0.4$, confirming that the standard model ($N_\nu = 3$) is only $\sim 1.5\sigma$ away. For the non-BBN CMB and LSS data, Simha & Steigman (2008) find $\eta_{10} = 6.14^{+0.16}_{-0.11}$ and $N_\nu = 2.9^{+1.0}_{-0.8}$. As shown in the left hand panel of Figure 4 (from Simha & Steigman (2008)), there is significant overlap between BBN and the CMB/LSS. BBN and the CMB agree and, at $\gtrsim 68\%$ confidence, they are consistent with $N_\nu = 3$. In the right hand panel of Figure 4 the likelihood $N_\nu - \eta_{10}$ contours are shown for the combined BBN/CMB/LSS data ($N_\nu = 2.5 \pm 0.4$, $\eta_{10} = 6.1 \pm 0.1$).

For the non-BBN constraints on the baryon density and expansion rate parameters, the BBN-inferred primordial abundances of D and ^4He are $y_{\text{DP}} = 2.5 \pm 0.3$ and $Y_{\text{P}} = 0.247^{+0.013}_{-0.011}$, in good agreement, within the errors, with the adopted relic abundances. However, it must be noted that for the non-BBN identified values of η_{10} and N_ν , the BBN-predicted lithium abundance, $[\text{Li}]_{\text{P}} = 2.72^{+0.05}_{-0.06}$, remains in serious conflict with the observationally inferred value. The lithium (^7Li) problem persists.

6. Conclusions

The very good agreement between the values of the baryon density and universal expansion rate parameters determined by BBN, when the Universe was ~ 20 minutes old, and by the CMB/LSS, some ~ 400 thousand to 14 billion years later, leads to constraints on some extensions of the standard models of particle physics and cosmology.

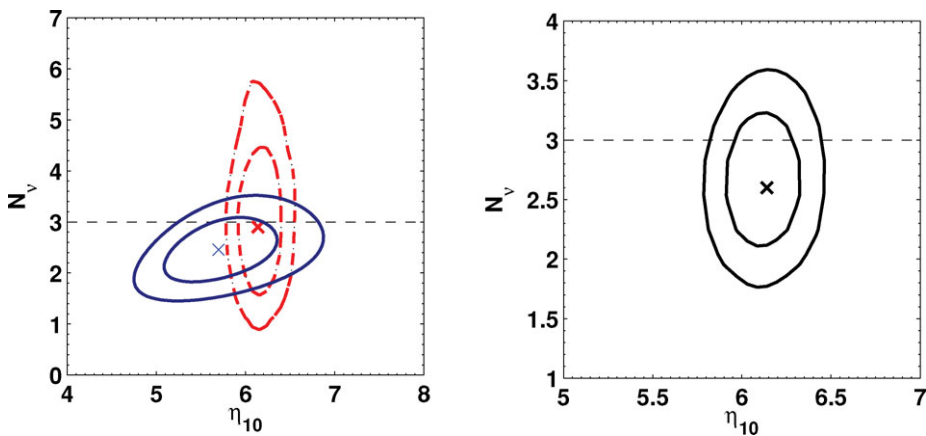


Figure 4. Left panel: The 68% and 95% contours in the N_ν vs. η_{10} plane (from Simha & Steigman (2008)) for BBN (D & ^4He) (solid) and for the CMB/LSS (dashed). The crosses indicate the best fit values (see the text). Right panel: The joint BBN/CMB/LSS contours; note the expanded scales for N_ν and η_{10} .

Entropy Conservation?

For example, the numbers of baryons and CMB photons in a comoving volume are related by the baryon density parameter, $N_B = \eta_B N_\gamma$. In the standard model of particle physics, N_B is unchanged from BBN to recombination and the present. Comparing η_B at BBN and at recombination leads to the constraint: $N_\gamma(\text{CMB})/N_\gamma(\text{BBN}) = 0.92 \pm 0.07$, limiting any post-BBN entropy production.

“Extra”, Post-BBN Radiation Density?

Since $\rho'_R/\rho_R = 1 + 7\Delta N_\nu/43$, the BBN and CMB constraints on N_ν limit the radiation energy densities at these widely separated epochs. In the absence of the creation of “new” radiation (e.g., by the late decay of a massive particle), $N_\nu(\text{BBN}) = N_\nu(\text{CMB})$. Comparing N_ν at BBN and at recombination constrains any possible difference between these values. This comparison reveals that $0.94 \leq N_\nu \leq 1.23$, consistent with **no** extra, post-BBN radiation density.

Variation Of the Gravitational Constant?

The BBN and CMB constraints on N_ν also limit any difference between the magnitude of the gravitational constant at BBN or at recombination and that observed today terrestrially since $G'/G = 1 + 7\Delta N_\nu/43$. From N_ν at BBN, $G_{\text{BBN}}/G_0 = 0.91 \pm 0.07$, while the CMB/LSS bound on N_ν leads to, $G_{\text{CMB}}/G_0 = 0.99 \pm 0.12$.

7. Summary

For $N_\nu \approx 3$, BBN agrees with the observations of the CMB (and LSS and H_0), confirming the consistency of the standard models of particle physics and cosmology. But, lithium remains a problem whose origin may lie with stellar depletion/dilution or with new particle physics and/or cosmology. When BBN is combined with the CMB and LSS, interesting constraints on some non-standard models of particle physics and cosmology can be obtained.

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