

CORRESPONDENCE.

ON MR. BAILEY'S ESTIMATE OF THE LIABILITIES OF CERTAIN
LIFE ASSURANCE COMPANIES.

To the Editor of the Assurance Magazine.

SIR,—It is not usual to discuss, in the *Assurance Magazine*, the accounts of individual Companies, or the merits of the various methods of division of profits employed by them; and it would be matter of regret if such discussions were generally allowed to appear in your pages. But when there is reason to think that an injustice has been done to a particular Company by one of your correspondents, I doubt not that you will allow your usual salutary rule to be relaxed so far as may be necessary to correct that injustice.

I refer to Mr. Bailey's table, which appeared in the *Assurance Magazine* for July, 1863 (vol. xi., pp. 111, 112), and purports to show the "estimated liability" of various Offices. I may say for myself that I entertain a very low opinion of the value of such general comparisons as are afforded by the table in question. Mr. Bailey has pointed out one disturbing element in the numerous recent amalgamations which have occurred; but this is far from being the only cause which interferes with the fairness of the comparison, or the most important one. It is obvious that if, of two Offices of the same standing, one has done a new business generally increasing in amount from year to year, while the new business of the other has been rather falling off, then the liability of the latter must be greater in proportion to the sum assured than is the case with the former. There is at least one marked instance of the effect of this cause apparent in Mr. Bailey's table (p. 112).

In order that such tables may have any value at all, it is of course essential that the figures given in them should be accurate, and that they should be deduced from the published statements of the Offices by a process which admits of no dispute. Now, there are two Offices, which I will call (A) and (B), included in Mr. Bailey's list, which appear to make a much smaller reserve for their liabilities than most other Offices of similar standing; but this is not really the case, as I will presently show. In fact, it appears to me that the accounts of these two Offices have been treated erroneously, and that the correct sums to be tabulated as the values of the estimated liability are much larger than those given in the table. In the Office A, which had been established for 54* years on 30th June, 1862, the sum assured at that date was £6,526,853; and the "estimated liability" is given by Mr. Bailey as only £775,002. Relying on the accuracy of these figures, I have on various occasions expressed myself, in the course of conversation, to the effect that "the Office in question makes a very small reserve for its liabilities;" and it cannot be disputed that this is the obvious conclusion to be drawn from the table. Having made the above statement to a friend who is insured in it, he challenged me to prove my assertion, and placed in my hands the printed accounts of the Association for a series of years; and I have now to acknowledge that an examination of these accounts has led me to a conclusion very different from Mr. Bailey's.

* More accurately 55½.—ED. A. M.

In order to ascertain the reserve actually made by the Office (A) for its liabilities, it will be convenient to give a summary of the balance sheet of the Association on 30th June, 1862. This is as follows:—

Liabilities.

Value of £4,758,993 assured on the lives of members—1st series	£	2,716,457
Value of £1,523,570 assured on the lives of members—2nd series		596,445
Value of £244,290 assured on the lives of persons not members		110,177
Sundry liabilities		102,444
		<u>£3,525,523</u>

Assets.

Sundry assets	2,830,539
Value of £5,612 premiums on non-members' policies	59,349
Value of such part of the annual premiums, amounting to £56,193, on the lives of members (2nd series), as they will be required to pay in full	171,355
Value of the future reduced premiums on the lives of members:—	
1st series, at $83\frac{1}{2}$ per cent. reduction	290,934
2nd „ $73\frac{1}{2}$ „ „	173,346
	<u>£3,525,523</u>

The first thing to be observed in this account is, that while the sum assured is, as stated by Mr. Bailey, £6,526,853, the realized assets, after allowing for all immediate liabilities, amount to £2,728,095, or no less than 41·8 per cent. of the sum assured. How, then, is Mr. Bailey's result obtained? It will be found that the amount tabulated by him (£775,002) is the difference between the value of the sums assured (£3,423,079) and the value of the future gross premiums (£2,648,077), the latter amount being deduced from the figures in the balance sheet by a few simple calculations which it is not necessary to describe. Or Mr. Bailey's result may be obtained in the following way:—The value of the future reduction of premium—at $83\frac{1}{2}$ per cent. on the 1st series and $73\frac{1}{2}$ per cent. on the second series—is found to be £1,953,093; and the difference between this amount and the net assets (£2,728,095) is £775,002—the amount tabulated by Mr. Bailey. The relation between these figures will be more clearly seen from the following statement:—

Liabilities.

Value of sums assured	£3,423,079
„ reduction of premium	1,953,093
	<u>£5,376,172</u>

Assets.

Value of gross premiums	£2,648,077
Net realized assets	2,728,095
	<hr/>
	£5,376,172

It thus appears that Mr. Bailey has deducted from the assurance fund of the Association the value of the future reduction of premium; and I presume he would justify this course by arguing, that although the reduction of premium is variable and uncertain, being declared from year to year, yet the hope is held out by the Association that the present reduction may be permanently maintained. Assuming for the present that the reduction is to be considered permanent in the case of (A) and (B), then I have to observe that the accounts of these Offices are treated in Mr. Bailey's table in a very different manner to those of the other Offices mentioned. In all the other instances the values of the bonuses declared, and of any permanent reductions of premium, are included in the "estimated liability"; and in one instance—the Equitable—it is pointed out that the magnitude of the bonuses has a great influence in raising the amount of the estimated liability. Consistency, therefore, certainly requires that if the reduction of premium in (A) is considered as a permanent thing, its value ought to be included in the estimated liability, which is therefore raised to £2,728,095; amounting, as already stated, to 41·8 per cent. on the sum assured.

But there can be no doubt that the above is not the correct way of regarding the reductions of premium declared by the two Offices in question. Those reductions are fixed every year in conformity with the results of the valuation then made; and are subject to increase or diminution, from year to year, according as the experience of the Office in the past year has been favourable, or the contrary. It is therefore more correct, as well as more convenient for the purpose of comparison with other Offices, to say that an annual cash bonus is declared, equal to the value of the abatement of premium allowed for the ensuing year. Thus then the directors of (A), in submitting to the members the balance sheet of which a summary has been given above, are to be regarded as saying—"We declare an abatement of premium for the year ending 30th June, 1863, of 83½ per cent. for the members of the 1st series, and 73½ per cent. for such members of the second series as are entitled to an abatement; and these abatements are calculated on such a scale that they may reasonably be expected to be maintained from year to year so long as the experience of the Society continues similar to its present experience." An examination of the balance sheet leads to the conclusion that the abatements may be maintained on the present scale, so long as the expenses of management are defrayed out of the excess of interest realized over the rate at which the valuations are made (which is currently reported to be 4 per cent.); and the incidental profits arising from surrenders, from the premiums upon non-members' policies, &c.;—provided the mortality experienced does not exceed that calculated upon in the table of mortality used in the valuations.

In order to reconstruct the balance sheet in a proper form for comparison with the accounts of other Societies, it is necessary to ascertain approximately the amount of the abatement of premium for the year

ending 30th June, 1863. It is stated in the printed account of the Association from which the above figures are taken, that the gross annual premiums on all existing policies amount to £228,800; that the premiums on non-members' policies are £5,612, and the premiums on the policies of members of the 2nd series are £56,193. It follows, therefore, that the annual premiums on the policies of members of the 1st series are £166,995, and the abatement at $83\frac{1}{2}$ per cent. amounts to £139,441. In the particular year under consideration it will be found that all the members of the 1st series were entitled to abatement, having paid seven premiums, but none of the second series were yet entitled to an abatement. Now, arranging the balance sheet in the form usually adopted when the value of the gross premiums is stated, it will be as follows:—

<i>Liabilities.</i>	
Value of sums assured	£3,423,079
Reserve for expenses and future bonuses	1,813,652
Balance, being divisible surplus	139,441
	£5,376,172
<i>Assets.</i>	
Investments	£2,728,095
Value of future premiums	2,648,077
	£5,376,172

It will be noticed, however, that in strict accuracy the surplus should be the value of the year's abatement of premium, instead of the amount of that abatement. It thus appears that when the accounts of this Office are treated in the same way as those of the other Societies considered by Mr. Bailey, it is found that, on 30th June, 1862, no less than $68\frac{1}{2}$ per cent. of the value of the future gross premiums was reserved for expenses and future bonuses, and the "estimated liability" was (£2,728,095—£139,441), or £2,588,654, instead of £775,002, as given by Mr. Bailey; or the estimated liability is 39·7 per cent. on the sum assured, instead of 11·9.

Similar remarks apply to the Office (B), which had been 28 years in existence on 4th April, 1863, and which is represented as making a reserve equal to 9·4 per cent. of the sum assured. In this case, the sum assured is £3,375,224, and the available assets, after making provision for immediate liabilities, are £964,275, or 28·6 of the sum assured. I gather from the published accounts of this Office, that the amount of the abatement allowed to members in the year 1863—4 was £43,226; and deducting this sum from the available assets, as above, it appears that the "reserve" of the Office is £921,049, or 27·3 on the sum assured, instead of 9·4.

There can thus, I think, be no doubt that the reserves made by (A) and (B) for their liabilities, are not unduly small as compared with the reserves of other Offices; but whether the method employed to ascertain the abatement of premium for each year, is the most suitable, is a totally distinct question, upon which it would not be proper to enter here.

That the method of treating the accounts adopted in the table is erroneous will perhaps be rendered more clear to some persons by the consideration of

the case of an Office which should make an annual valuation and declare a reversionary bonus thereupon each year. Such an Office might declare, for a series of years, a reversionary bonus at the rate of $1\frac{1}{2}$ per cent. per annum on the sum assured, and the expectation might be held out to the assured that this rate of bonus would be maintained. In this case it would be manifestly improper to deduct from the assurance fund of the Society the value of the future reversionary bonus at the above rate, and style the balance the "estimated liability" of the Office. But this is precisely analogous to what has been done with the Offices (A) and (B).

I have already trespassed much on your space, but as I believe that a general interest is felt in our profession on these points, I will proceed to the consideration of another method by which the sufficiency and the magnitude of the reserve made by the Offices in question may be tested—viz., the valuation of individual policies. Take the case of a policy for £100 effected in the Office (A) at the age 40, at the annual premium £3. 17s., which has been in force m years ($m < 7$); then the reserve made for this policy by the method of valuation pursued, is

$$100A_{40+m} - 3.85 \times \left(1 + \frac{1}{6-m} a_{40+m}\right) - 1.02a_{40+m|6-m}$$

$$= 100A_{40+m} - 3.85(1 + a_{40+m}) + 2.83a_{40+m|6-m}$$

i.e., $V_{40|m} = 100A_{40+m} - 3.85(1 + a_{40+m}) + 2.83 \frac{N_{46}}{D_{40+m}} \dots (1)$

For the premium £3.85 is payable in advance for $7 - m$ years, and afterwards the premium is supposed to be reduced 73.5 per cent., or to become £1.02.

Again, for a similar policy on the life of a member of the first series, which has been in force 7 years or more (say $7 + n$ years), and upon which, therefore, there is a reduction of 83.5 per cent. on the premiums, the reduction being £3.21475, and the reduced premium £.63524, the value of the policy is

$$V_{40|7+n} = 100A_{47+n} - .63525(1 + a_{47+n}) \dots (2)$$

By means of the formulæ (1) and (2), the values in columns (2) and (5) of the following table have been calculated (at Carlisle 4 per cent.).

Table of Values of Policies.

POLICIES ON THE LIVES OF MEMBERS OF 2ND SERIES.			POLICIES ON THE LIVES OF MEMBERS OF 1ST SERIES.		
	Formula (1).	Carlisle 3 per Cent.		Formula (2).	Carlisle 3 per Cent.
$V_{40 1}$	7.790	1.394	$V_{40 7}$	27.863	10.191
$V_{40 2}$	10.882	2.773	$V_{40 8}$	28.955	11.889
$V_{40 3}$	14.089	4.155	$V_{40 9}$	30.143	13.718
$V_{40 4}$	17.451	5.585	$V_{40 10}$	31.418	15.653
$V_{40 5}$	20.983	7.056	$V_{40 11}$	32.776	17.698
$V_{40 6}$	24.713	8.589	$V_{40 12}$	34.155	19.760
			$V_{40 20}$	52.243	36.664
			$V_{40 30}$	65.452	55.228
			$V_{40 40}$	76.772	70.429

From a comparison of these values, two important conclusions may be drawn. Firstly, it should seem that the reserve made by the Office is greatly in excess, in the instances here given, of the values of the policies as given by the Carlisle 3 per cent. table; and it will be found, I believe, throughout the whole of life, that the reserve made by the valuation of the Office is in excess of that required by the Carlisle 3 per cent. valuation; from which it results that if a valuation of the Society were made by the Carlisle 3 per cent. table, there would be a much larger cash bonus divided than is now allowed in reduction of premium. Secondly, it will be noticed that the reserve made for recent policies is for several years greater than the amount of the premiums received, so that in fact every new policy issued causes loss on the subsequent valuations—reduces the divisible surplus—and makes the abatement of the premium less than it would otherwise be.

This last observation opens up a wide and tempting field of investigation, but one which cannot be considered suitable for these pages. I therefore abstain from proceeding any further in that direction.

It will, of course, be understood that the values in the preceding table are not to be taken as the actual amounts reserved by the valuation of the Company. I believe that valuation is not conducted by the Carlisle table; and without being in possession of the table of mortality by which the valuations are conducted, it is impossible to assign the actual values of the policies. If the table in use is one which gives throughout a greater expectation of life than the Carlisle, then the values of the policies will be less than those given above; but it cannot be supposed that any table of mortality whatever would give such results as to vitiate the conclusions I have ventured to draw from a comparison of the values in the above table.

In conclusion, I should wish to add, that in writing these remarks I have not been in any way actuated by a desire to recommend the system pursued by the two Offices I have alluded to. I do not feel at liberty to express in these pages any opinion as to the merits of that system; and in all that I have said I have been careful to abstain from any expression of opinion, and to confine myself strictly to the discussion of questions of fact.

I am, Sir,

Your obedient servant,

Equity and Law Life Assurance Society,
18, *Lincoln's Inn Fields,*
August, 1864.

T. B. SPRAGUE.

ON MR. HODGE'S REMARKS UPON THREE-LIFE SURVIVORSHIPS.

To the Editor of the Assurance Magazine.

SIR,—I must beg the favour of a small space in your columns for a word or two in reference to Mr. Hodge's comments, at the last meeting of the Institute, upon Mr. Gray's account of my "Solutions of survivorship problems."

Mr. Hodge informed us that it was at one time his practice to calculate